



Partonic quasi-distributions

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IFUJ, 27 November 2017

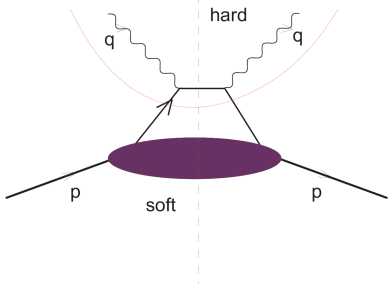
Research with **Enrique Ruiz Arriola**
details in PLB 773(2017)385 and in arXiv:1711.03377

Outline

- Partonic **quasi-distributions** (QDFs) of J_i , designed to extract PDFs from Euclidean QCD lattices
(cf. past seminars by P. Korcyl and K. Cichy)
- Lorentz invariance and **Radyushkin's relation** of QDFs to TMDs
- **Sum rules** for moments of distributions
- Longitudinal-transverse **factorization** and its **breaking** by evolution
- **Kwieciński's** adaptation of CCFM
- Phenomenological results for QDFs from TMDs
- Comparison to lattice results for the proton from **ETMC** [Alexandrou, Cichy et al. 2015-17]
- Predictions for QDFs and QDAs of the **pion** and for the **gluonic** components

Introduction

Parton distribution



$$Q^2 = -q^2, \quad x = \frac{Q^2}{2p \cdot q}, \quad Q^2 \rightarrow \infty$$

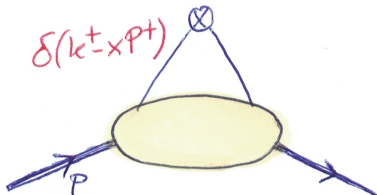
Factorization of soft and hard processes,
Wilson's OPE

$$\langle J(q)J(-q) \rangle = \sum_i C_i(Q^2; \mu) \langle \mathcal{O}_i(\mu) \rangle$$

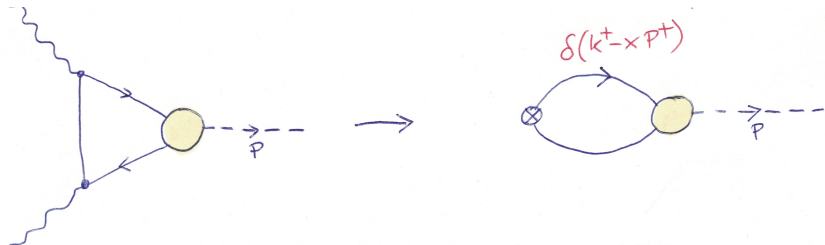
Twist expansion $\rightarrow F(x, Q) = F_0(x, \alpha(Q)) + \frac{F_2(x, \alpha(Q))}{Q^2} + \dots$

Bjorken limit \rightarrow light-cone
momentum is constrained:

$$k^+ \equiv k^0 + k^3 = xP^+ \quad x \in [0, 1]$$



Distribution amplitude (DA) of the pion



Enters various measures of exclusive processes,
e.g., pion-photon transition form factor

Field-theoretic definitions

Parton Distribution Function (PDF), leading twist, nucleon or pion:

$$q(x) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle P | \bar{\psi}(0) \gamma^+ U[0, z] \psi(z) | P \rangle \Big|_{z^+=0, z_T=0}$$

Parton Distribution Amplitude (PDA), leading twist, pion:

$$\phi(x) = \frac{i}{F_\pi} \int \frac{dz^-}{2\pi} e^{i(x-1)P^+z^-} \langle P | \bar{\psi}(0) \gamma^+ \gamma_5 U[0, z] \psi(z) | \text{vac} \rangle \Big|_{z^+=0, z_T=0}$$

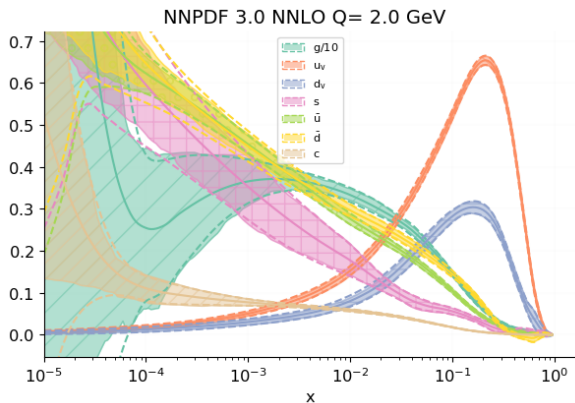
(spin-averaged, isospin suppressed)

P - hadron's momentum, $v^\pm \equiv v^0 \pm v^3$ - light-cone basis

$U[z_1, z_2] = \exp\left(-ig_s \int_{z_1}^{z_2} d\xi \lambda^a A_a^+(\xi)\right)$ - Wilson's gauge link

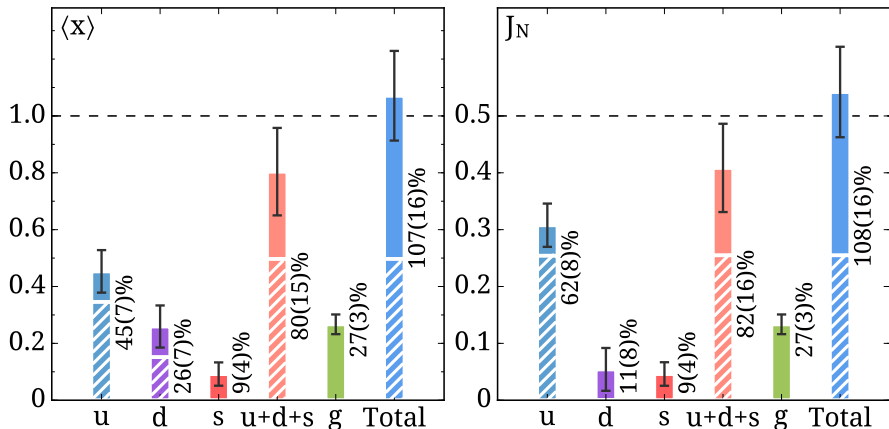
x - fraction of P^+ carried by the quark, $x \in [0, 1]$ (or $x \in [-1, 1]$)

Phenomenology



Moments from the lattice

Decomposition of the momentum and spin in the proton (scale 2 GeV)



[Alexandrou et al 2017]

Quasi-distributions of J_i

PDF and QDF

Parton Distribution Function (PDF):

$$q(x) = \int \frac{dz_-}{4\pi} e^{ixP_+z_-} \langle P | \bar{\psi}(0) \gamma^+ U[0, z] \psi(z) | P \rangle \Big|_{z_+ = 0, z_T = 0}$$

(impossible to put on Euclidean lattice, as $t^2 - z^2 = 0$ becomes one point,
 $t_E^2 + z^2 = 0$)

Quasi-Distribution Function (QDF): [Ji 2013]

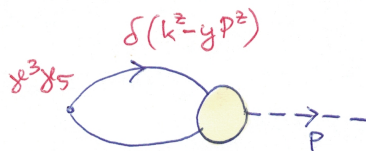
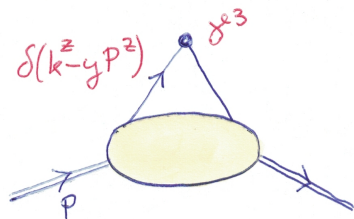
$$\tilde{q}(y; P_3) = \int \frac{dz_3}{4\pi} e^{-iyP_3z_3} \langle P | \bar{\psi}(0) \gamma^3 U[0, z] \psi(z) | P \rangle \Big|_{z_0 = 0, z_T = 0}$$

y - fraction of pion's P_3 carried by the quark, y is not constrained

Limit:

$$\lim_{P_3 \rightarrow \infty} \tilde{q}(x; P_3) = q(x)$$

QDF and QDA in the momentum representation



Constrained longitudinal momenta, but $y \in (-\infty, \infty)$
(partons can move “backwards”)

Covariant formulation

Lorentz covariance

$$\langle P | \bar{\psi}(0) \gamma^\mu U[0, z] \psi(z) | P \rangle = P^\mu h(P \cdot z, z^2) + z^\mu h_z(P \cdot z, z^2)$$

($\nu = -P \cdot z$ is referred to as the loffe time)

$$\tilde{q}(y, P_3) = P_3 \int \frac{dz_3}{2\pi} e^{-iyP_3 z_3} h(-P_3 z_3, -z_3^2)$$

$$q(x) = P_+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} h(P_+ z_-, 0)$$

Transverse momentum Distribution (TMD):

$$\hat{q}(x, \mathbf{z}_T) = P^+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} h(P_+ z_-, -\mathbf{z}_T^2)$$

or:

$$q(x, \mathbf{k}_T) = P^+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} \int \frac{dz_T^2}{(2\pi)^2} e^{i\mathbf{k}_T \cdot \mathbf{z}_T} h(P_+ z_-, -\mathbf{z}_T^2)$$

Radyushkin's QDF-TMD relation – pedestrian derivation

[Radyushkin 2017]

Choose $k_2 = (x - y)P_3$ in the definition of TMD:

$$\begin{aligned} & \int dk_1 \int dx q(x, k_1, (y - x)P_3) \\ &= \int dk_1 \int dx P^+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} \int \frac{dz_1 dz_2}{(2\pi)^2} e^{ik_1 z_1 + i(x-y)P_3 z_2} h(P_+ z_-, -z_1^2 - z_2^2) \\ &= P^+ \int dz_- \delta(P_+ z_- + P_3 z_2) \int dz_1 \delta(z_1) \int \frac{dz_2}{2\pi} e^{-iyP_3 z_2} h(P_+ z_-, -z_1^2 - z_2^2) \\ &= \int \frac{dz_2}{2\pi} e^{-iyP_3 z_2} h(-P_3 z_2, -z_2^2) \quad [z_2 \rightarrow z_3] \text{ (rot. symmetry)} \\ &\equiv \frac{1}{P_3} \tilde{q}(y, P_3) \end{aligned}$$

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(straight line gauge link assumed instead of a infinite-staple shape [Collins])

QDF-TMD relations (cont.)

$$\tilde{q}(y, P_3) = P_3 \int dk_1 \int dx q(x, k_1, (y-x)P_3)$$

Equivalent form:

$$\tilde{q}(y, P_3) = P_3 \int dx \int \frac{dz_2}{2\pi} e^{-i(y-x)z_2 P_3} \hat{q}(x, 0, z_2)$$

Inverse relation:

$$\hat{q}(x, 0, z_2) = z_2 \int dy \int dP_3 e^{i(y-x)z_2 P_3} \tilde{q}(y, P_3)$$

Theoretical and practical significance

lattice QDFs \leftrightarrow vast knowledge of TMDs (also from lattice [Musch et al. 2011])

Transverse-longitudinal factorization

Factorization ansatz

$$q(x, k_T) = q(x)F(k_T), \quad \hat{q}(x, z_T) = q(x)\hat{F}(z_T)$$

$$\tilde{q}(y, P_3) = P_3 \int dx F[(x - y)P_3] q(x)$$

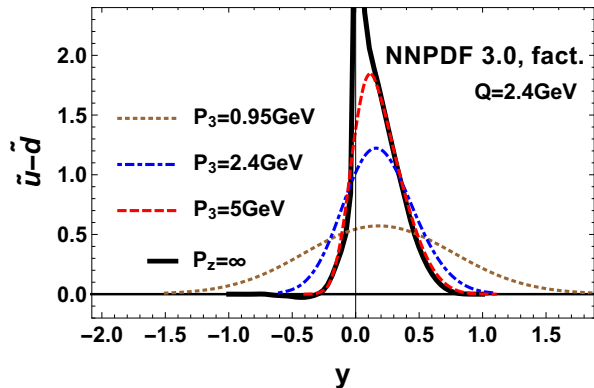
Gaussian:

$$F(k_T) = \frac{e^{-\frac{k_T^2}{\langle k_T^2 \rangle}}}{\pi \langle k_T^2 \rangle}, \quad \hat{F}(z_T) = e^{-\frac{z_T^2 k_T^2}{4}}$$

$$\tilde{q}(y, P_3) = \frac{1}{\sqrt{2\pi}\Sigma} \int dx e^{-\frac{(x-y)^2}{2\Sigma^2}} q(x), \quad \Sigma^2 = \frac{\langle k_T^2 \rangle}{2P_3^2}$$

Factorization (at $m_\pi = 600$ MeV) seen in the lattice TMD studies [Musch et al. 2011] and the quenched QDF studies [Orginos et al. 2017]

Result of folding: washed out QDF from PDF



solid – PDF limit, dashed – QDF

To achieve a few-percent agreement with PDF for $x > 0.15$ one needs $P_3 > 5\text{ GeV}$

Results at finite P_3 are interesting on their own!

Sum rules

Sum rules for Ioffe-Time Distributions (ITDs)

Primary object from the lattice:

[WB, ERA 2017]

$$h(-P_3 z_3, -z_3^2) = \int_{-1}^1 dx e^{iP_3 z_3 x} \hat{q}(x, -z_3^2) = \int_{-\infty}^{\infty} dy e^{iP_3 z_3 y} \tilde{q}(y, P_3)$$

Use $\nu = P_3 z_3$ - the **Ioffe time** (strictly speaking, $\tau_I = \nu/M$)

Differentiation wrt. ν at the origin \rightarrow

Slope:

$$\left. \frac{d}{d\nu} h\left(-\nu, -\frac{\nu^2}{P_3^2}\right) \right|_{\nu=0} = i\langle x \rangle_q = i\langle y \rangle_q(P_3)$$

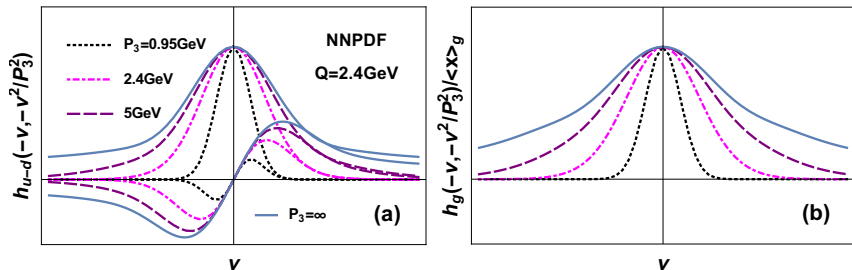
Curvature:

$$\left. \frac{d^2}{d\nu^2} h\left(-\nu, -\frac{\nu^2}{P_3^2}\right) \right|_{\nu=0} = -\langle x^2 \rangle_q - \frac{1}{P_3^2} \int_{-1}^1 dx \langle k_T^2(x) \rangle q(x) = -\langle y^2 \rangle_q(P_3)$$

(x - and k_T -moments enter)

Similarly for higher moments, gluon distributions

Sum rules from ITDs - example



real (symmetric) and imaginary (antisymmetric) parts
solid lines: $h(-\nu, 0)$

Sum rules from reduced ITDs

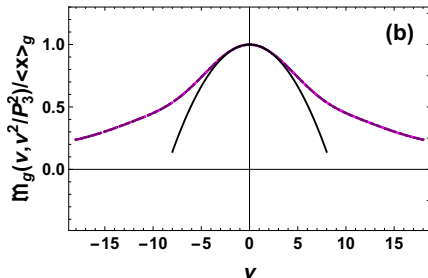
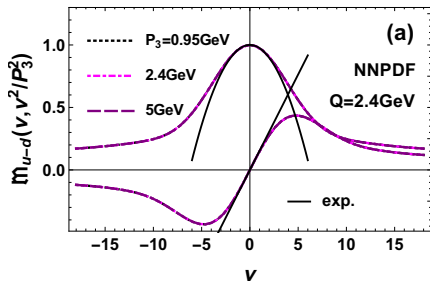
Reduced ITDs [Munsch et al. 2011, Orginos et al. 2017]

$$\begin{aligned}\mathfrak{M}(\nu, z_3^2) &= \frac{h(-\nu, -z_3^2)}{h(0, -z_3^2)} = \frac{\int dx e^{i\nu x} \hat{q}(x, -\frac{\nu^2}{P_3^2})}{\int dx \hat{q}(x, -\frac{\nu^2}{P_3^2})} = \\ &= (\text{factorization}) = \frac{\hat{F}(\frac{\nu}{P_3}) \int dx e^{i\nu x} q(x)}{\hat{F}(\frac{\nu}{P_3}) \int dx q(x)} = \int dx e^{i\nu x} q(x) = h(-\nu, 0)\end{aligned}$$

(in the factorization model it is independent of P_3 !)

$$\begin{aligned}\left. \frac{d}{d\nu} \mathfrak{M}(\nu, \nu^2/P_3^2) \right|_{\nu=0} &= i\langle x \rangle_q = i\langle y \rangle_q(P_3), \\ \left. \frac{d^2}{d\nu^2} \mathfrak{M}(\nu, \nu^2/P_3^2) \right|_{\nu=0} &= -\langle x^2 \rangle_q = -\langle y^2 \rangle_q(P_3) + \frac{1}{P_3^2} \int_{-1}^1 dx \langle k_T^2(x) \rangle q(x)\end{aligned}$$

Sum rules from reduced ITDs - example



Lowest moment approximate well at low $|\nu|$

Long ν tail results from the low- x singularity in PDF: $\sim x^{-\alpha} \rightarrow \sim \nu^{-1+\alpha}$

Lattice: $\nu < LP_3$, $P_3 = \frac{2\pi n}{L}$, $x > \frac{2\pi}{LP_3} = \frac{1}{n} \sim 0.1 - 0.2$

L – lattice size

QCD evolution

QCD evolution for QDF from TMD

One needs to specify the scale:

$$\tilde{q}(y, P_3; Q) = P_3 \int dx \int \frac{dz_2}{2\pi} e^{-i(y-x)z_2 P_3} \hat{q}(x, 0, z_2; Q)$$

Kwieciński's one-loop CCFM, structure like the DGLAP equations for the integrated PDFs, but with a modified kernel, diagonal in z_T :

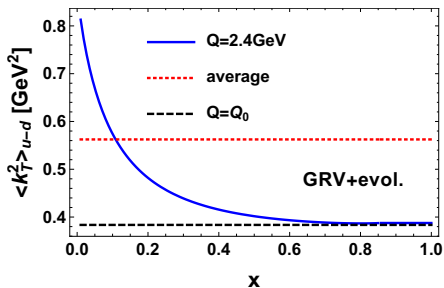
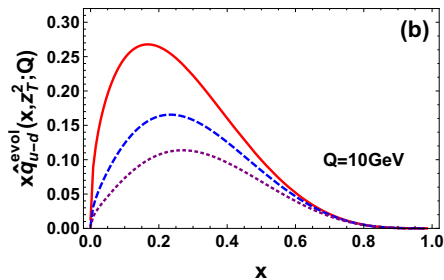
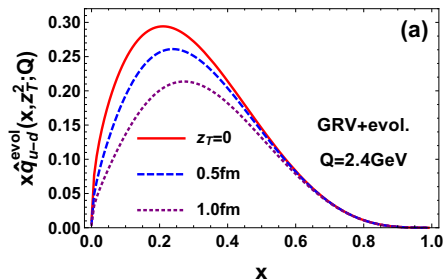
$$Q^2 \frac{\partial \hat{q}(x, z_T; Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 d\xi P_{qq}(\xi) \left[\Theta(\xi - x) \right. \\ \left. \times J_0[(1 - \xi)Qz_T] \hat{q}\left(\frac{x}{\xi}, z_T; Q\right) - \hat{q}(x, z_T; Q) \right]$$

The initial condition at the scale Q_0 in a factorized form

$$\hat{q}(x, z_T; Q_0) = \hat{F}(z_T^2) q(x)$$

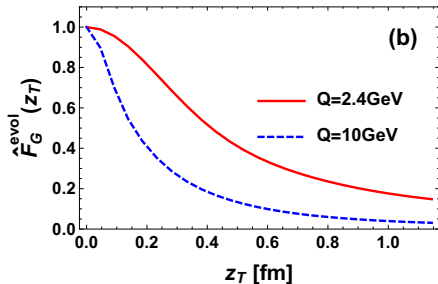
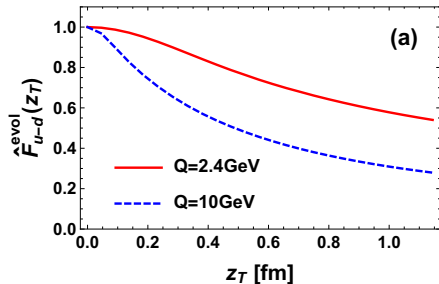
$$\hat{q}(x, z_T; Q) = \hat{F}(z_T^2) \hat{q}^{\text{evol}}(x, z_T; Q)$$

k_T -spreading



Evolution-generated form factor

$$\hat{F}^{\text{evol}}(z_T; Q) = \int dx \hat{q}^{\text{evol}}(x, z_T; Q)$$

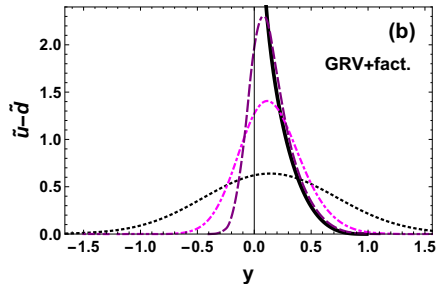
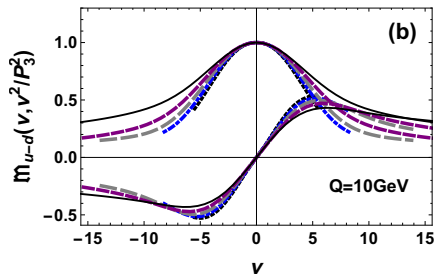
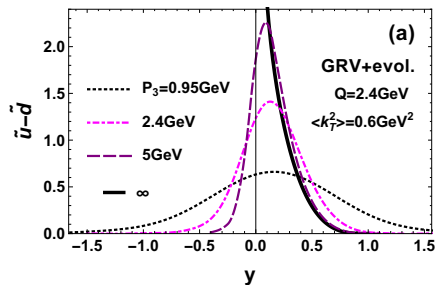
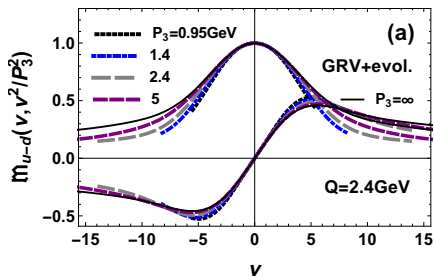


asymptotically quarks $\sim z_T^{-4 \frac{C_F}{\beta_0} \log \frac{\alpha_s(Q_0)}{\alpha_s(Q)}}$, gluons $\sim z_T^{-4 \frac{N_c}{\beta_0} \log \frac{\alpha_s(Q_0)}{\alpha_s(Q)}}$
 ($C_F = 4/3$, $N_c = 3$, $\beta_0 = 9$)

[WB, ERA 2004]

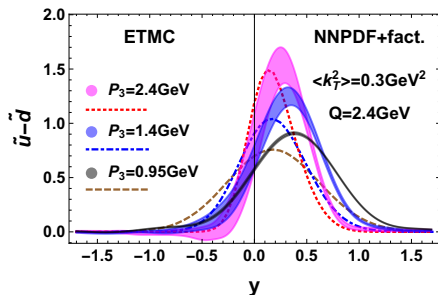
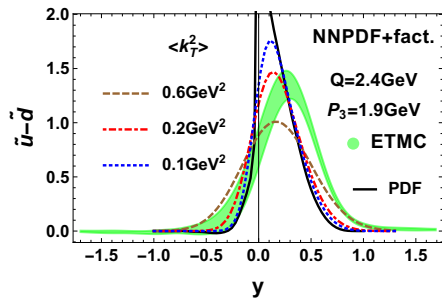
... back to quasi \rightarrow

Factorization breaking from QCD evolution



Comparison to lattice data

Comparison to ETMC lattice



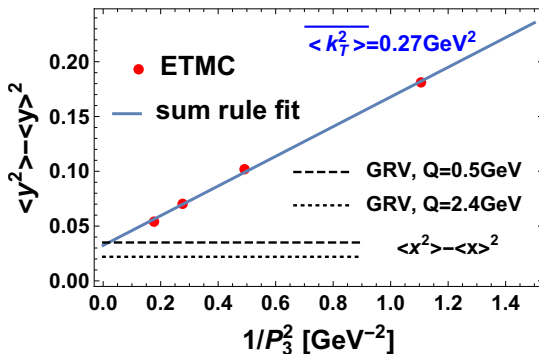
[Alexandrou, Cichy et al. 2015-2017] use a correlator retaining the sub-leading structure $\sim z^\mu$, mixing with a twist-3 scalar, $m_\pi = 370$ MeV + typical lattice problems: finite lattice spacing, volume effects ...

[Orginos et al. 2017] extraction from the (quenched, $m_\pi = 600$ MeV) lattice is also visibly to the right of the phenomenological PDF

Sum rules with the ETMC data

For the second **central** moment the sum rules yield

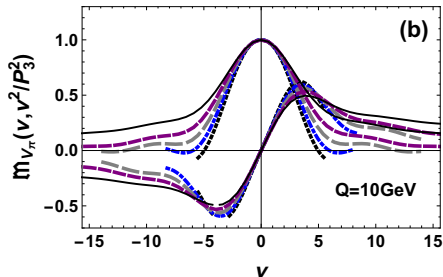
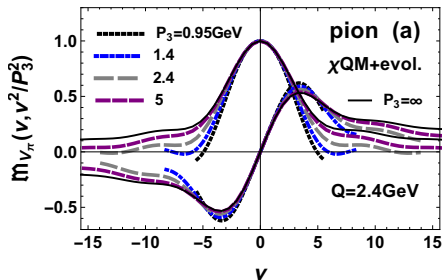
$$\langle y^2 \rangle - \langle y \rangle^2 = \langle x^2 \rangle - \langle x \rangle^2 + \frac{\overline{\langle k_T^2 \rangle}}{2P_3^2}$$



(for $u - d$)

Pion

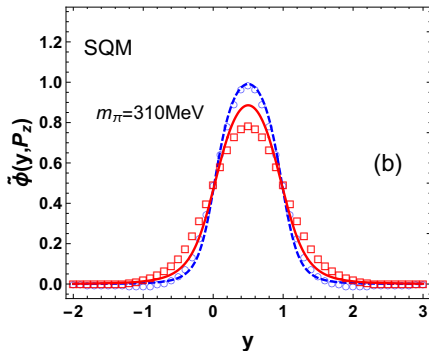
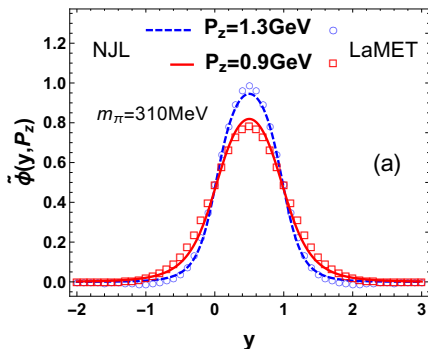
Most factorization breaking from QCD evolution: pion in chiral quark models



Here evolution goes over a notoriously large span, from $Q_0 \sim 320\text{ MeV}$, which leads to large factorization breaking seen in the reduced ITDs

Comparison of model QDA to lattice

[details in PLB 773(2017)385]



Quark QDA of the pion in NJL (a) and SQM (b) (for $m_\pi = 310 \text{ MeV}$) vs. the LaMET lattice data at $Q = 2 \text{ GeV}$

Conclusions

Conclusions

- QDF, TMD, ITD, pseudo-distributions – all related
- QDFs at **finite** and not necessarily large P_3 interesting on their own, can be used to verify models and methods
- Radyushkin's relation \rightarrow **TMD evolution** for the **QDF evolution**
- **Factorization breaking** from evolution, can be large when the evolution ratio is large, best seen in the clever **reduced ITDs** of Musch et al. / Orginos et al.
- **Sum rules**, relating longitudinal moments of QDFs, PDFs, and k_T moments of TMDs – work **encouragingly well** for the ETMC data!
- **QDA of the pion** in agreement with the LaMET lattice data

Hopes: With limitations (x above ~ 0.1 , low scales), Euclidean lattices will produce useful results related to partonic distributions (QDF, PDF, TMD, ...) of the nucleon and pion, test **factorization**

Doubts: Can we get more than, effectively, lowest x and k_T moments?

Extras

Some lattice details [Alexandrou, Cichy et al.]

Matrix elements from the ratio of 3- and 2-point functions correlators:

$$C^3(t, \tau, 0; \vec{P}) = \left\langle N_\alpha(\vec{P}, t) \mathcal{O}(\tau) \bar{N}_\alpha(\vec{P}, 0) \right\rangle$$

$$C^2(t, \tau, 0; \vec{P}) = \left\langle N_\alpha(\vec{P}, t) \bar{N}_\alpha(\vec{P}, 0) \right\rangle$$

Boosted nucleon field:

$$N_\alpha(\vec{P}, t) = \Gamma_{\alpha\beta} \sum_{\vec{x}} e^{-i\vec{P}\cdot\vec{x}} \epsilon^{abc} u_\beta^a(x) \left(d^{bT}(x) \mathcal{C} \gamma_5 u^c(x) \right)$$

where $\mathcal{C} = i\gamma_0\gamma_2$ and $\Gamma = \frac{1+\gamma_4}{2}$ is the parity projector.

Operator:

$$\mathcal{O}(z, \tau) = \sum_{\vec{y}} \bar{\psi}(y + \hat{e}_3 z) \gamma_3 W_3(y + \hat{e}_3 z, y) \psi(y)$$

Wilson link: $W_j(y + z\hat{e}_j, y) = U_j(y + (z-1)\hat{e}_j) \dots U_j(y + \hat{e}_j) U_j(y)$

Matrix element:

$$\frac{C^{3pt}(t, \tau, 0; \vec{P})}{C^{2pt}(t, 0; \vec{P})} \stackrel{0 \ll \tau \ll t}{=} \frac{-iP_3}{\sqrt{P_3^2 + M^2}} h(P_3, \Delta z)$$

Transversity relations for the pion DA, based on the Lorentz covariance

For the pion distribution amplitude

[WB, Prelovsek, Šantelj, ERA 2010, Miller, Tiburzi 2009]

$$\Psi(q \cdot z, z^2) = \int_0^1 d\alpha e^{i(2\alpha-1)q \cdot z} \Phi(\alpha, z^2)$$

$$\Psi^{\text{ET}}(0, -r^2) = \int_0^1 d\alpha \Phi^{\text{LC}}(\alpha, -r^2)$$

lattice simulations \leftrightarrow light-cone wave functions

In the case of the distribution function, the analog of $\Phi(\alpha, z^2)$ are the **pseudo-distributions** [Radyushkin 2017]

→

[for a summary see Golec-Biernat et al. 2007]:

CCFM:

- for the unintegrated gluon distribution
- all-loop approximation – angular ordering (coherence) for both large and small values of x
- a new non-Sudakov form factor that sums virtual corrections for small x

Kwieciński:

- one-loop approximation – angular ordering at small x and the corresponding virtual corrections not included
- coherence only in the real parton emissions for large x
- at small x the standard DGLAP transverse momentum ordering
- both **quark** and gluon unintegrated distributions