# Partonic quasi-distributions from TMDs

**Wojciech Broniowski**[1,2] **and Enrique Ruiz Arriola**

[1]Institute of Nuclear Physics PAN & [2]Jan Kochanowski U.

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# Outline

- Partonic quasi-distributions (QDFs) of Ji, designed for Euclidean QCD lattices, finite $P_3$
- Lorentz invariance and Radyushkin's relation of QDFs to TMDs
- Sum rules from the Ioffe-time distributions (ITDs) and moments of distributions
- Longitudinal-transverse factorization and its breaking by evolution
- Kwieciński's adaptation of CCFM
- Phenomenological results for QDFs at finite $P_3$ from TMDs
- Comparison to lattice results for the proton from ETMC [Alexandrou, Cichy et al. 2015-17]
- Extraction of information on TMDs from ETMC QDFs (!) via sum rules
- Predictions for QDFs of the pion, for the gluonic components

# PDF and QDF

Parton Distribution Function (PDF):

$$q(x) = \int \frac{dz_-}{4\pi} e^{ixP_+ z_-} \left. \langle P|\bar{\psi}(0)\gamma^+ \, U[0,z]\psi(z)|P\rangle \right|_{z_+=0, z_\perp=0}$$

(impossible to put on Euclidean lattice,
as $t^2 - z^2 = 0$ shrinks to one point, $t_E^2 + z^2 = 0$)

Quasi-Distribution Function (QDF): [Ji 2013]

$$\tilde{q}(y; P_3) = \int \frac{dz_3}{4\pi} e^{-iyP_3 z_3} \left. \langle P|\bar{\psi}(0)\gamma^3 \, U[0,z]\psi(z)|P\rangle \right|_{z_0=0, z^\perp=0}$$

$y$ - fraction of pion's $P_3$ carried by the quark $y$ is not constrained

Limit:

$$\lim_{P_3 \to \infty} \tilde{q}(x; P_3) = q(x)$$

Power corrections vanish asymptotically

# Covariant formulation

Lorentz covariance (all for spin-averaged quantities):

$$\langle P|\bar{\psi}(0)\gamma^\mu U[0,z]\psi(z)|P\rangle = P^\mu h(P\cdot z, z^2) + z^\mu h_z(P\cdot z, z^2)$$

($\nu = -P\cdot z$ is referred to as the Ioffe time)

$$\begin{aligned}\tilde{q}(y,P_3) &= P_3\int\frac{dz_3}{2\pi}e^{-iyP_3z_3}h(-P_3z_3,-z_3^2)\\ q(x) &= P_+\int\frac{dz_-}{2\pi}e^{ixP_+z_-}h(P_+z_-,0)\end{aligned}$$

TMD:

$$q(x,\boldsymbol{k_T}) = P^+\int\frac{dz_-}{2\pi}e^{ixP_+z_-}\int\frac{dz_T^2}{(2\pi)^2}e^{i\boldsymbol{k_T}\cdot\boldsymbol{z_T}}h(P_+z_-,-\boldsymbol{z_T^2})$$

or:

$$\hat{q}(x,\boldsymbol{z_T}) = P^+\int\frac{dz_-}{2\pi}e^{ixP_+z_-}h(P_+z_-,-\boldsymbol{z_T^2})$$

# Radyushkin's QDF-TMD relation - pedestrian derivation

[Radyushkin 2017]
(for DA of the pion [WB, Prelovsek, Šantelj, ERA 2010, Miller, Tiburzi 2009])

Choose the specific value $k_2 = (x - y)P_3$ in the definition of TMD. Then

$$\begin{aligned}
&\int dk_1 \int dx \, q(x, k_1, (y-x)P_3) \\
&= \int dk_1 \int dx \, P^+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} \int \frac{dz_1 dz_2}{(2\pi)^2} e^{ik_1 z_1 + i(x-y)P_3 z_2} h(P_+ z_-, -z_1^2 - z_2^2) \\
&= P^+ \int dz^- \delta(P_+ z_- + P_3 z_2) \int dz_1 \delta(z_1) \int \frac{dz_2}{2\pi} e^{-iyP_3 z_2} h(P_+ z_-, -z_1^2 - z_2^2) \\
&= \int \frac{dz_2}{2\pi} e^{-iyP_3 z_2} h(-P_3 z_2, -z_2^2) \qquad [\boldsymbol{z_2 \to z_3}] \\
&\equiv \frac{1}{P_3} \tilde{q}(y, P_3)
\end{aligned}$$

(subtleties related to the path in the gauge link are ignored, straight line gauge link assumed instead of a infinite-staple shape)

# QDF-TMD relations (cont.)

$$\tilde{q}(y, P_3) = P_3 \int dk_1 \int dx \, q(x, k_1, (y-x)P_3)$$

Equivalent form:

$$\tilde{q}(y, P_3) = P_3 \int dx \int \frac{dz_2}{2\pi} e^{-i(y-x)z_2 P_3} \hat{q}(x, 0, z_2)$$

Inverse relation:

$$\hat{q}(x, 0, z_2) = z_2 \int dy \int dP_3 e^{i(y-x)z_2 P_3} \tilde{q}(y, P_3)$$

Theoretical and phenomenological significance
lattice QDFs $\leftrightarrow$ vast knowledge of TMDs (also from lattice [Munsch et al. 2011])

# Factorization ansatz

$$q(x, k_T) = q(x)F(k_T), \quad \hat{q}(x, z_T) = q(x)\hat{F}(z_T)$$

$$\tilde{q}(y, P_3) = P_3 \int dx F[(x-y)P_3]\, q(x)$$

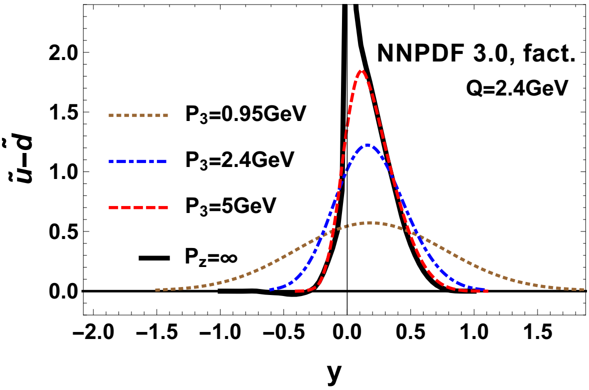
Gaussian:

$$F(k_T) = \frac{e^{-\frac{k_T^2}{\langle k_T^2 \rangle}}}{\pi \langle k_T^2 \rangle}, \quad \hat{F}(z_T) = e^{-\frac{z_T^2 k_T^2}{4}}$$

$$\tilde{q}(y, P_3) = \frac{1}{\sqrt{2\pi}\Sigma} \int dx\, e^{-\frac{(x-y)^2}{2\Sigma^2}} q(x), \quad \Sigma^2 = \frac{\langle k_T^2 \rangle}{2P_3^2}$$

Factorization (at $m_\pi = 600$ MeV) seen in the lattice TMD studies [Munsch et al. 2011] and the quenched QDF studies [Orginos et al. 2017]

# Result of folding: QDA for TMD




solid – PDF limit, dashed – QDF, negative $x$ in PDF - antiquarks

The needed values for $P_3$ to achieve a few-percent agreement with PDF for $x > 0.15$ are $P_3 > 5$ GeV

Results at finite $P_3$ are interesting on their own (lattice, models)

# Sum rules from ITDs

Primary object from the lattice:

$$h(-P_3 z_3, -z_3^2) = \int_{-1}^{1} dx\, e^{iP_3 z_3 x} \hat{q}(x, -z_3^2) = \int_{-\infty}^{\infty} dy\, e^{iP_3 z_3 y} \tilde{q}(y, P_3)$$

with $\nu = P_3 z_3$ - the Ioffe time. Differentiation wrt. $\nu$ at the origin $\rightarrow$ Slope:

$$\frac{d}{d\nu} h\left(-\nu, -\frac{\nu^2}{P_3^2}\right)\Bigg|_{\nu=0} = i\langle x\rangle_q = i\langle y\rangle_q(P_3)$$

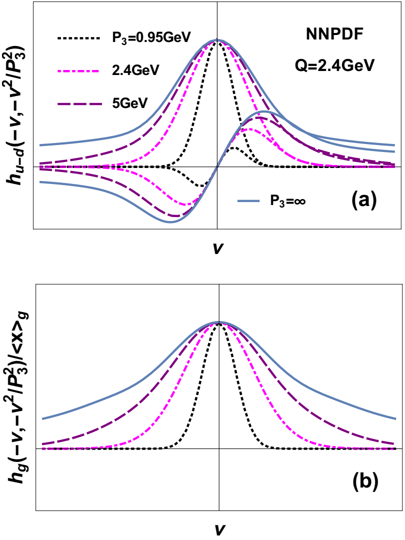
Curvature:

$$\frac{d^2}{d\nu^2} h\left(-\nu, -\frac{\nu^2}{P_3^2}\right)\Bigg|_{\nu=0} = -\langle x^2\rangle_q - \frac{1}{P_3^2}\int_{-1}^{1} dx \langle k_T^2(x)\rangle q(x) = -\langle y^2\rangle_q(P_3)$$

($x$-moments and $k_T$-moments enter)

Similarly for gluon distributions

# Sum rules from ITDs - example




real (symmetric) and imaginary (antisymmetric) parts

# Sum rules from reduced ITDs

Reduced ITDs [Munsch et al. 2011, Orginos et al. 2017]

$$\mathfrak{M}(\nu, -z^2) = \frac{h(-\nu, z^2)}{h(0, z^2)} = \frac{\int dx\, e^{i\nu x} \hat{q}(x, -\frac{\nu^2}{P_3^2})}{\int dx\, \hat{q}(x, -\frac{\nu^2/}{P_3^2})} =$$

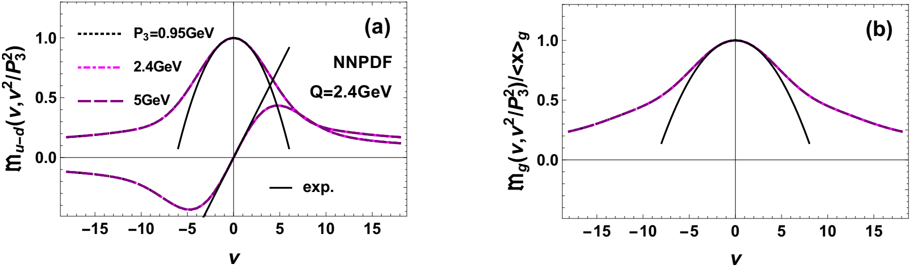
$$= (\text{factorization}) = \frac{\hat{F}(\frac{\nu}{P_3}) \int dx\, e^{i\nu x} q(x)}{\hat{F}(\frac{\nu}{P_3}) \int dx\, q(x)} = \int dx\, e^{i\nu x} q(x)$$

(in the factorization model it is independent of $P_3$!)

$$\frac{d}{d\nu} \mathfrak{M}(\nu, -\nu^2/P_3^2)\Big|_{\nu=0} = i\langle x \rangle_q = i\langle y \rangle_q (P_3),$$

$$\frac{d^2}{d\nu^2} \mathfrak{M}(\nu, -\nu^2/P_3^2)\Big|_{\nu=0} = -\langle x^2 \rangle_q = -\langle y^2 \rangle_q (P_3) + \frac{1}{P_3^2} \int_{-1}^{1} dx \langle k_T^2(x) \rangle q(x)$$

# Sum rules from reduced ITDs - example




Long-tail in $\nu$ is the result of the low-$x$ (integrable) singularity in PDF:
$\sim x^{-\alpha} \to \sim \nu^{-1+\alpha}$

Lattice: $\nu < LP_3$, $P_3 = \frac{2\pi n}{L}$, $x > \frac{2\pi}{LP_3} = \frac{1}{n} \sim 0.1 - 0.2$
$L$ is the lattice size

# QCD evolution

One needs to specify the scale:

$$\tilde{q}(y, P_3; Q) = P_3 \int dx \int \frac{dz_2}{2\pi} e^{-i(y-x)z_2 P_3} \hat{q}(x, 0, z_2; Q)$$

Use Kwieciński's one-loop CCFM, diagonal in $z_T$, structure very much like the DGLAP equations for the integrated PDFs, but with a modified kernel:

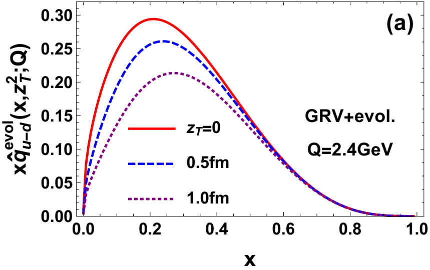
$$Q^2 \frac{\partial \hat{q}(x, z_T; Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 d\xi \, P_{qq}(\xi) \Big[ \Theta(\xi - x) \times J_0[(1-\xi) Q z_T] \, \hat{q}\left(\frac{x}{\xi}, z_T; Q\right) - \hat{q}(x, z_T; Q) \Big]$$

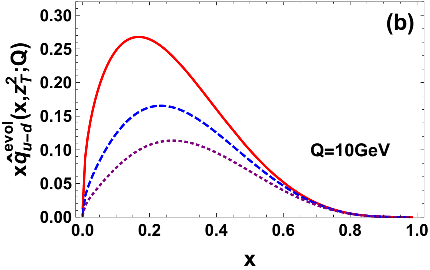
The initial condition at the scale $Q_0$ is provided with a factorized form

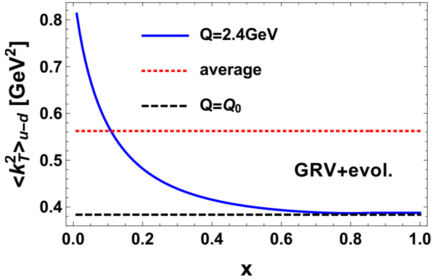
$$\hat{q}(x, z_T; Q_0) = \hat{F}(z_T^2) q(x)$$

$$\hat{q}(x, z_T; Q) = \hat{F}(z_T^2) \hat{q}^{\text{evol}}(x, z_T; Q)$$

# $k_T$-spreading

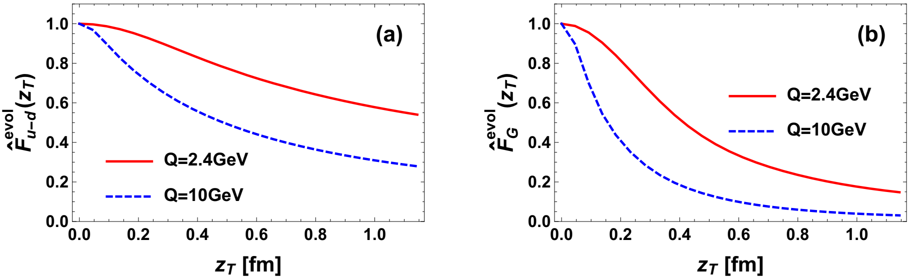


# Evolution-generated form factor

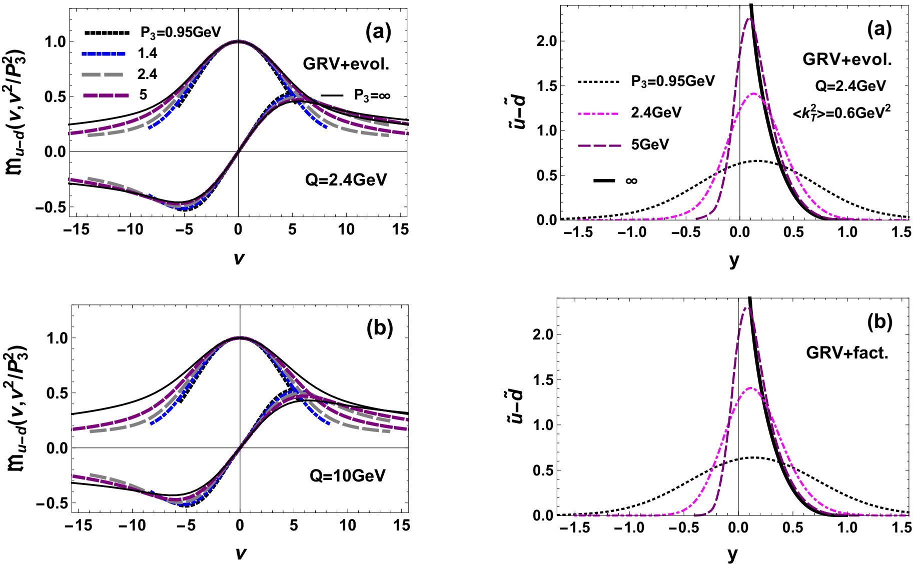
$$F^{\text{evol}}(z_T; Q) = \int dx\, \hat{q}^{\text{evol}}(x, z_T; Q)$$



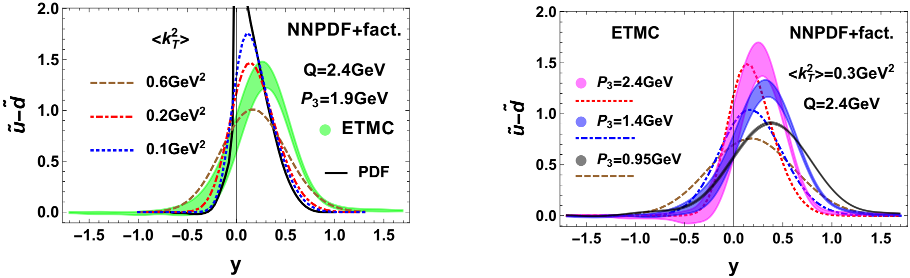

asympotically quarks $\sim z_T^{-4\frac{C_F}{\beta_0}\log\frac{\alpha_s(Q_0)}{\alpha_s(Q)}}$, gluons $\sim z_T^{-4\frac{N_c}{\beta_0}\log\frac{\alpha_s(Q_0)}{\alpha_s(Q)}}$
$(C_F = 4/3,\ N_c = 3,\ \beta_0 = 9)$ [WB, ERA 2004]

... back to quasi →

# Factorization breaking



# Comparison to ETMC lattice



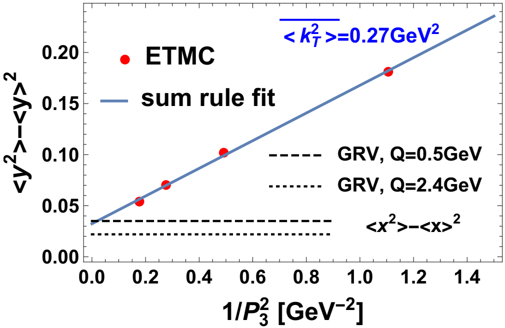

[Alexandrou, Cichy et al. 2015-2017] use a correlator retaining the sub-leading structure $\sim z^\mu$ (see slide 4), mixing with a twist-3 scalar, $m_\pi = 370$ MeV, target-mass corrections (relevant at low scales) + typical lattice problems: finite cut-off from the lattice spacing, volume effects, the source-sink separation issue, etc.

[Orginos et al. 2017] use $m_\pi = 600$ MeV, and the PDF extracted from the (quenched) lattice is also visibly to the right of the phenomenological PDF

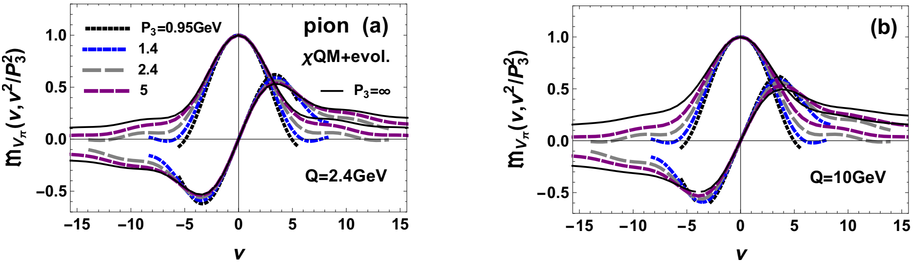
# Sum rules with the ETMC data

For the second central moment the sum rules yield

$$\langle y^2 \rangle - \langle y \rangle^2 = \langle x^2 \rangle - \langle x \rangle^2 + \frac{\overline{\langle k_T^2 \rangle}}{2P_3^2}$$



# Most factorization breaking from QCD evolution: pion in chiral quark models (see next talk by ERA)




Here evolution goes over a notoriously large span, from $Q_0 \sim 320$ MeV, which leads to large factorization breaking seen in the reduced ITDs

# Conclusions

- Results for QDFs at finite $P_3$ interesting on their own, can be used to verify models and methods
- Radyushkin's relation allows us to use the TMD evolution for the QDF evolution
- Factorization breaking from evolution, can be large when the evolution ratio is large
- Best seen in the clever measure of the reduced ITDs of Orginos et al.
- Sum rules, relating low $\nu$ ITDs to moments of QDFs, PDFs, and $k_T$ moments of TMDs – work encouragingly well for the ETMC data!

Hope: With limitations ($x$ above $\sim 0.1$, low $Q$), the Euclidean lattices should be able to produce useful results related to partonic distributions (QDF, PDF, TMD) of hadrons

# Extras

# Some lattice details [Alexandrou, Cichy et al.]

Matrix elements from the ratio of 3- and 2-point functions correlators:

$$C^3(t,\tau,0;\vec{P}) = \left\langle N_\alpha(\vec{P},t)\mathcal{O}(\tau)\overline{N}_\alpha(\vec{P},0)\right\rangle$$
$$C^2(t,\tau,0;\vec{P}) = \left\langle N_\alpha(\vec{P},t)\overline{N}_\alpha(\vec{P},0)\right\rangle$$

Boosted nucleon field:

$$N_\alpha(\vec{P},t) = \Gamma_{\alpha\beta}\sum_{\vec{x}} e^{-i\vec{P}\cdot\vec{x}}\epsilon^{abc}u^a_\beta(x)\left(d^{b^T}(x)\mathcal{C}\gamma_5 u^c(x)\right)$$

where $\mathcal{C} = i\gamma_0\gamma_2$ and $\Gamma = \frac{1+\gamma_4}{2}$ is the parity projector.
Operator:

$$\mathcal{O}(z,\tau) = \sum_{\vec{y}} \overline{\psi}(y+\hat{e}_3 z)\gamma_3 W_3(y+\hat{e}_3 z, y)\psi(y)$$

Wilson link: $W_j(y+z\hat{e}_j,y) = U_j(y+(z-1)\hat{e}_j)\ldots U_j(y+\hat{e}_j)U_j(y)$
Matrix element:

$$\frac{C^{3pt}(t,\tau,0;\vec{P})}{C^{2pt}(t,0;\vec{P})} \overset{0\ll\tau\ll t}{=} \frac{-iP_3}{\sqrt{P_3^2+M^2}}h(P_3,\Delta z)$$

# Transversity relations for the pion DA

For the pion distribution amplitude
[WB, Prelovsek, Šantelj, ERA 2010, Miller, Tiburzi 2009]

$$\Psi(q \cdot z, z^2) = \int_0^1 d\alpha e^{i(2\alpha-1)q\cdot z} \Phi(\alpha, z^2)$$

$$\Psi^{\mathrm{ET}}(0, -r^2) = \int_0^1 d\alpha \Phi^{\mathrm{LC}}(\alpha, -r^2)$$

lattice simulations $\leftrightarrow$ light-cone wave functions

In the case of the distribution function, the analog of $\Phi(\alpha, z^2)$ are the pseudo-distributions [Radyushkin 2017]

$\rightarrow$

# CCFM

[Golec-Biernat et al. 2007]:

CCFM:

- for the unintegrated gluon distribution
- all-loop approximation – angular ordering (coherence) for both large and small values of $x$
- a new non-Sudakov form factor that sums virtual corrections for small $x$

Kwieciński:

- one-loop approximation – the angular ordering at small $x$ and the corresponding virtual corrections not included
- coherence only in the real parton emissions for large $x$, including only the Sudakov form factor which resummes virtual corrections
- at small $x$ the standard DGLAP transverse momentum ordering appears
- both quark and gluon unintegrated distributions