Partonic quasi-distributions

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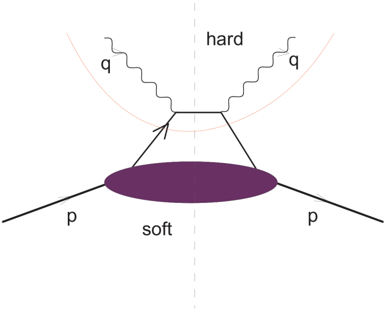
Research with **Enrique Ruiz Arriola**
details in PLB 773(2017)385 and in arXiv:1711.03377

Outline

- Partonic quasi-distributions (QDFs) of Ji, designed to extract PDFs from Euclidean QCD lattices (cf. seminar by K. Cichy)
- Lorentz invariance and Radyushkin's relation of QDFs to TMDs
- Sum rules for moments of distributions
- Longitudinal-transverse factorization and its breaking by evolution
- Kwieciński's adaptation of CCFM
- Comparison to lattice results for the proton from ETMC
- QDA of the pion vs. LaMET

Introduction

Parton distributions



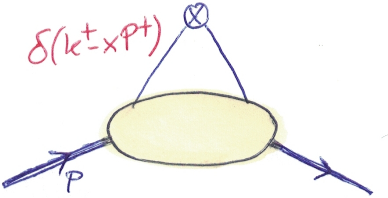
$Q^2 = -q^2, \;\; x = \frac{Q^2}{2p\cdot q}, \;\; Q^2 \to \infty$

Factorization of soft and hard processes, Wilson's OPE

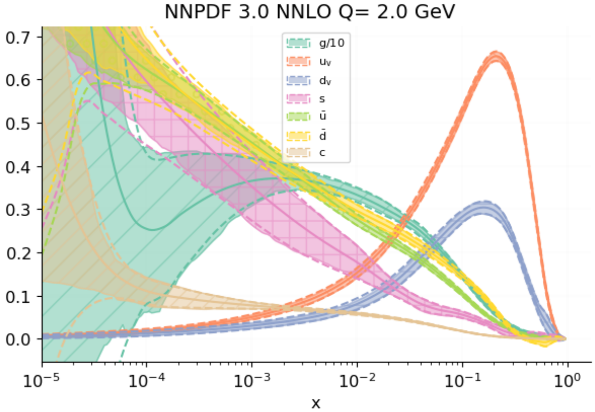
$$\langle J(q)J(-q)\rangle = \sum_i C_i(Q^2;\mu)\langle \mathcal{O}_i(\mu)\rangle$$

Twist expansion $\to$ $F(x,Q) = F_0(x,\alpha(Q)) + \frac{F_2(x,\alpha(Q))}{Q^2} + \ldots$

Bjorken limit $\to$ light-cone momentum is constrained:
$k^+ \equiv k^0 + k^3 = xP^+ \qquad x \in [0,1]$



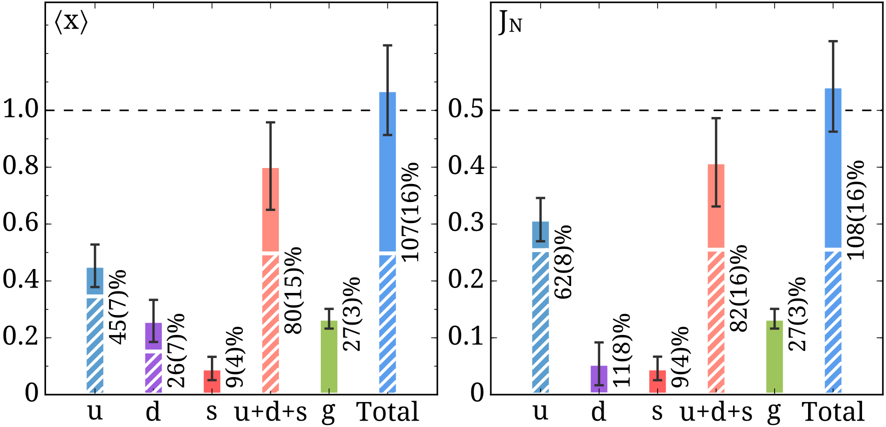
Phenomenological parametrizations for the proton PDFs



probabilistic interpretation

Moments from the lattice

Decomposition of the momentum and spin in the proton (scale 2 GeV)



strips – valence

(only up to second moment)

[Alexandrou et al 2017]

Field-theoretic definitions

Parton Distribution Function (PDF), leading twist, nucleon or pion:

$$q(x) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle P|\bar{\psi}(0)\gamma^+ U[0,z]\psi(z)|P\rangle\Big|_{z^+=0, z_T=0}$$

Parton Distribution Amplitude (PDA), leading twist, pion:

$$\phi(x) = \frac{i}{F_\pi} \int \frac{dz^-}{2\pi} e^{i(x-1)P^+z^-} \langle P|\bar{\psi}(0)\gamma^+\gamma_5 U[0,z]\psi(z)|\mathrm{vac}\rangle\Big|_{z^+=0, z_T=0}$$

(spin-averaged, isospin suppressed)

P - hadron's momentum, $v^\pm \equiv v^0 \pm v^3$ - light-cone basis
$U[z_1, z_2] = \exp\left(-ig_s \int_{z_1}^{z_2} d\xi \lambda^a A_a^+(\xi)\right)$ - Wilson's gauge link
x - fraction of P^+ carried by the quark, $x \in [0,1]$ (or $x \in [-1,1]$)

Quasi-distributions of Ji

PDF and QDF

Parton Distribution Function (PDF):

$$q(x) = \int \frac{dz_-}{4\pi} e^{ixP_+ z_-} \langle P|\bar{\psi}(0)\gamma^+ U[0,z]\psi(z)|P\rangle\Big|_{z_+=0, z_T=0}$$

(impossible to put on Euclidean lattice, as $t^2 - z^2 = 0$ becomes one point, $t_E^2 + z^2 = 0$)

Quasi-Distribution Function (QDF): [Ji 2013]

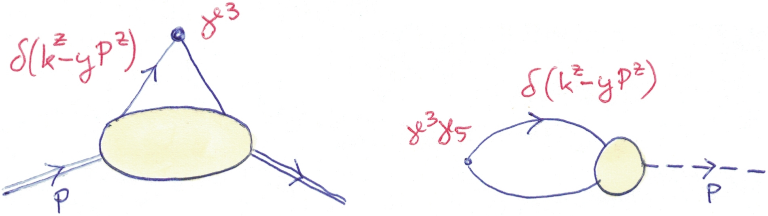
$$\tilde{q}(y; P_3) = \int \frac{dz_3}{4\pi} e^{-iyP_3 z_3} \langle P|\bar{\psi}(0)\gamma^3 U[0,z]\psi(z)|P\rangle\Big|_{z_0=0, z_T=0}$$

y - fraction of pion's P_3 carried by the quark, y is not constrained

Limit:

$$\lim_{P_3\to\infty} \tilde{q}(x; P_3) = q(x)$$

QDF and QDA in the momentum representation



Constrained longitudinal momenta, but $y \in (-\infty, \infty)$

(partons can move “backwards”)

Covariant formulation

Lorentz covariance

$$\langle P|\bar{\psi}(0)\gamma^\mu U[0,z]\psi(z)|P\rangle = P^\mu h(P\cdot z, z^2) + z^\mu h_z(P\cdot z, z^2)$$

($\nu = -P\cdot z$ is referred to as the Ioffe time)

$$\tilde{q}(y,P_3) = P_3 \int \frac{dz_3}{2\pi} e^{-iyP_3 z_3} h(-P_3 z_3, -z_3^2) \qquad (*)$$

$$q(x) = P_+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} h(P_+ z_-, 0)$$

Transverse momentum Distribution (TMD):

$$\hat{q}(x,\boldsymbol{z_T}) = P^+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} h(P_+ z_-, -\boldsymbol{z_T^2}) \qquad (**)$$

or:

$$q(x,\boldsymbol{k_T}) = P^+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} \int \frac{dz_T^2}{(2\pi)^2} e^{i\boldsymbol{k_T}\cdot\boldsymbol{z_T}} h(P_+ z_-, -\boldsymbol{z_T^2})$$

Radyushkin's QDF-TMD relation – pedestrian derivation

[Radyushkin 2017]

Choose $k_2 = (x - y)P_3$ in the definition of TMD:

$$
\begin{aligned}
&\int dk_1 \int dx\, q(x, k_1^2 + (y-x)^2 P_3^2) \\
&= \int dk_1 \int dx\, P^+ \int \frac{dz_-}{2\pi} e^{ixP_+ z_-} \int \frac{dz_1 dz_2}{(2\pi)^2} e^{ik_1 z_1 + i(x-y)P_3 z_2} h(P_+ z_-, -z_1^2 - z_2^2) \\
&= P^+ \int dz^- \delta(P_+ z_- + P_3 z_2) \int dz_1 \delta(z_1) \int \frac{dz_2}{2\pi} e^{-iyP_3 z_2} h(P_+ z_-, -z_1^2 - z_2^2) \\
&= \int \frac{dz_2}{2\pi} e^{-iyP_3 z_2} h(-P_3 z_2, -z_2^2) \qquad [\boldsymbol{z_2 \to z_3}]\ \text{(rot. symmetry)} \\
&\equiv \frac{1}{P_3} \tilde{q}(y, P_3)
\end{aligned}
$$

(straight line gauge link assumed instead of a infinite-staple shape [Collins])

QDF-TMD relations (cont.)

Equivalent form:

$$\tilde{q}(y, P_3) = P_3 \int dx \int \frac{dz_2}{2\pi} e^{-i(y-x)z_2 P_3} \hat{q}(x, z_2^2)$$

Inverse relation:

$$\hat{q}(x, z_2^2) = z_2 \int dy \int dP_3 e^{i(y-x)z_2 P_3} \tilde{q}(y, P_3)$$

(can replace z_2 with z_3, pseudo-distributions)

Theoretical and practical significance

lattice QDFs $\leftrightarrow$ vast knowledge of TMDs (also from lattice [Musch et al. 2011])

Transverse-longitudinal factorization

Factorization ansatz to simplify

$$q(x, k_T) = q(x)F(k_T), \quad \hat{q}(x, z_T) = q(x)\hat{F}(z_T)$$

$$\tilde{q}(y, P_3) = P_3 \int dx F[(x-y)P_3]\, q(x)$$

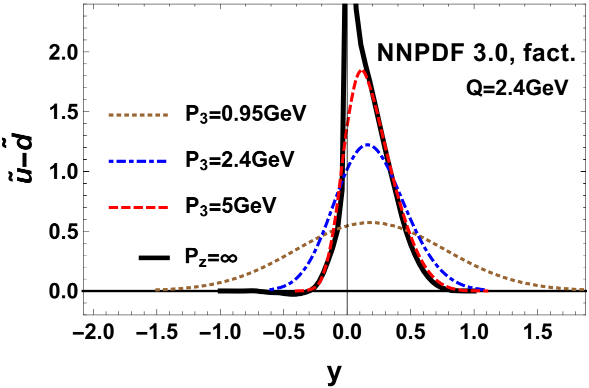
Gaussian:

$$F(k_T) = \frac{e^{-\frac{k_T^2}{\langle k_T^2 \rangle}}}{\pi \langle k_T^2 \rangle}, \quad \hat{F}(z_T) = e^{-\frac{z_T^2 k_T^2}{4}}$$

$$\tilde{q}(y, P_3) = \frac{1}{\sqrt{2\pi}\Sigma} \int dx\, e^{-\frac{(x-y)^2}{2\Sigma^2}} q(x), \quad \Sigma^2 = \frac{\langle k_T^2 \rangle}{2P_3^2}$$

Factorization (at $m_\pi = 600$ MeV and low scales) seen in the lattice TMD studies [Musch et al. 2011] and the quenched QDF studies [Orginos et al. 2017]

Result of folding: washed out QDF from PDF



solid – PDF limit, dashed – QDF $\langle k_T^2 \rangle = 0.6\ \mathrm{GeV}^2$

For a few-percent agreement with PDF for $x > 0.15$ one needs $P_3 > 5\ \mathrm{GeV}$

Results at finite P_3 are interesting on their own!

Sum rules

Sum rules for Ioffe-Time Distributions (IDFs)

Primary object from the lattice: [WB, ERA 2017]

$$h(-P_3 z_3, -z_3^2) = \int_{-1}^{1} dx\, e^{iP_3 z_3 x} \hat{q}(x, -z_3^2) = \int_{-\infty}^{\infty} dy\, e^{iP_3 z_3 y} \tilde{q}(y, P_3)$$

Use $\nu = P_3 z_3$ - the Ioffe time (strictly speaking, $\tau_I = \nu/M$)
Differentiation wrt. ν at the origin $\rightarrow$
Slope:

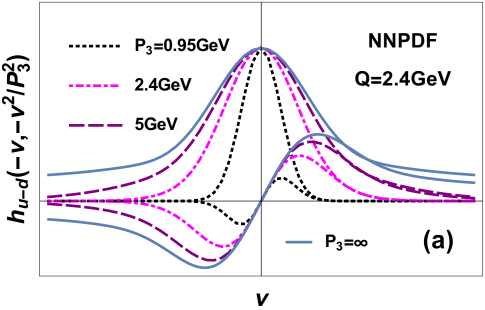
$$\frac{d}{d\nu} h\left(-\nu, -\frac{\nu^2}{P_3^2}\right)\Bigg|_{\nu=0} = i\langle x\rangle_q = i\langle y\rangle_q$$

Curvature:

$$\frac{d^2}{d\nu^2} h\left(-\nu, -\frac{\nu^2}{P_3^2}\right)\Bigg|_{\nu=0} = -\langle x^2\rangle_q - \frac{1}{P_3^2}\int_{-1}^{1} dx \langle k_T^2(x)\rangle q(x) = -\langle y^2\rangle_q(P_3)$$

Similarly for higher moments, gluon distributions

IDFs from TMDs



real (symmetric) and imaginary (antisymmetric) parts
solid lines: $h(-\nu, 0)$

Sum rules from reduced IDFs

Reduced IDFs [Munsch et al. 2011, Orginos et al. 2017]

$$\mathfrak{M}(\nu, z_3^2) = \frac{h(-\nu, -z_3^2)}{h(0, -z_3^2)} = \frac{\int dx\, e^{i\nu x} \hat{q}(x, z_3^2)}{\int dx\, \hat{q}(x, z_3^2)} =$$

$$= (\text{factorization}) = \frac{\hat{F}(z_3) \int dx\, e^{i\nu x} q(x)}{\hat{F}(z_3) \int dx\, q(x)} = \int dx\, e^{i\nu x} q(x) = h(-\nu, 0)$$

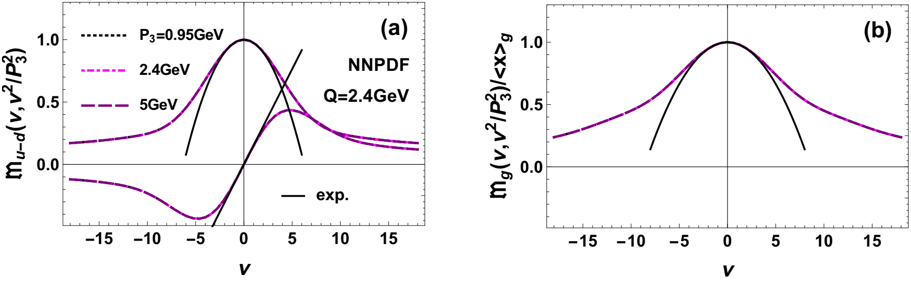
(in the factorization model it is independent of P_3!)

In general

$$\frac{d}{d\nu} \mathfrak{M}(\nu, \nu^2/P_3^2)\Big|_{\nu=0} = i\langle x \rangle_q = i\langle y \rangle_q,$$

$$\frac{d^2}{d\nu^2} \mathfrak{M}(\nu, \nu^2/P_3^2)\Big|_{\nu=0} = -\langle x^2 \rangle_q = -\langle y^2 \rangle_q(P_3) + \frac{1}{P_3^2} \int_{-1}^{1} dx \langle k_T^2(x) \rangle q(x)$$

Sum rules from reduced IDFs - example



Lowest moments approximate well at low $|\nu|$

Long ν tails result from the low-x singularity in PDF: $\sim x^{-\alpha} \to \sim \nu^{-1+\alpha}$

QCD evolution

QCD evolution for QDF from TMD

One needs to specify the scale:

$$\tilde{q}(y, P_3; Q) = P_3 \int dx \int \frac{dz_2}{2\pi} e^{-i(y-x)z_2 P_3} \hat{q}(x, 0, z_2; Q)$$

Kwieciński's one-loop CCFM, structure like the DGLAP equations for the integrated PDFs, but with a modified kernel, diagonal in z_T:

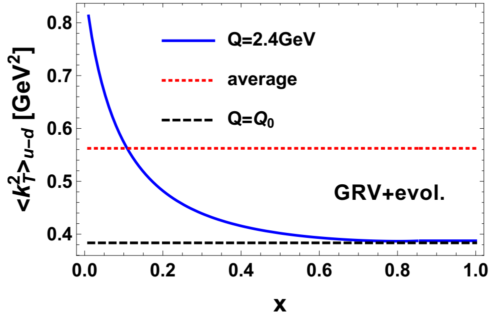
$$Q^2 \frac{\partial \hat{q}(x, z_T; Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 d\xi \, P_{qq}(\xi) \Big[\Theta(\xi - x) \times J_0[(1-\xi)Q z_T] \, \hat{q}\left(\frac{x}{\xi}, z_T; Q\right) - \hat{q}(x, z_T; Q) \Big]$$

k_T-spreading

The initial condition at the scale Q_0 in a factorized form

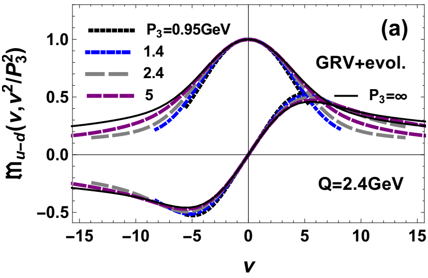
$$\hat{q}(x, z_T; Q_0) = \hat{F}(z_T^2) q(x)$$

$$\hat{q}(x, z_T; Q) = \hat{F}(z_T^2) \hat{q}^{\text{evol}}(x, z_T; Q)$$

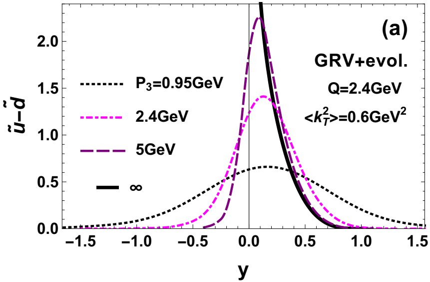


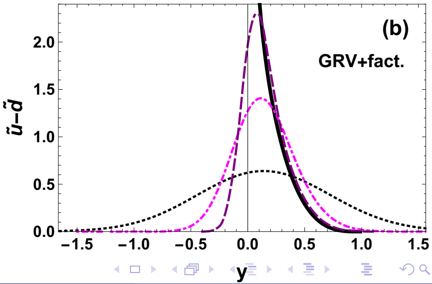
Factorization breaking from QCD evolution

seen



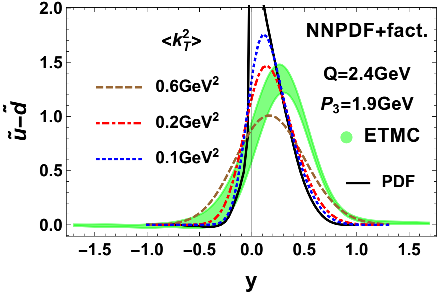
not seen

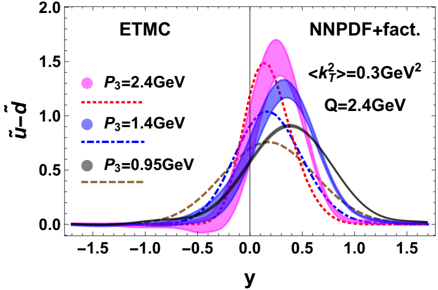




Comparison to lattice data

Comparison to ETMC lattice





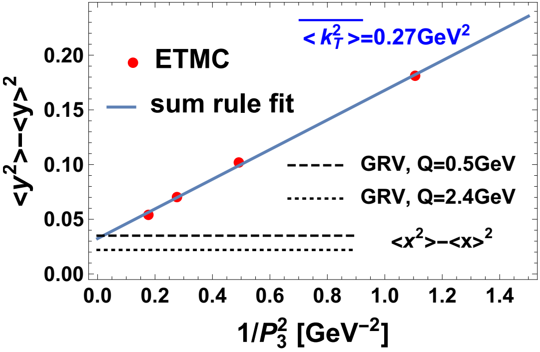
[Alexandrou, Cichy et al. 2015-2017] use a correlator retaining the sub-leading structure $\sim z^{\mu}$, mixing with a twist-3 scalar, $m_\pi = 370$ MeV + typical lattice problems: finite lattice spacing, volume effects ...

[Orginos et al. 2017] extraction from the (quenched, $m_\pi = 600$ MeV) lattice is also visibly to the right of the phenomenological PDF

Sum rules with the ETMC data

For the second central moment the sum rules yield

$$\langle y^2\rangle - \langle y\rangle^2 = \langle x^2\rangle - \langle x\rangle^2 + \frac{\overline{\langle k_T^2\rangle}}{2P_3^2}$$



(for $u-d$)

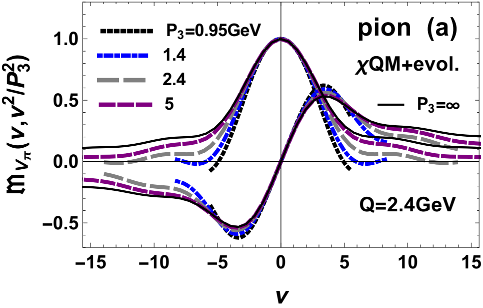
Pion

Analytic formulas at QM scale, $m_\pi = 0$

[WB+ERA arXiv:1711.03377]

Name Symbol	Nambu–Jona-Lasinio	Spectral quark model
PDA, PDF $\phi(x)$, $q(x)$	$\theta[x(1-x)]$	$\theta[x(1-x)]$
QDA, QDF $\tilde{\phi}(y,P_3)$, $\tilde{q}(y,P_3)$	$\frac{N_c M^2}{4\pi^2 f^2} \mathrm{sgn}(y) \ln \left. \frac{P_3\lvert y\rvert + \sqrt{M^2+P_3^2 y^2}}{M} \right\rvert_{\mathrm{reg}} + (y \leftrightarrow 1-y)$	$\frac{1}{\pi}\left[\frac{2m_\rho P_3 y}{m_\rho^2 + 4{P_3}^2 y^2} + \mathrm{arctg}\left(\frac{2P_3 y}{m_\rho}\right)\right] + (y \leftrightarrow 1-y)$
LCWF, TMD $\Psi(x,k_T^2)$, $q(x,k_T^2)$	$\frac{N_c M^2}{4\pi^2 f^2} \left. \frac{1}{k_T^2 + M^2} \right\rvert_{\mathrm{reg}} \theta[x(1-x)]$	$\frac{6m_\rho^3}{\pi\left(4k_\perp^2 + m_\rho^2\right)^{5/2}} \theta[x(1-x)]$
pseudo-DA, DF $\mathcal{P}(x,\lvert\boldsymbol{z}\rvert)$, $\hat{q}(x,\lvert\boldsymbol{z}\rvert)$	$\frac{N_c M^2}{4\pi^3 f^2} K_0(M\lvert\boldsymbol{z}\rvert)\Big\rvert_{\mathrm{reg}} \theta[x(1-x)]$	$\frac{1}{2} e^{-\frac{m_\rho\lvert\boldsymbol{z}\rvert}{2}} \left(m_\rho\lvert\boldsymbol{z}\rvert + 2\right) \theta[x(1-x)]$
IDA, IDF $\mathcal{M}(\nu,\lvert\boldsymbol{z}\rvert)$	$\frac{N_c M^2}{2\pi^3 f^2} \frac{\sin\left(\frac{\nu}{2}\right)}{\nu} K_0(M\lvert\boldsymbol{z}\rvert)\Big\rvert_{\mathrm{reg}}$	$\frac{\sin\left(\frac{\nu}{2}\right)}{\nu} e^{-\frac{m_\rho\lvert\boldsymbol{z}\rvert}{2}} \left(m_\rho\lvert\boldsymbol{z}\rvert + 2\right)$
VDA, VDF $\Phi(x,\mu)$	$\frac{N_c M^2}{4\pi^2 f^2} \mu\, e^{-\mu M^2}\Big\rvert_{\mathrm{reg}} \theta[x(1-x)]$	$\frac{\mu^{5/2} m_\rho^3 e^{-\frac{1}{4}\mu m_\rho^2}}{4\sqrt{\pi}} \theta[x(1-x)]$

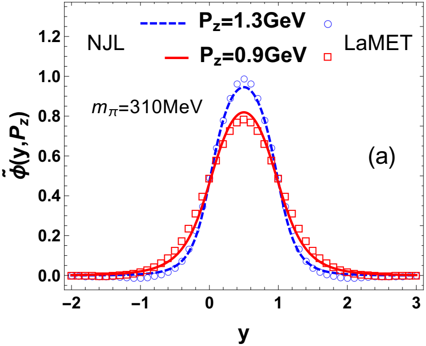
Larger factorization breaking from QCD evolution



Here evolution goes over a notoriously large span, from $Q_0 \sim 320$ MeV, which leads to large factorization breaking seen in the reduced IDFs

Comparison of model QDA to lattice

[details in PLB 773(2017)385]



Quark QDA of the pion in NJL (for $m_\pi = 310$ MeV, no evolution) vs. LaMET lattice data at $Q = 2$ GeV

Conclusions

Conclusions

- QDFs at finite and not necessarily large P_3 interesting on their own, can be used to verify models and methods
- Radyushkin's relation $\rightarrow$ TMD $\leftrightarrow$ QDF
- Factorization breaking from evolution, can be large when the evolution ratio is large, best seen in the clever reduced IDFs of Musch et al. / Orginos et al.
- Sum rules, relating y moments of QDFs, and x and k_T moments of TMDs – work encouragingly well for the ETMC data!
- QDA of the pion in agreement with the LaMET lattice data

Hopes: With limitations (x above ~ 0.1, low scales), Euclidean lattices will produce useful complementary results for QDFs, PDFs, TMDs, IDFs ... of the nucleon and pion, test factorization

Doubts: Can we get more than, effectively, a few lowest x and k_T moments?

Extras

Some lattice details [Alexandrou, Cichy et al.]

Matrix elements from the ratio of 3- and 2-point functions correlators:

$$C^3(t,\tau,0;\vec{P}) = \left\langle N_\alpha(\vec{P},t)\mathcal{O}(\tau)\overline{N}_\alpha(\vec{P},0)\right\rangle$$
$$C^2(t,\tau,0;\vec{P}) = \left\langle N_\alpha(\vec{P},t)\overline{N}_\alpha(\vec{P},0)\right\rangle$$

Boosted nucleon field:

$$N_\alpha(\vec{P},t) = \Gamma_{\alpha\beta}\sum_{\vec{x}} e^{-i\vec{P}\cdot\vec{x}}\epsilon^{abc}u^a_\beta(x)\left(d^{b^T}(x)\mathcal{C}\gamma_5 u^c(x)\right)$$

where $\mathcal{C} = i\gamma_0\gamma_2$ and $\Gamma = \frac{1+\gamma_4}{2}$ is the parity projector.
Operator:

$$\mathcal{O}(z,\tau) = \sum_{\vec{y}} \overline{\psi}(y+\hat{e}_3 z)\gamma_3 W_3(y+\hat{e}_3 z, y)\psi(y)$$

Wilson link: $W_j(y+z\hat{e}_j, y) = U_j(y+(z-1)\hat{e}_j)\ldots U_j(y+\hat{e}_j)U_j(y)$
Matrix element:

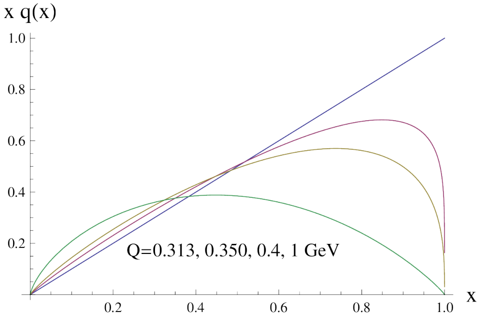
$$\frac{C^{3pt}(t,\tau,0;\vec{P})}{C^{2pt}(t,0;\vec{P})} \overset{0\ll\tau\ll t}{=} \frac{-iP_3}{\sqrt{P_3^2+M^2}} h(P_3,\Delta z)$$

Scale and evolution

QM provide non-perturbative result at a low scale Q_0

$$F(x, Q_0)|_{\rm model} = F(x, Q_0)|_{\rm QCD}\,, \quad Q_0 - \text{the matching scale}$$

Determination of Q_0 via momentum fraction: quarks carry 100% of momentum at Q_0. One adjusts Q_0 in such a way that when evolved to $Q = 2$ GeV, the quarks carry the experimental value of 47%



LO DGLAP evolution:

$$Q_0 = 313^{+20}_{-10} \text{ MeV}$$

[Davidson, Arriola 1995]:

$$q(x; Q_0) = 1$$

CCFM

[for a summary see Golec-Biernat et al. 2007]:

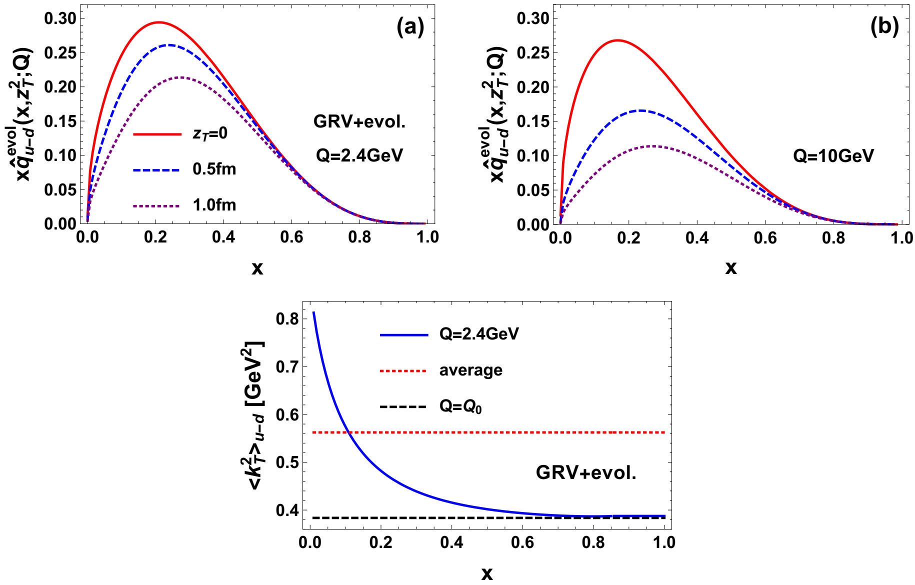
CCFM:

- for the unintegrated gluon distribution
- all-loop approximation – angular ordering (coherence) for both large and small values of x
- a new non-Sudakov form factor that sums virtual corrections for small x

Kwieciński:

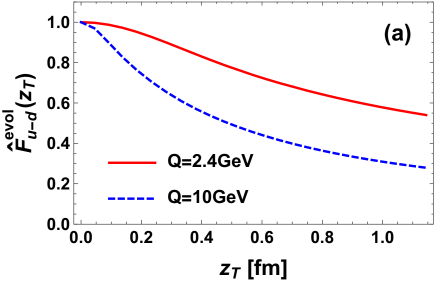
- one-loop approximation – angular ordering at small x and the corresponding virtual corrections not included
- coherence only in the real parton emissions for large x
- at small x the standard DGLAP transverse momentum ordering
- both quark and gluon unintegrated distributions

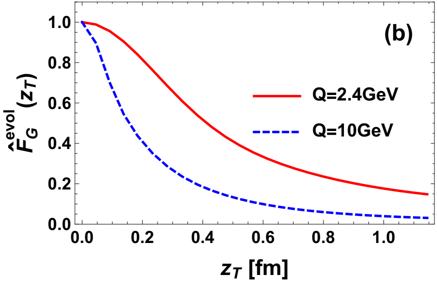
k_T-spreading



Evolution-generated form factor

$$\hat{F}^{\rm evol}(z_T;Q) = \int dx\, \hat{q}^{\rm evol}(x,z_T;Q)$$





asymptotically quarks $\sim z_T^{-4\frac{C_F}{\beta_0}\log\frac{\alpha_s(Q_0)}{\alpha_s(Q)}}$, gluons $\sim z_T^{-4\frac{N_c}{\beta_0}\log\frac{\alpha_s(Q_0)}{\alpha_s(Q)}}$
$(C_F = 4/3,\ N_c = 3,\ \beta_0 = 9)$ [WB, ERA 2004]