Size fluctuations cause p_T fluctuations

Ł. Obara, M. Chojnacki, P. Bożek, WB [UJK & IFJ PAN]

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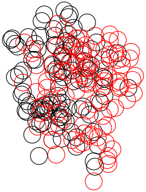
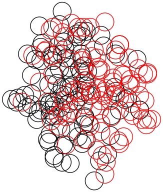
$\langle r \rangle = 2.95$ fm



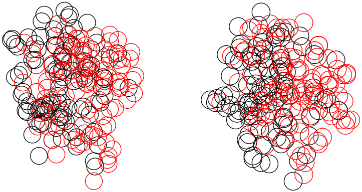


$\langle r \rangle = 2.83$ fm

$^{197}Au + {}^{197}Au,\ N_w = 198$

Basic idea

$\langle r \rangle = 2.95$ fm



$\langle r \rangle = 2.83$ fm

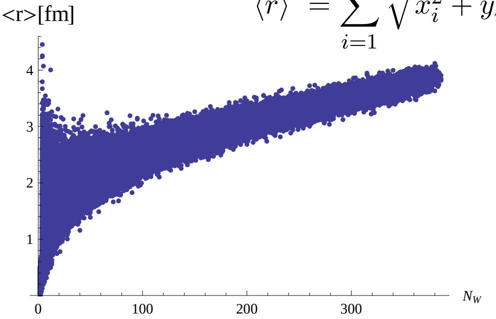
$$^{197}Au + ^{197}Au, \; N_w = 198$$

- An event with the same number of wounded nucleons N_w may have a different shape and size
- Smaller initial size $\rightarrow$ larger hydrodynamic flow $\rightarrow$ larger p_T (and vice versa)
- Thus size fluctuations cause event-by-event p_T fluctuations
- How strong? ... PRC C80:051902,2009, arXiv:0907.3216

Size fluctuations

- average transverse size in a given event:

$$\langle r \rangle = \sum_{i=1}^{N_w} \sqrt{x_i^2 + y_i^2}$$



- event-by-event average of transverse sizes at fixed N_w:

$$\langle\langle r \rangle\rangle = \frac{1}{N_{events}} \sum_{k=1}^{N_{events}} \langle r \rangle_k$$

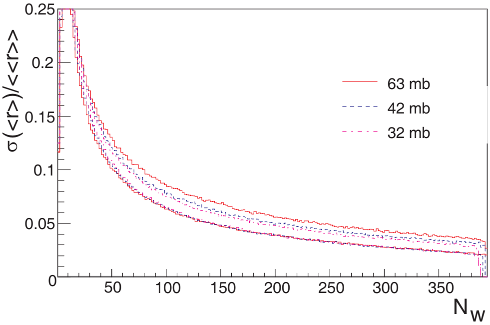
GLISSANDO

Scaled σ at fixed N_w:

$$\sigma_{scaled} = \frac{\sigma\left(\langle r\rangle\right)}{\langle\langle r\rangle\rangle}$$

bottom: wounded, top: mixed

$(N_{\rm prod} \sim \alpha N_w/2 + (1-\alpha)N_{\rm bin})$



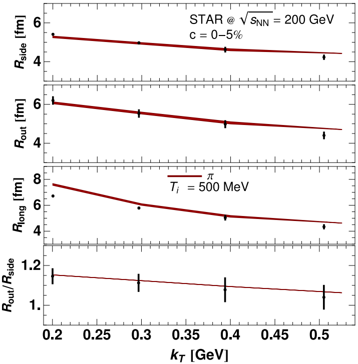
In the wounded nucleon model the σ_{scaled} is insensitive σ_{NN}, hence insensitive to the collision energy. In the mixed model some dependence comes from α, ranging from 0.12 to 0.3.

Hydrodynamics with statistical hadronization

Hydro carries over the initial size fluctuation to (observed) $\langle p_T \rangle$ fluctuations "hydrodynamic push"

- Initial state $\rightarrow$ hydrodynamics $\rightarrow$ freezeout $\rightarrow$ hadrons
- More compressed initial condition leads to a faster build-up of flow, and then higher transverse velocity at freezeout, which in turn leads to higher $\langle p_T \rangle$
- It will turn out that $\sigma(\langle p_T \rangle)/\langle\langle p_T \rangle\rangle \simeq A\sigma(\langle r \rangle)/\langle\langle r \rangle\rangle$
- We estimate the proportionality constant via simulations with Lhyquid (Chojnacki, Florkowski) and viscous hydro (Bożek and Wyskiel) and use THERMINATOR

2+1 perfect hydro (solution of the HBT puzzle)



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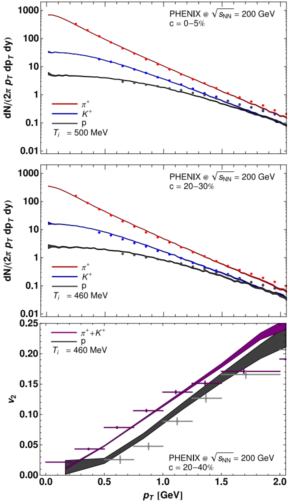
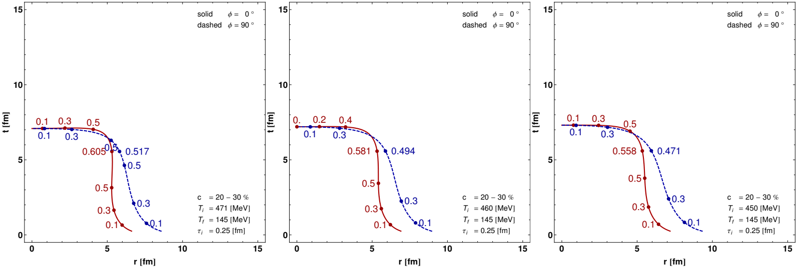


Fluctuations of the FO surface

Fluctuations of the size of the initial condition $\longrightarrow$ hydro $\longrightarrow$ fluctuations of the freezeout surface and velocity



initial source: 5% squeezed optimum 5% stretched

Event-by-event hydrodynamics our way

Instead of 1 000 000 events, just two are enough!

The distribution of the $\langle r \rangle$ (at fixed N_w) is to a very good approximation Gaussian:

$$f(\langle r \rangle) \sim \exp\left(-\frac{(\langle r \rangle - \langle\langle r \rangle\rangle)^2}{2\sigma^2(\langle r \rangle)} \right)$$

Imagine we ran simulations with fixed $\langle r \rangle$ (no size fluctuations). Then particles would have some average momentum $\bar{p}_T$. Since hydrodynamic evolution is deterministic, $\bar{p}_T$ is a (very complicated) function of $\langle r \rangle$. We can now use the Taylor expansion around $\langle\langle r \rangle\rangle$:

$$\bar{p}_T - \langle\langle p_T \rangle\rangle = \left.\frac{d\bar{p}_T}{d\langle r \rangle}\right|_{\langle r \rangle = \langle\langle r \rangle\rangle} (\langle r \rangle - \langle\langle r \rangle\rangle) + \ldots$$

The distribution of $\langle \bar{p}_T \rangle$ becomes

$$f(\bar{p}_T) \sim \exp\left(-\frac{(\bar{p}_T - \langle\langle p_T \rangle\rangle)^2}{2\sigma^2(\langle r \rangle)\left(\frac{d\bar{p}_T}{d\langle r \rangle}\right)^2} \right)$$

Dynamical fluctuations

The full statistical distribution $f(\langle p_T \rangle)$ is a folding of the statistical distribution of $\langle p_T \rangle$ at a fixed initial size, centered around a certain $\bar{p}_T$, with the distribution of $\bar{p}_T$ centered around $\langle\langle p_T \rangle\rangle$:

$$f(\langle p_T \rangle) \sim \int d^2\bar{p}_T \exp\left(-\frac{(\langle p_T \rangle - \bar{p}_T)^2}{2\sigma_{stat}^2}\right) \exp\left(-\frac{(\bar{p}_T - \langle\langle p_T \rangle\rangle)^2}{2\sigma_{dyn}^2}\right)$$

$$\sim \exp\left(-\frac{(\langle p_T \rangle - \langle\langle p_T \rangle\rangle)^2}{2\left(\sigma_{stat}^2 + \sigma_{dyn}^2\right)}\right), \text{where} \quad \sigma_{dyn} = \sigma(\langle r \rangle) \left.\frac{d\bar{p}_T}{d\langle r \rangle}\right|_{\langle r \rangle = \langle\langle r \rangle\rangle}$$

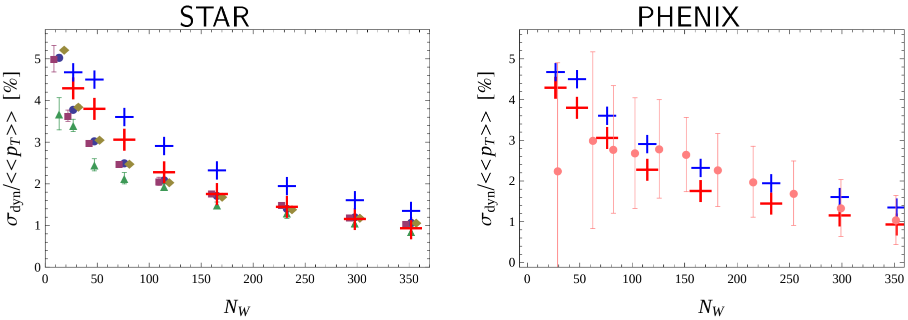
The scaled dynamical variance is

$$\frac{\sigma_{dyn}}{\langle\langle p_T \rangle\rangle} = \frac{\sigma(\langle r \rangle)}{\langle\langle r \rangle\rangle} \frac{\langle\langle r \rangle\rangle}{\langle\langle p_T \rangle\rangle} \left.\frac{d\bar{p}_T}{d\langle r \rangle}\right|_{\langle r \rangle = \langle\langle r \rangle\rangle}$$

Other measures

The whole thing can be done with other measures of fluctuations, *e.g.*, Φ_{p_T}, F_{p_T}, ...

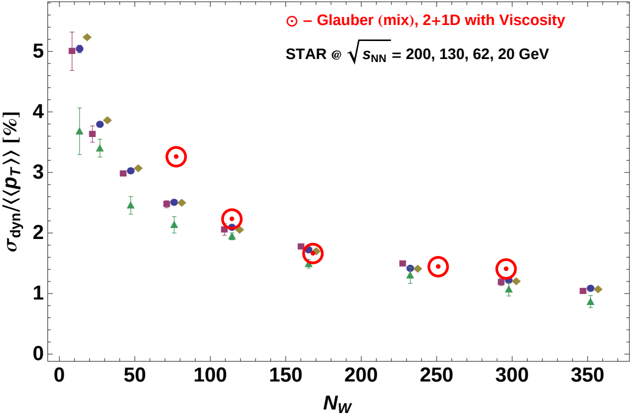
Results



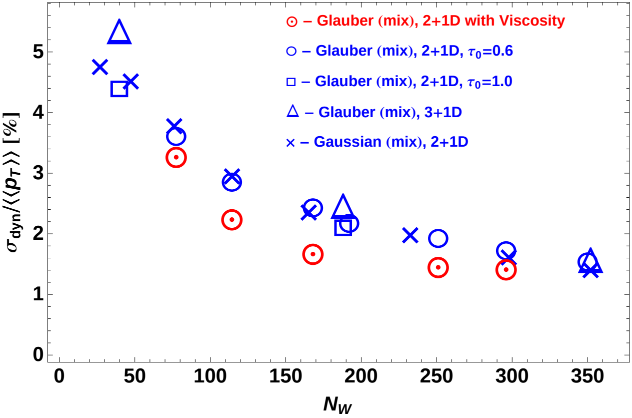
- wounded-nucleon model (red crosses)
  mixed model (blue crosses)
- mixed model overshoots the data by 20% which can perhaps be reduced with weake hydro push (*e.g.* viscosity, 3+1)
- proper centrality dependence is approx. reproduced: $\sigma_{dyn}/\langle\langle p_T\rangle\rangle \sim 1/\sqrt{N_W}$

Viscous hydro

[Bożek & Wyskiel 2009]



Various hydro calculations



Connection to EoS

Scaled standar deviation of $\langle p_T \rangle$ is connected to thermodynamic properties (Ollitrault '91)

$$\frac{\sigma_{dyn}}{\langle\langle p_T \rangle\rangle} = \frac{P}{\varepsilon}\frac{\sigma(\langle s \rangle)}{\langle\langle s \rangle\rangle} = 2\frac{P}{\varepsilon}\frac{\sigma(\langle r \rangle)}{\langle\langle r \rangle\rangle}$$

s – entropy density, ε – energy density, P – pressure (last equality follows from $\langle s \rangle \sim 1/\langle r \rangle^2$)

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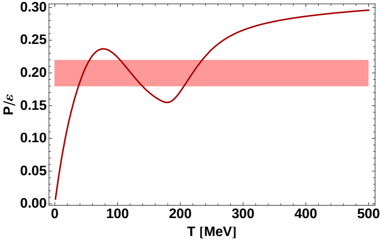

One can study average properties of the equation of state (its stiffness)

Conclusions

- A few percent fluctuations of the initial size, present in Glauber approaches, explain via hydro the experimentally observed $\langle p_T \rangle$ fluctuations.
- The effect is robust
- Proper scaling with the number of wounded nucleons – proper dependence on centrality
- A very weak dependence on the incident energy – as in the experiment
- Little left for further effects, such as minijets, clusters, *etc.*
- Viscosity helps to reduce by 20% and leads to excellent agreement with the data
- Average information on P/ε according to Ollitrault's formula