Fluktuacje v_2 – fizyka czy statystyka?*

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*based on WB+**P. Bożek**+**M. Rybczyński**, *Fluctuating initial conditions in heavy-ion collisions from the Glauber approach*, Phys. Rev. C76 (2007) 054905 [arXiv:0706.4266]

Introduction

- Elliptic flow, measure:

$$v_2 = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle}$$

- Initial shape asymmetry:

$$\epsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle x^2 \rangle + \langle y^2 \rangle}$$

- Hydro:

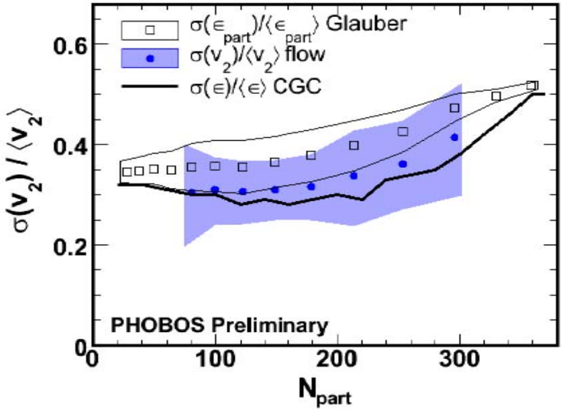
$$v_2 \sim \epsilon$$

  (linearity of perturbation)

- Event-by-event fluctuations of v_2 measured (PHOBOS, STAR)

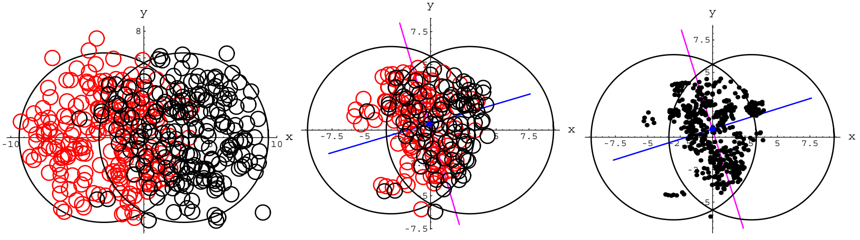
$$\Delta v_2 / v_2 \simeq \Delta\epsilon / \epsilon$$

Wosiek@WPCF'08



⟵ all end here!

A typical gold-gold event



all nucleons — wounded nucleons — binary collisions

Sizable fluctuations of the principal axes

Aguiar+Kodama+Osada+Hama 2001, Miller+Snellings 2003, PHOBOS 2005, Bhalerao+Blaizot+Borghini+Ollitrault 2005, Andrade+Grassi+Hama+Kodama+Socolowski 2006, Voloshin 2006, ...

Toy model - n points on a unit circle

uniform distribution

$f(\phi) = 1$

fixed axes (standard)

$\varepsilon = \frac{1}{n} \sum_{i=k}^{n} (y_k^2 - x_k^2) = \frac{1}{n} \sum_{k=1}^{n} \cos(2\phi_k)$

Average over (infinitely many) events: $\langle \varepsilon \rangle = 0$

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variable axes (participant)

$\varepsilon^* = \frac{1}{n} \sum_{k=1}^{n} \cos[2(\phi_k - \phi^*)]$

$$Y = \frac{1}{n} \sum_{k=1}^{n} \cos(2\phi_k), \quad X = \frac{1}{n} \sum_{k=1}^{n} \sin(2\phi_k)$$

ϕ^* : quantity $\frac{1}{n} \sum_{k=1}^{n} \cos[2(\phi_k - \phi^*)]$ maximized in each event

$\Rightarrow \quad \cos(2\phi^*) = Y/\sqrt{Y^2 + X^2}, \quad \sin(2\phi^*) = X/\sqrt{Y^2 + X^2}$

Central limit theorem

$$\varepsilon^* = \sqrt{\left(\frac{1}{n}\sum_{k=1}^{n}\cos(2\phi_k)\right)^2 + \left(\frac{1}{n}\sum_{k=1}^{n}\sin(2\phi_k)\right)^2} = \sqrt{X^2+Y^2}$$

Unusual measure!

$\langle X\rangle = \langle Y\rangle = 0, \quad \langle X^2\rangle = \langle Y^2\rangle = \frac{1}{2n}, \quad \langle XY\rangle = 0$ (uncorrelated!)

For large n the distribution of Y and X is Gaussian:

$$f(X,Y) = \frac{n}{\pi}\exp\left[-n\left(X^2+Y^2\right)\right]$$

$Y = q\cos\alpha,\ X = q\sin\alpha, \quad f(q,\alpha) = \frac{n}{\pi}\exp\left[-nq^2\right]$

Moments:

$$\begin{aligned}
\int_0^{2\pi} d\alpha \int_0^{\infty} q\,dq\, f(q,\alpha) &= 1 \\
\int_0^{2\pi} d\alpha \int_0^{\infty} q\,dq\, q f(q,\alpha) &= \sqrt{\frac{\pi}{4n}} \equiv \langle \varepsilon^* \rangle \\
\int_0^{2\pi} d\alpha \int_0^{\infty} q\,dq\, q^2 f(q,\alpha) &= \frac{1}{n} \equiv \langle \varepsilon^{*2} \rangle
\end{aligned}$$

$$\mathrm{var}(\varepsilon^*) = \frac{1}{n}\left(1 - \frac{\pi}{4}\right)$$

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$$\mathrm{var}(\varepsilon^*) = \frac{1}{n}\left(1 - \frac{\pi}{4}\right)$$

$$\frac{\sigma(\varepsilon^*)}{\langle \varepsilon^* \rangle} = \sqrt{\frac{4}{\pi} - 1} \simeq 0.52$$

Independent of n (for large n, *i.e.* in practice $n > 5$)

Toy model, $\varepsilon \neq 0$

non-uniform distribution

$f(\phi) = 1 + 2\epsilon \cos(2\phi)$

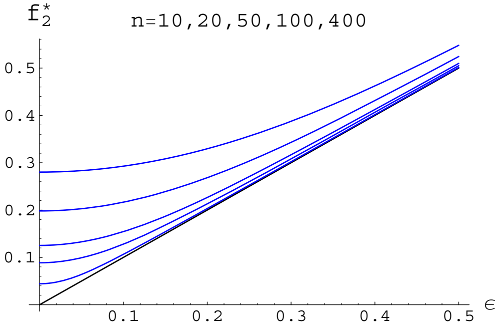
$$\varepsilon^* = \frac{1-2\epsilon^2}{\sqrt{n\pi}} \sum_{j=0}^{\infty} \left(2\epsilon^2\right)^j \frac{\Gamma(j+\frac{1}{2})\Gamma(j+\frac{3}{2})}{j!^2} {}_1F_1\left(-\frac{1}{2}, j+1; -\frac{n\epsilon^2}{1-2\epsilon^2}\right)$$

$$\langle \varepsilon^{*2} \rangle = \frac{1+(n-1)\epsilon^2}{n}$$

limit of $\epsilon = 0$

$$\varepsilon^*(\epsilon=0) = \frac{\sqrt{\pi}}{2\sqrt{n}}, \quad \frac{\sigma(\varepsilon^*)}{\varepsilon^*} = \sqrt{\frac{4}{\pi} - 1}$$

ε^* as a function of ε (toy model)



ε^* in the general two-dimensional case

(under the assumption of no correlations of locations of sources)

$$\varepsilon^* = \frac{\sqrt{2}\sigma_Y^2}{I_{k,0}\sqrt{\pi}\sigma_X} \sum_{j=0}^{\infty} (2\delta\sigma_Y^2)^j \frac{\Gamma\left(j+\frac{1}{2}\right)\Gamma\left(j+\frac{3}{2}\right)\ {}_1F_1\left(-\frac{1}{2}; j+1; -\frac{\bar{Y}^2}{2\sigma_Y^2}\right)}{j!^2}$$

$$\bar{Y} = I_{2,2},\ \sigma_Y^2 = \frac{1}{2n}(I_{4,0} - 2I_{2,2}^2 + I_{4,4}),\ \sigma_X^2 = \frac{1}{2n}(I_{4,0} - I_{4,4}),$$

$$\delta = \frac{1}{2\sigma_Y^2} - \frac{1}{2\sigma_X^2},\quad I_{k,l} = \int \rho d\rho f_l(\rho)\rho^k / \int \rho d\rho f_0(\rho)$$

at $b = 0$ very simple results (independent of A, energy, model, ...)

$$\varepsilon^* = \frac{\sqrt{\pi I_{4,0}}}{2I_{2,0}\sqrt{n}},\quad \frac{\Delta\varepsilon^*}{\varepsilon^*} = \sqrt{\frac{4}{\pi} - 1},\quad f_2^*(\rho) = \frac{1}{2}\sqrt{\frac{\pi}{nI_{2k,0}}}\,\rho^k f_0(\rho)$$

Glauber-like models tested, GLISSANDO

- wounded nucleons, $\sigma_w = 42$ mb, $d = 0.4$ fm

one goal:

compare various Glauber-like models

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- hot spots: $\sigma_w = 42$ mb, $\sigma_{\rm bin} = 0.5$ mb. When a rare binary collision occurs it produces on the average a large amount of the transverse energy $= 14.5\% \times \sigma_{\rm w}/\sigma_{\rm bin}$

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- hot spots + Γ: Sources may deposit the transverse energy with a certain probability distribution. We superimpose the Γ distribution with $\kappa = 0.5$ over the distribution of sources,

$$g(w, \kappa) = w^{\kappa-1} \kappa^{\kappa} \exp(-\kappa w)/\Gamma(\kappa),$$

where $\bar{w} = 1$ and $\mathrm{var}(w) = 1/\kappa$

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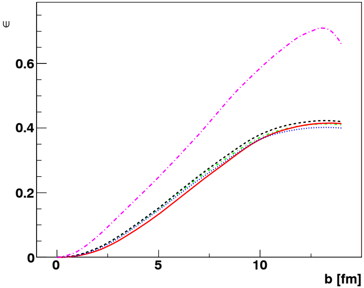
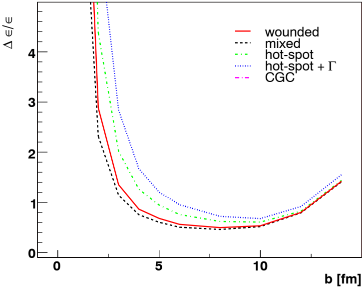
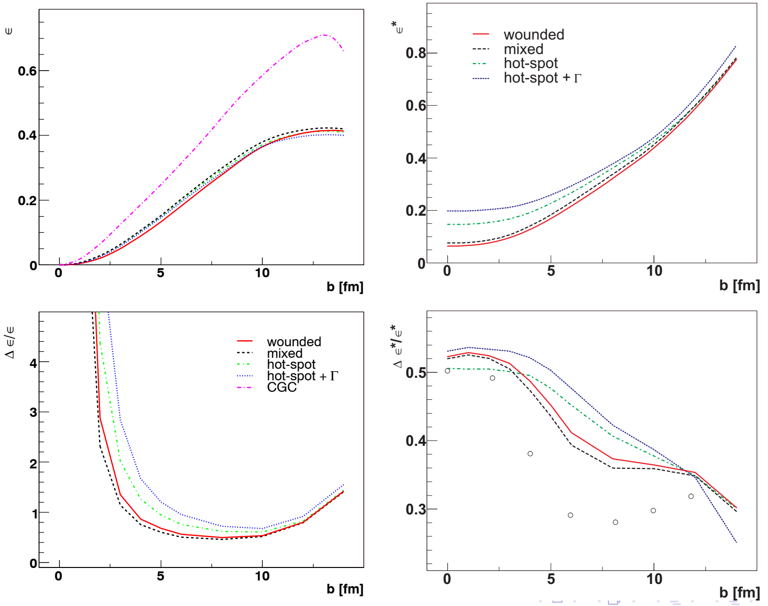

ϵ
0.6
0.4
0.2
0
0
5
10
b [fm]


Δ ϵ/ϵ
4
3
2
1
0
0
5
10
b [fm]
wounded
mixed
hot-spot
hot-spot + Γ
CGC

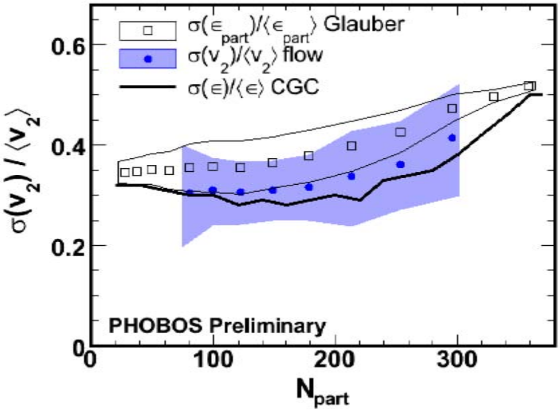

ε
0.6
0.4
0.2
0
0
5
10
b [fm]
ε*
0.8
0.6
0.4
0.2
0
wounded
mixed
hot-spot
hot-spot + Γ
0
5
10
b [fm]
Δ ε/ε
4
3
2
1
0
wounded
mixed
hot-spot
hot-spot + Γ
CGC
0
5
10
b [fm]
Δ ε*/ε*
0.5
0.4
0.3
0
5
10
b [fm]

Event-by-event fluctuations of v_2

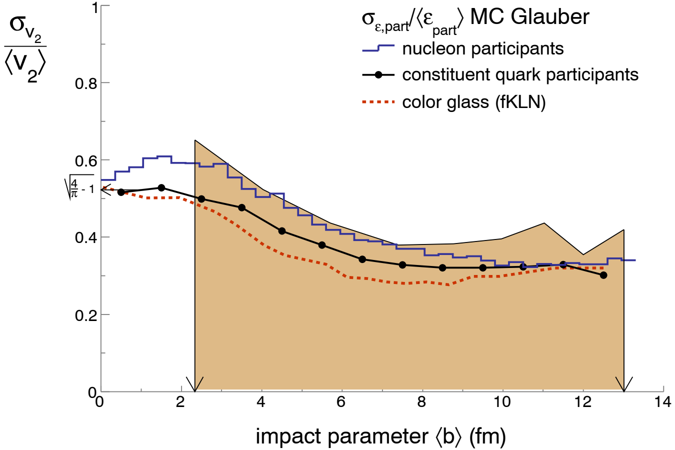
At low azimuthal asymmetry one expects on hydrodynamical grounds

$$\frac{\Delta v_2^*}{v_2^*} = \frac{\Delta \varepsilon^*}{\varepsilon^*}$$

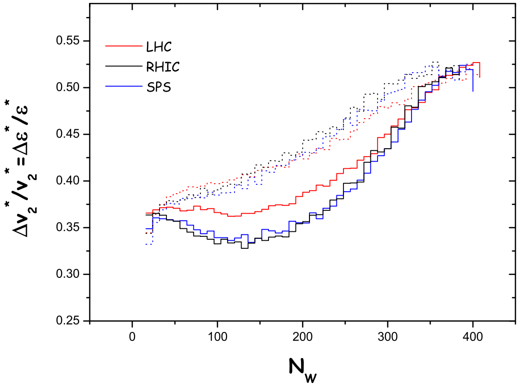
$$\frac{\Delta v_2^*}{v_2^*}(b=0) \simeq \frac{\Delta \varepsilon^*}{\varepsilon^*}(b=0) \simeq \sqrt{\frac{4}{\pi} - 1} \simeq 0.52$$



hydro: $\Delta\varepsilon^*/\varepsilon^* \simeq \Delta v_2^*/v_2^*$(Au+Au, central)$\simeq 0.5$



[Sorensen, STAR]



from SPS (32mb), through RHIC (42mb) to LHC (63mb)

solid - wounded, dashed - hot-spot+Γ [Rybczyński, Epiphany'08]

Summary

- Analytic formulas explain why at $b = 0$ we have $\Delta\varepsilon^*/\varepsilon^* \simeq 0.52$, insensitive of the model used or the mass number of the colliding nuclei
- At $b > 0$ some sensitivity of $\Delta\varepsilon^*/\varepsilon^*$ on the model, small sensitivity on collision energy
- CGS [Drescher+Nara] has lower ε^* fluctuations than Glauber
- Comparison to data shows that the Glauber-like models leave little or no room for an increase of fluctuations during the (pre-hydro) evolution
- Not discussed: $\Delta v_4^*/v_4^* = 2\Delta v_2^*/v_2^*$