

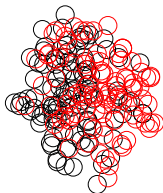
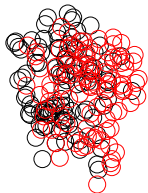
Fluktuacje rozmiaru a fluktuacje pędu

Łukasz Obara, Mikołaj Chojnacki, WB

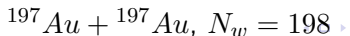
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$$\langle r \rangle = 2.95 \text{ fm}$$

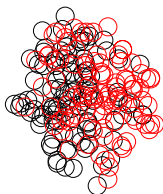
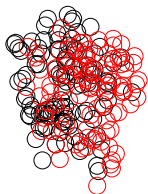


$$\langle r \rangle = 2.83 \text{ fm}$$



Basic idea

$$\langle r \rangle = 2.95 \text{ fm}$$



$$\langle r \rangle = 2.83 \text{ fm}$$

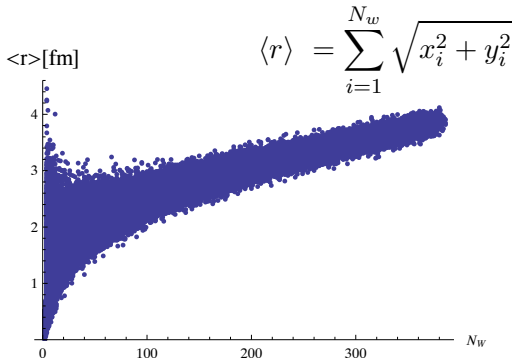


- An event with the same number of wounded nucleons N_w may have a different shape and **size**
- Smaller initial size \rightarrow larger hydrodynamic flow \rightarrow larger p_T (and vice versa)
- Thus size fluctuations cause event-by-event p_T fluctuations
- How strong? ...

[arXiv:0907.3216 \[nucl-th\]](https://arxiv.org/abs/0907.3216)

Size fluctuations

- average transverse size in a given event:



- event-by-event average of transverse sizes at fixed N_w :

$$\langle\langle r \rangle\rangle = \frac{1}{N_{events}} \sum_{k=1}^{N_{events}} \langle r \rangle_k$$

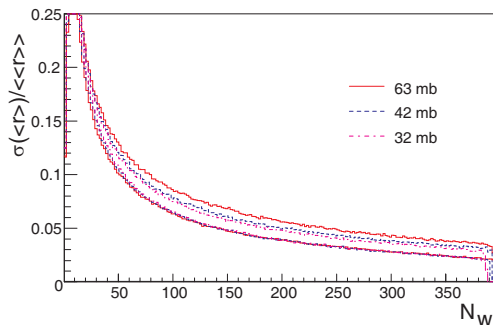
GLISSANDO

Scaled σ at fixed N_w :

$$\sigma_{scaled} = \frac{\sigma(\langle r \rangle)}{\langle \langle r \rangle \rangle}$$

bottom: wounded, top: mixed

$$(N_{prod} \sim \alpha N_w / 2 + (1 - \alpha) N_{bin})$$



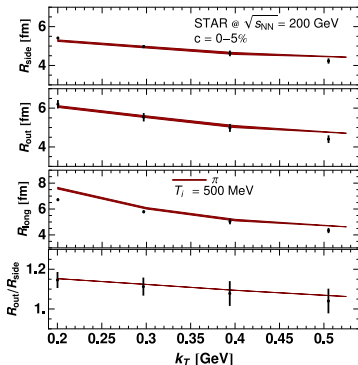
In the wounded nucleon model the σ_{scaled} is insensitive σ_{NN} , hence **insensitive to the collision energy**. In the mixed model some dependence comes from α , ranging from 0.12 to 0.3.

Hydrodynamics with statistical hadronization

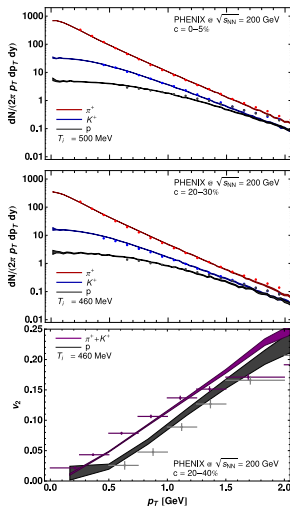
Hydro carries over the initial size fluctuation to (observed) $\langle p_T \rangle$ fluctuations “hydrodynamic push”

- Initial state \rightarrow hydrodynamics \rightarrow freezeout \rightarrow hadrons
- More **compressed** initial condition leads to a **faster build-up of flow**, and then **higher transverse velocity** at freezeout, which in turn leads to **higher $\langle p_T \rangle$**
- $\sigma(\langle p_T \rangle) / \langle \langle p_T \rangle \rangle \simeq A \sigma(\langle r \rangle) / \langle \langle r \rangle \rangle$
- We estimate the proportionality constant via simulations with **Lhyquid** (Chojnacki, Florkowski) and **THERMINATOR**

2+1 perfect hydro (solution of the HBT puzzle)

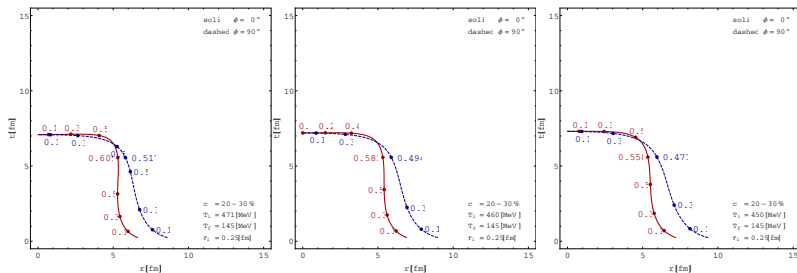


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Fluctuations of the FO surface

Fluctuations of the size of the initial condition \rightarrow hydro \rightarrow fluctuations of the freezeout surface and velocity



initial source: 5% squeezed

optimum

5% stretched

Event-by-event hydrodynamics our way

Instead of 1 000 000 events, just **two** are enough!

The distribution of the $\langle r \rangle$ (at fixed N_w) is to a very good approximation Gaussian:

$$f(\langle r \rangle) \sim \exp \left(-\frac{(\langle r \rangle - \langle \langle r \rangle \rangle)^2}{2\sigma^2(\langle r \rangle)} \right)$$

Imagine we ran simulations with fixed $\langle r \rangle$ (no size fluctuations). Then particles would have some average momentum \bar{p}_T . Since hydrodynamic evolution is **deterministic**, \bar{p}_T is a (very complicated) function of $\langle r \rangle$. We can now use the Taylor expansion around $\langle \langle r \rangle \rangle$:

$$\bar{p}_T - \langle \langle p_T \rangle \rangle = \left. \frac{d\bar{p}_T}{d\langle r \rangle} \right|_{\langle r \rangle = \langle \langle r \rangle \rangle} (\langle r \rangle - \langle \langle r \rangle \rangle) + \dots$$

The distribution of $\langle \bar{p}_T \rangle$ becomes

$$f(\bar{p}_T) \sim \exp \left(-\frac{(\bar{p}_T - \langle \langle p_T \rangle \rangle)^2}{2\sigma^2(\langle r \rangle) \left(\frac{d\bar{p}_T}{d\langle r \rangle} \right)^2} \right)$$

Dynamical fluctuations

The full statistical distribution $f(\langle p_T \rangle)$ is a folding of the statistical distribution of $\langle p_T \rangle$ at a fixed initial size, centered around a certain \bar{p}_T , with the distribution of \bar{p}_T centered around $\langle\langle p_T \rangle\rangle$:

$$f(\langle p_T \rangle) \sim \int d^2 \bar{p}_T \exp\left(-\frac{(\langle p_T \rangle - \bar{p}_T)^2}{2\sigma_{stat}^2}\right) \exp\left(-\frac{(\bar{p}_T - \langle\langle p_T \rangle\rangle)^2}{2\sigma_{dyn}^2}\right)$$

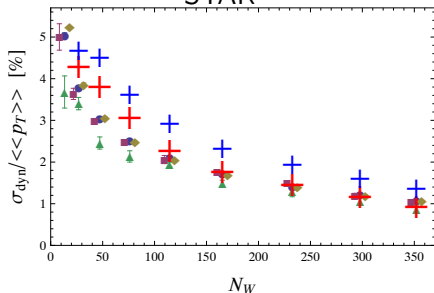
$$\sim \exp\left(-\frac{(\langle p_T \rangle - \langle\langle p_T \rangle\rangle)^2}{2(\sigma_{stat}^2 + \sigma_{dyn}^2)}\right), \text{ where } \sigma_{dyn} = \sigma(\langle r \rangle) \left. \frac{d\bar{p}_T}{d\langle r \rangle} \right|_{\langle r \rangle = \langle\langle r \rangle\rangle}$$

The scaled dynamical variance is

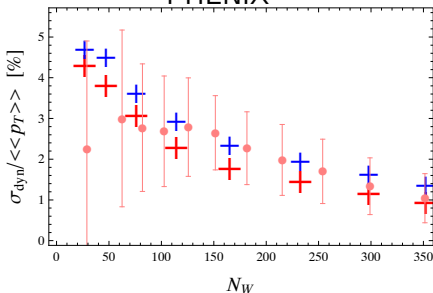
$$\frac{\sigma_{dyn}}{\langle\langle p_T \rangle\rangle} = \frac{\sigma(\langle r \rangle)}{\langle\langle r \rangle\rangle} \frac{\langle\langle r \rangle\rangle}{\langle\langle p_T \rangle\rangle} \left. \frac{d\bar{p}_T}{d\langle r \rangle} \right|_{\langle r \rangle = \langle\langle r \rangle\rangle}$$

Results

STAR



PHENIX



- wounded-nucleon model (red crosses)
- mixed model (blue crosses)
- mixed model overshoots the data by 20% which can perhaps be reduced with weaker hydro push (e.g. viscosity, 3+1)
- proper centrality dependence is approx. reproduced:

$$\sigma_{\text{dyn}}/\langle\langle p_T \rangle\rangle \sim 1/\sqrt{N_W}$$

Connection to EoS

Scaled standard deviation of $\langle p_T \rangle$ is connected to thermodynamic properties (Ollitrault '91)

$$\frac{\sigma_{dyn}}{\langle \langle p_T \rangle \rangle} = \frac{P \sigma(\langle s \rangle)}{\varepsilon \langle \langle s \rangle \rangle} = 2 \frac{P}{\varepsilon} \frac{\sigma(\langle r \rangle)}{\langle \langle r \rangle \rangle}$$

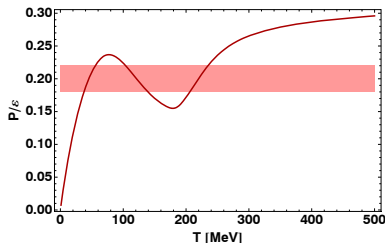
s – entropy density, ε – energy density, P – pressure (last equality follows from $\langle s \rangle \sim 1/\langle r \rangle^2$)

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One can study average properties of the equation of state (its stiffness) that way

Conclusions

- A few percent fluctuations of the initial size, present in Glauber approaches, explain in a natural way the experimentally observed $\langle p_T \rangle$ fluctuations
- Proper scaling with the number of wounded nucleons $\sigma_{dyn}/\langle\langle p_T \rangle\rangle \sim 1/\sqrt{N_W}$ – proper dependence on centrality
- A very weak dependence on the incident energy – as in experiments
- Our $\langle p_T \rangle$ fluctuations should be considered as “background” for studying further effects, such as minijets, clusters, temperature fluctuations, etc.
- Average information on P/ε according to Ollitrault’s formula