Hollowness in pp scattering at the LHC

Wojciech Broniowski

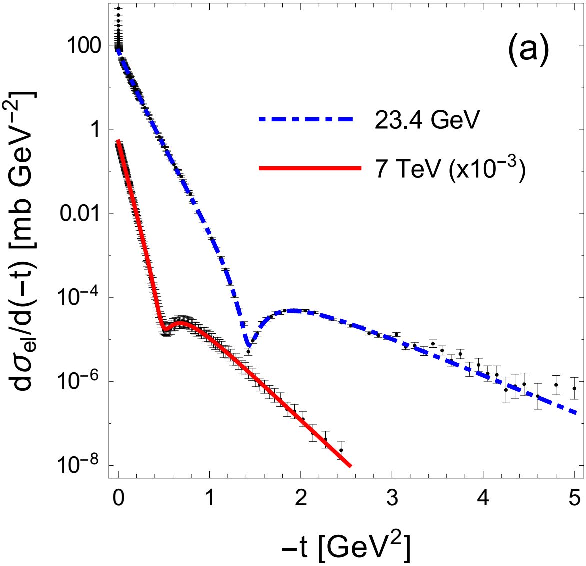
Jan Kochanowski U.
Institute of Nuclear Physics PAN

Various Faces of QCD 2
NCBJ, Świerk, 8-9 October 2016

Research with **Enrique Ruiz Arriola**

Based on [arXiv:1609.05597]

From ISR to TOTEM



Parametrization of the scattering amplitude

Parametrization by [Fagundes 2013], based on [Barger-Phillips 1974], motivated by the Regge asymptotics:

$$\frac{f(s,t)}{p} = \sum_n c_n(s) F_n(t) s^{\alpha_n(t)} = \frac{i\sqrt{A}e^{\frac{Bt}{2}}}{\left(1-\frac{t}{t_0}\right)^4} + i\sqrt{C}e^{\frac{Dt}{2}+i\phi}$$

s-dependent (real) parameters are fitted separately to all known differential pp cross sections for $\sqrt{s} = 23.4$, 30.5, 44.6, 52.8, 62.0, and 7000 GeV with $\chi^2/\mathrm{d.o.f} \sim 1.2 - 1.7$

$$\frac{d\sigma_{\rm el}}{dt} = \frac{\pi}{p^2}|f(s,t)|^2, \quad \sigma_T = \frac{4\pi}{p}\mathrm{Im} f(s,0)$$

Eikonal approximation

$$
\begin{aligned}
f(s,t) &= \sum_{l=0}^{\infty}(2l+1) f_l(p) P_l(\cos\theta) \\
&= \frac{p^2}{\pi}\int d^2b\, h(\vec{b},s)\, e^{i\vec{q}\cdot\vec{b}} = 2p^2 \int_0^{\infty} bdbJ_0(bq)h(b,s)
\end{aligned}
$$

$t=-\vec{q}^2$, $q=2p\sin(\theta/2)$, $bp = l+1/2+\mathcal{O}(s^{-1})$, $P_l(\cos\theta)\to J_0(qb)$,
hence the amplitude in the impact-parameter representation becomes

$$
h(b,s) = \frac{i}{2p}\left[1-e^{i\chi(b)}\right] = f_l(p) + \mathcal{O}(s^{-1})
$$

The eikonal approximation works well for $b < 2\ \mathrm{fm}$ and $\sqrt{s} > 20\ \mathrm{GeV}$

Procedure: $f(s,t)\to h(b,s)\to \chi(b)$...

Eikonal approximation 2

The standard formulas for the total, elastic, and total inelastic cross sections read

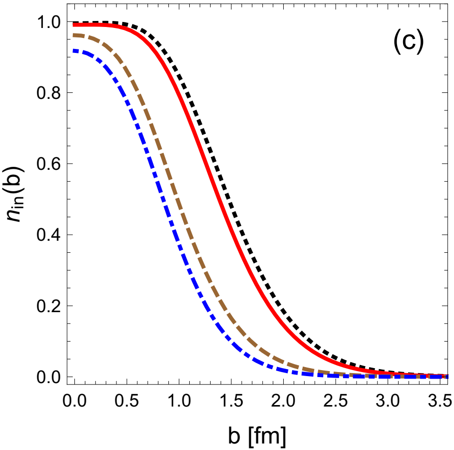
$$\begin{aligned}
\sigma_T &= \frac{4\pi}{p}\mathrm{Im} f(s,0) = 4p\int d^2b \mathrm{Im} h(\vec{b},s) = 2\int d^2b\left[1-\mathrm{Re}\, e^{i\chi(b)}\right] \\
\sigma_{\mathrm{el}} &= \int d\Omega |f(s,t)|^2 = 4p^2\int d^2b |h(\vec{b},s)|^2 = \int d^2b |1-e^{i\chi(b)}|^2 \\
\sigma_{\mathrm{in}} &\equiv \sigma_T - \sigma_{\mathrm{el}} = \int d^2b n_{\mathrm{in}}(b) = \int d^2b\left[1-e^{-2\mathrm{Im}\chi(b)}\right]
\end{aligned}$$

The inelasticity profile

$$n_{\mathrm{in}}(b) = 4p\mathrm{Im} h(b,s) - 4p^2|h(b,s)|^2$$

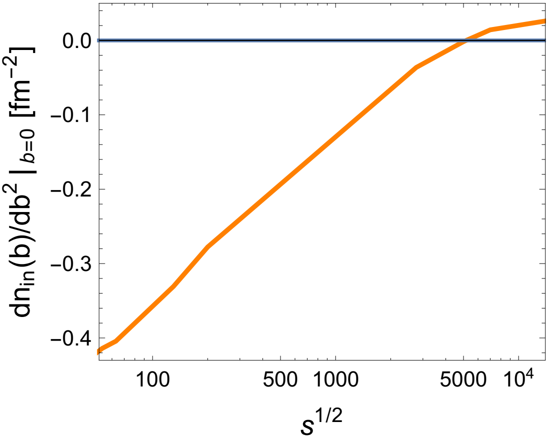
satisfies $n_{\mathrm{in}}(b) \leq 1$ (unitarity)

Dip in the inelasticity profile



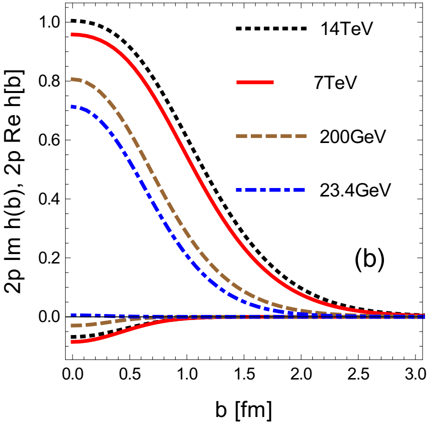
From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV

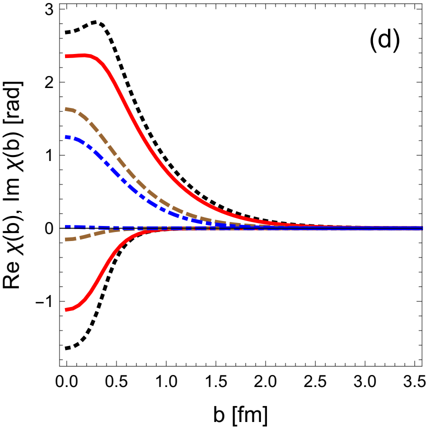
Slope of the inelasticity profile



Transition around $\sqrt{s} = 5$ TeV

Amplitude and eikonal phase



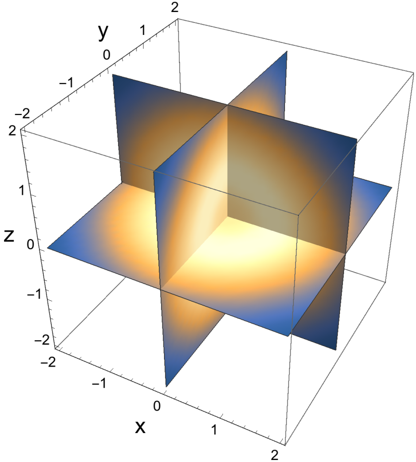


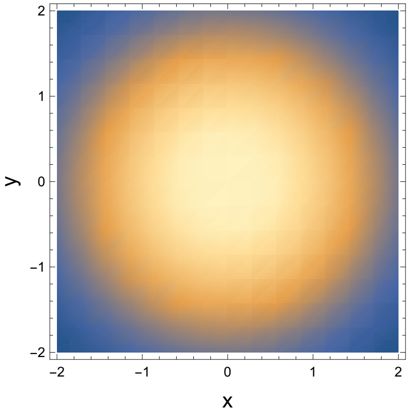
$$2p\,h(b) = i\left[1 - e^{i\chi(b)}\right]$$

(top curves - Im, bottom - Re)

2D vs 3D opacity

Projection of 3D on 2D covers up the hollow: $f(x,y,z)$ vs $\int_{-\infty}^{\infty} dz\, f(x,y,z)$





The hollow is covered up

Optical potential

Phenomenological optical potential introduced by [Allen, Payne, Polyzou 2000] via the total squared mass operator for the pp system:

$$\mathcal{M}^2 = P^\mu P_\mu \stackrel{CM}{=} 4(p^2 + M_N^2) + \mathcal{V}$$

P^μ – total four-momentum, p – CM momentum of each nucleon, M_N – nucleon mass, $\mathcal{V}$ – invariant interaction, determined in the CM frame by matching in the non-relativistic limit to a non-relativistic potential, i.e., $\mathcal{V} = 4U = 4M_N V$
The prescription transforms the relativistic Schrödinger equation $\hat{\mathcal{M}}^2\Psi = s\Psi$, into an equivalent non-relativistic Schrödinger equation

$$(-\nabla^2 + U)\Psi = (s/4 - M_N^2)\Psi$$

with the reduced potential $U = M_N V$ (to be determined by inverse scattering)

Eikonal limit and optical potential

As in WKB $-\hbar^2\Psi = 2m(E-V)\Psi$, where $\Psi = Ae^{iS/\hbar}$

$$(\nabla S)^2 - \cancel{i\hbar\nabla^2 S} = 2m(E-V)$$

$$\nabla S/\hbar = \sqrt{p^2 - 2mV/\hbar^2}$$

In one dimension and for $k \gg$ other scales

$$S/\hbar = pz - \frac{m}{\hbar^2 p}\int_{-\infty}^{z} dz' V(z')$$

Inverse scattering and optical potential

In the eikonal approximation one has

$$\Psi(\vec{x}) = \exp\left[ipz - \frac{i}{2p}\int_{-\infty}^{z} U(\vec{b}, z')dz'\right]$$

$$\chi(b) = -\frac{1}{2p}\int_{-\infty}^{\infty} U(\sqrt{b^2+z^2})dz = -\frac{1}{p}\int_{b}^{\infty} \frac{rU(r)\,dr}{\sqrt{r^2-b^2}}$$

is the (complex) eikonal phase [Glauber 1959]. This Abel-type equation can be inverted:

$$U(r) = M_N V(r) = \frac{2p}{\pi}\int_{r}^{\infty} db \frac{\chi'(b)}{\sqrt{b^2-r^2}}$$

On-shell optical potential

From the definition of the inelastic cross section

$$\sigma_{\rm in} = -\frac{1}{p}\int d^3x \, {\rm Im}\, U(\vec{x})|\Psi(\vec{x})|^2$$

$\rightarrow$ density of inelasticity is proportional to the absorptive part of the optical potential times the square of the modulus of the wave function. One can identify the *on-shell optical potential* as

$$\mathrm{Im}\, W(\vec{x}) = \mathrm{Im}\, U(\vec{x})|\Psi(\vec{x})|^2$$

Upon z integration,

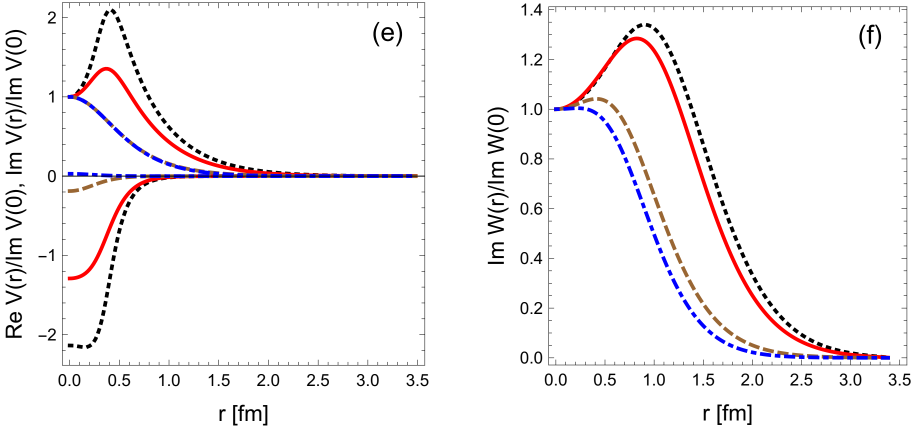
$$-\frac{1}{p}\int dz \mathrm{Im}\, W(\vec{b}, z) = n_{in}(b)$$

Inversion yields

$$\mathrm{Im} W(r) = \frac{2p}{\pi}\int_r^\infty db \frac{n'(b)}{\sqrt{b^2 - r^2}}$$

Results of inverse scattering

exp. amplitude $\to$ eikonal phase $\to U(r) = M_N V(r)$
exp. amplitude $\to$ inelasticity profile $\to W(r)$



From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV
Large dip in the absorptive parts, in $W(r)$ starts already at RHIC!

No classical folding of absorptive parts

The hollowness effect cannot be reproduced by by folding of uncorrelated proton structures. We would then get, small r

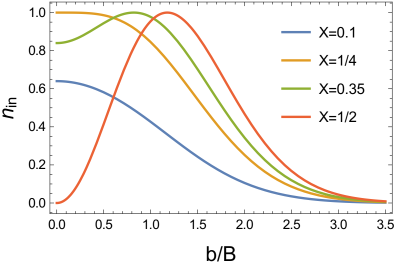
$$
\begin{aligned}
W(r) &= \int d^3y \rho(\vec{y}+\vec{r}/2)\rho(\vec{y}-\vec{r}/2) \\
&= \int d^3y \rho(\vec{y})^2 - \frac{1}{4}\int d^3y [\vec{r}\cdot\nabla\rho(\vec{y})]^2 + \ldots
\end{aligned}
$$

$\rightarrow$ $W(r)$ would necessarily have a local maximum at $r=0$, in contrast to the phenomenological result
$\rightarrow$ not possible to obtain hollowness classically by folding the absorptive parts from uncorrelated constituents

Aspects of unitarity: model of [Dremin 2014]

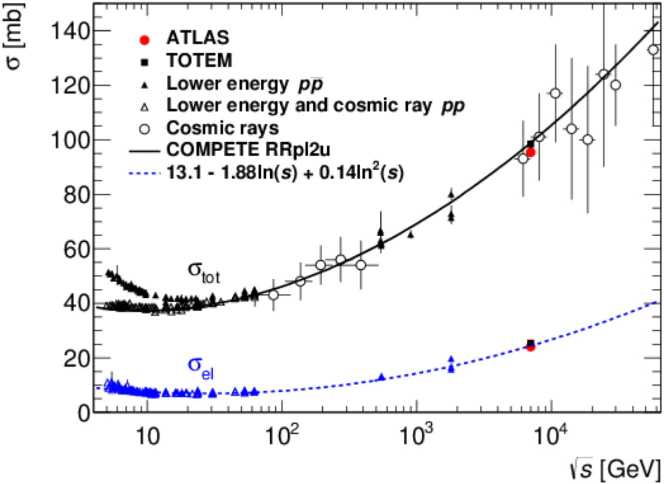
$$2p\mathrm{Im}h(b) \equiv k(b) = 4Xe^{-b^2/(2B)}, \;\; \mathrm{Re}h(b) = 0, \;\; X = \sigma_{el}/\sigma_T$$

$$n_{in}(b) = 2k(b) - k(b)^2 = 8Xe^{-b^2/(2B^2)} - 16X^2e^{-b^2/B^2}$$



- $X > 1/4$: $n_{in}(b)$ has a maximum at $b_0 = \sqrt{2}B\log(4X) > 0$, with $k(b_0) = 1$
- $X = 1/2$: black disk limit
- $W(r)$ develops a dip when $X > \sqrt{2}/8 = 0.177$

Cross sections



Ratio goes above 1/4 as energy increases!

Aspects of unitarity 2

If $2ph(b) = k(b)$ is not necessarily Gaussian but purely imaginary, then

$$n_{in}(b) = 2k(b) - k(b)^2$$

$$\frac{dn_{in}(b)}{db^2} = 2\frac{dk(b)}{db^2}[1 - k(b)]$$

hence the minimum of $n(b)$ moves away from the origin when $k(0) > 1$

The real part of the amplitude, which is $\sim 10\%$, brings in corrections at the level of 1%

Conclusions

- Hollowness in $n_{in}(b)$ inferred from the parametrization of the data
- Quantum effect, rise of $2p\mathrm{Im}h(b)$ above 1
- Not possible to obtain classically by folding the absorptive parts from uncorrelated constituents
- 2D $\rightarrow$ 3D magnifies the effect (flat in 2D means hollow in 3D)
- Interpretation via optical potential in the relativized problem
- Microscopic/dynamical explanations open [Alba Soto, Albacete 2016]
- Similar hollowness effect in low-energy n-A scattering