Production of resonance in a thermal model

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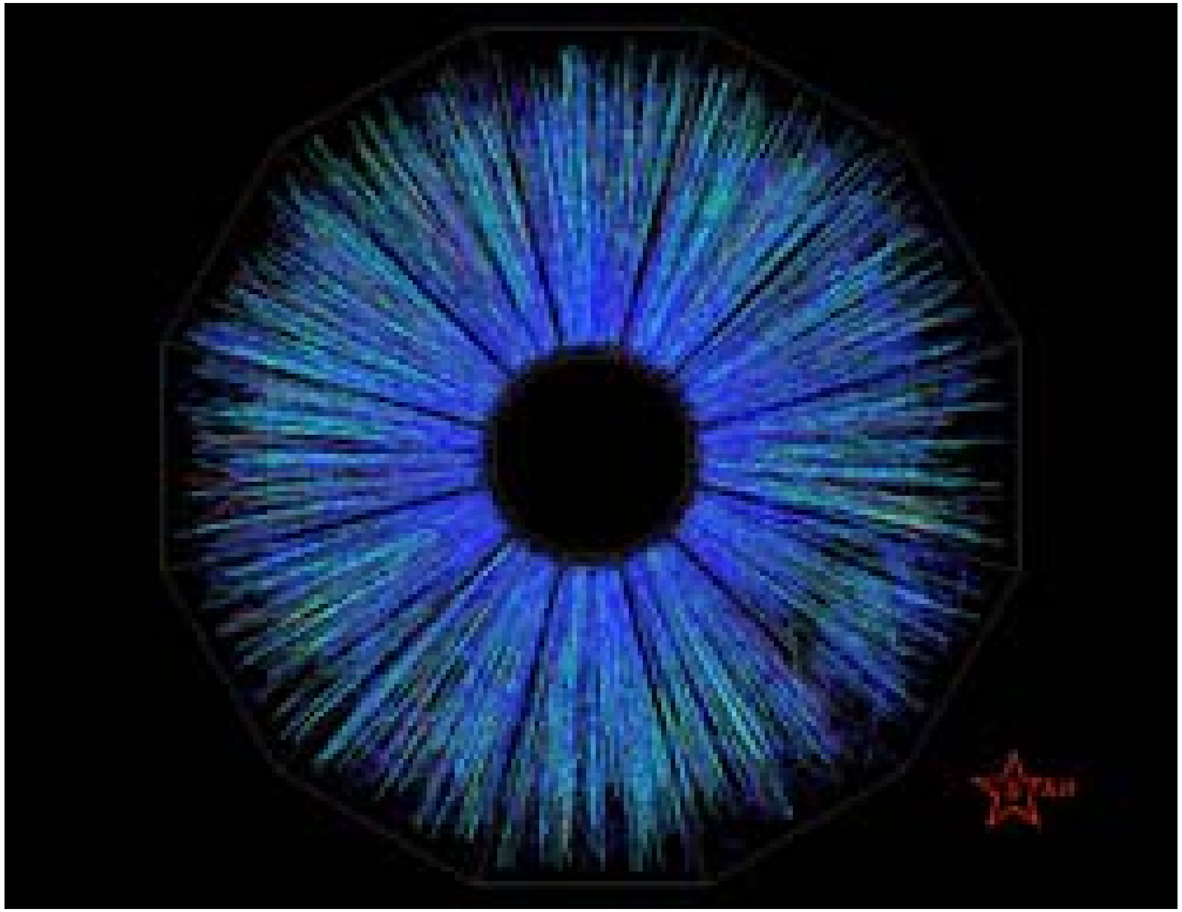
(WB+W. Florkowski+Brigitte Hiller, Phys. Rev. C **68** (2003) 034911,
Piotr Bożek+WB+WF, *Balance functions in a thermal model with resonances*, `nucl-th/0310062`)

The iris of RHIC



Motivation and scope

- RHIC is a "major data provider": soft physics, hard physics, "tomography", ...

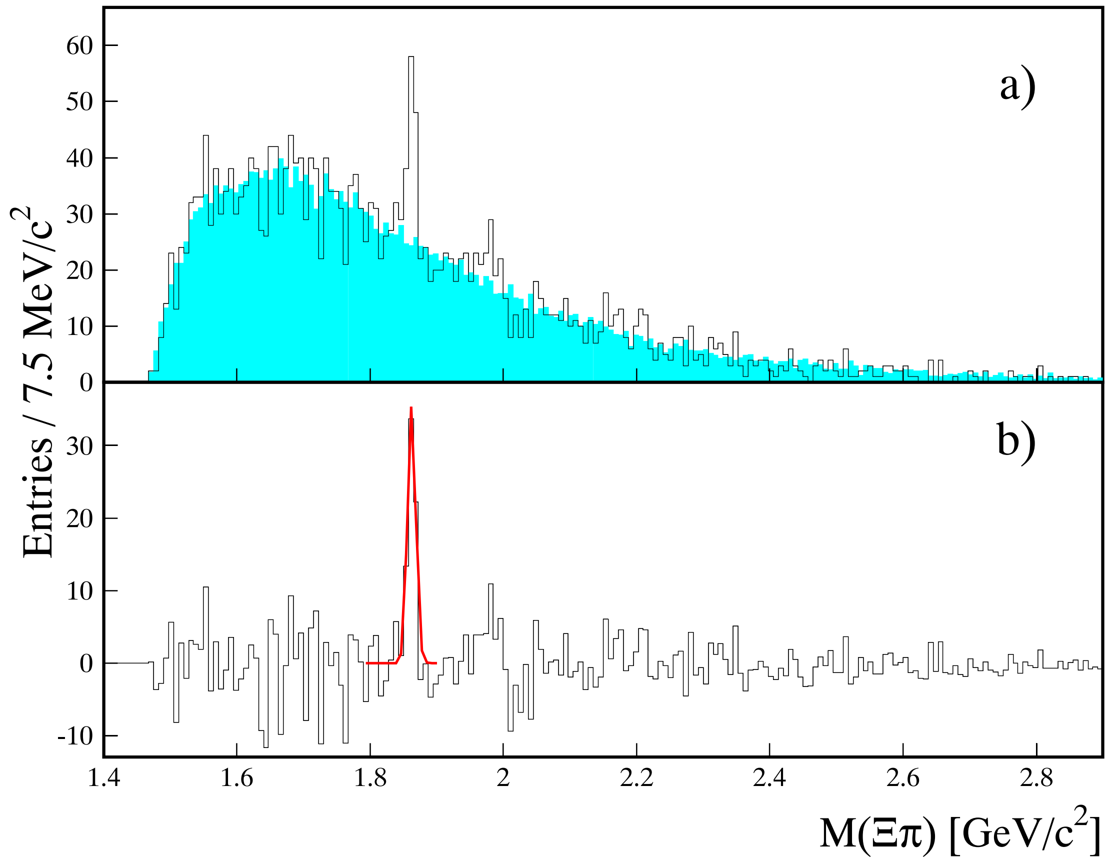
- New spectroscopy: NA49 at CERN SPS found a very narrow $\Xi^{--}_{3/2}(1862)$ in $\Xi\pi$ correlations, which is a $ddss\bar{u}$ state. Possible search of $\theta^+(1540)$, *i.e.* $uudd\bar{s}$, ...)

- Hadronic resonances are important in particle production

- Appear in measurements of correlations of identified particles ($K^*(892)$, ρ, $\Delta^{++}(1232)$, ...)

- Reveals clues on the evolution of the system formed: hadronization, duration of the hadronic phase, equation of state of hot matter, size/shape at freeze-out, degree of rescattering afterwards, medium modification of particle properties, ...

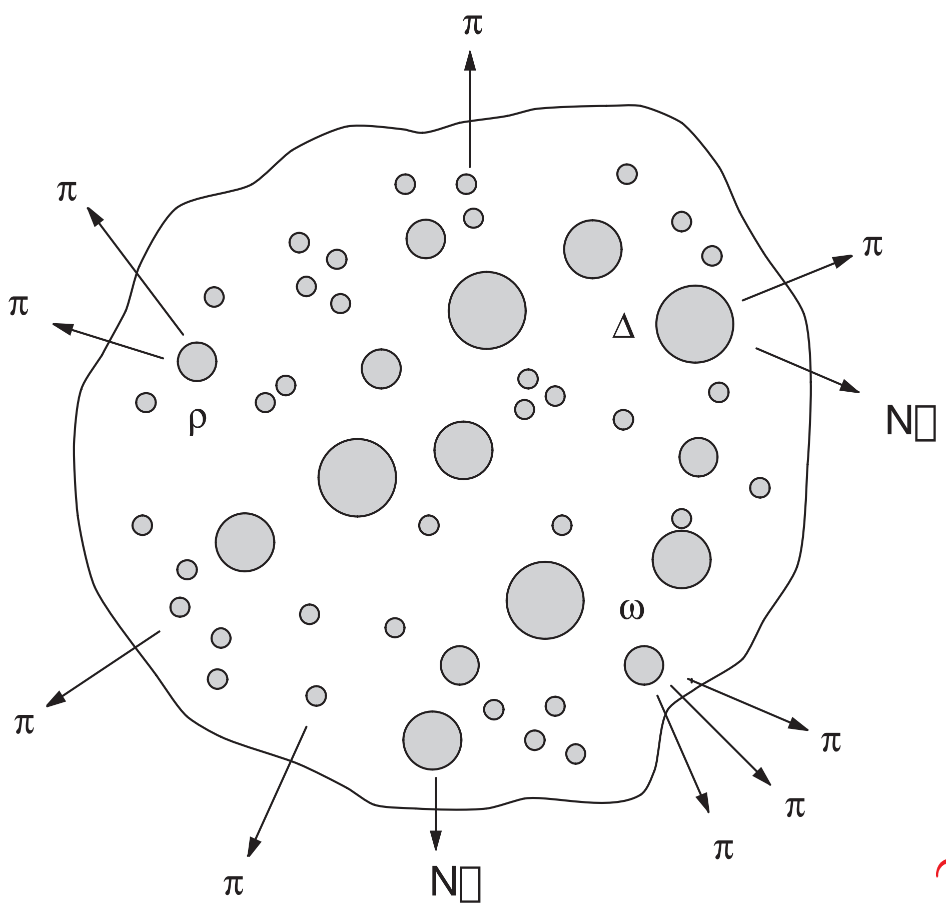
$ddss\bar{u}$



(NA49, hep-ex/0310014)

Thermal (statistical) models

Koppe (1948), Fermi (1950), Landau, Hagedorn, Rafelski, Bjorken, ...



$$\sim e^{-(E-\mu)/T}$$

WB + WF, PRL **87** (2001) 272302; PRC **65** (2002) 064905 (our variant of the model)
WB + Anna Baran + WF, Acta Phys. Pol. B **33** (2002) 4235 (review)

Our approach in a capsule

1. $T_{\rm chem} = T_{\rm kin} \equiv T$, single freeze-out (a radical simplification, supported by recent results: $R_{\rm out}/R_{\rm side} \sim 1$, $R_{\rm side}(\phi)$ has out-of-plane deformation, resonances seen abundantly)

2. Complete treatment of resonances (important due to the Hagedorn-like exponential growth of the number of states)

3. Assumed simple freezeout hypersurface (longitudinal and transverse flow)

4. 4 parameters: T,μ_B (fixed by the ratios of the particle abundances), invariant time at freeze-out τ (controls the overall normalization), transverse size $\rho_{\rm max}$ ($\rho_{\rm max}/\tau$ controls the slopes of the $p_\perp$ spectra)

5. Hubble-like flow, $u^\mu = x^\mu/\tau$ (supported by the so-called *scaling* solution to hydrodynamics)

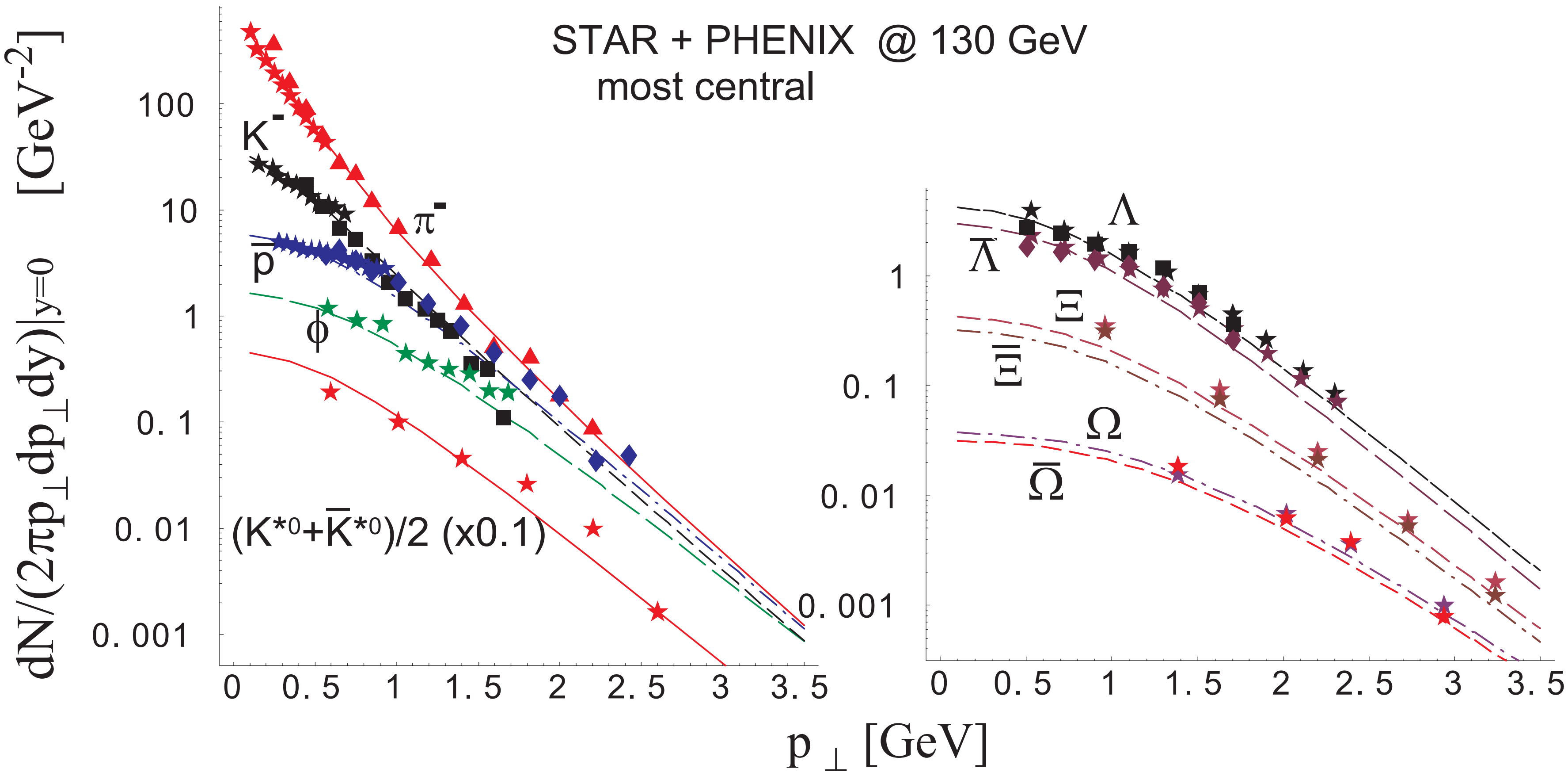
... and it works very well! $\rightarrow$

Particle ratios

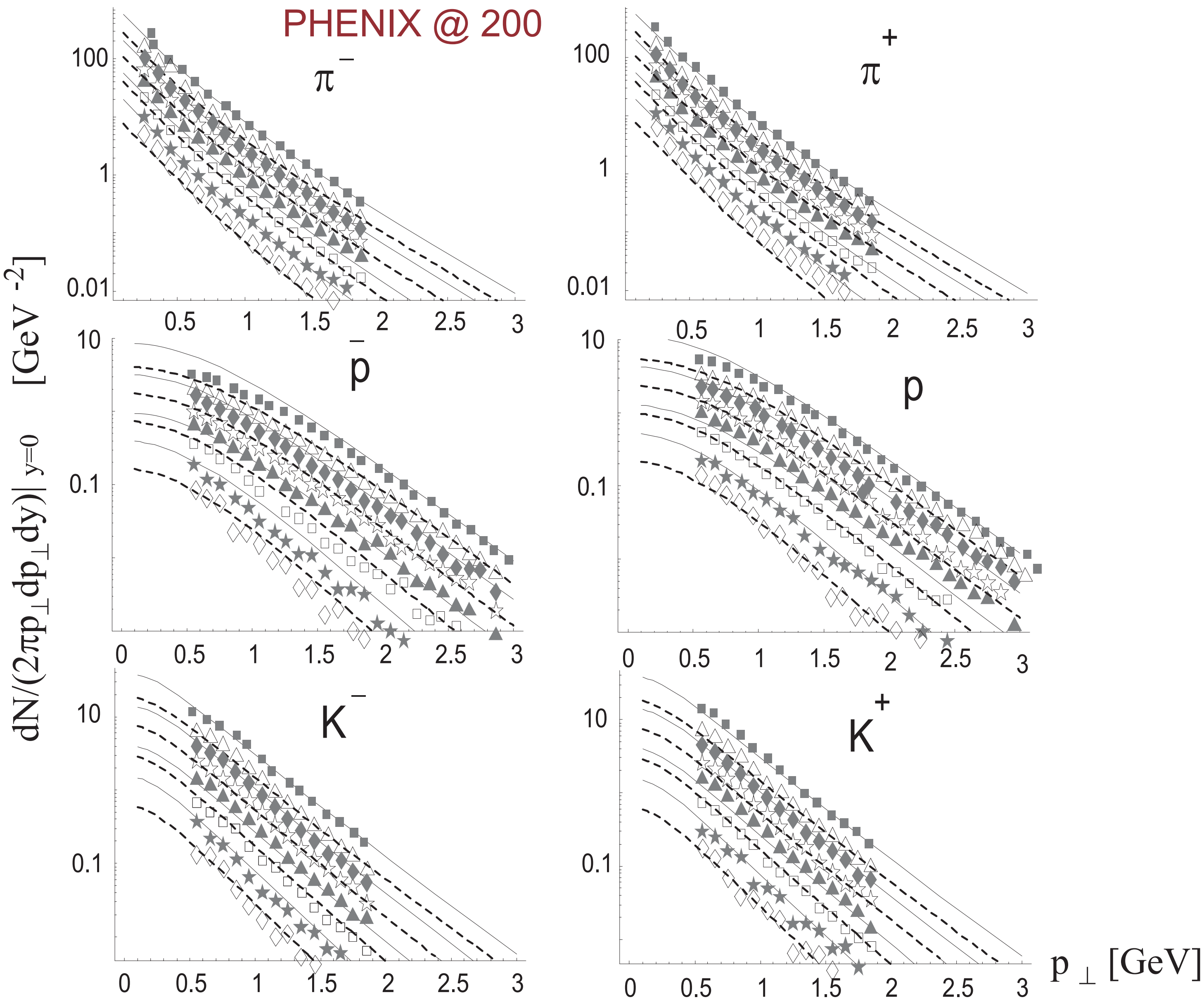
$\sqrt{s_{NN}}$ [GeV]	130	200
T [MeV]	165 ± 7	160±5
μ_B [MeV]	41 ± 5	26±4
μ_S [MeV]	9	5
μ_I [MeV]	-1	-1
χ^2/DOF	1.0	1.5

	Model	Experiment
Ratios used in the thermal analysis for 200 GeV		
π^-/π^+	1.009 ± 0.003	$1.025 \pm 0.006 \pm 0.018$ $1.02 \pm 0.02 \pm 0.10$
K^-/K^+	0.939 ± 0.008	$0.95 \pm 0.03 \pm 0.03$ $0.92 \pm 0.03 \pm 0.10$
$\overline{p}/p$	0.74 ± 0.04	$0.73 \pm 0.02 \pm 0.03$ $0.70 \pm 0.04 \pm 0.10$ 0.78 ± 0.05
$\overline{p}/\pi^-$	0.104 ± 0.010	0.083 ± 0.015
K^-/π^-	0.174 ± 0.001	0.156 ± 0.020
$\Omega/h^- \quad \times 10^3$	0.990 ± 0.120	$0.887 \pm 0.111 \pm 0.133$
$\overline{\Omega}/h^- \quad \times 10^3$	0.900 ± 0.124	$0.935 \pm 0.105 \pm 0.140$

Transverse-momentum spectra



(experimental Ξ's went down by ∼ a factor of 2)

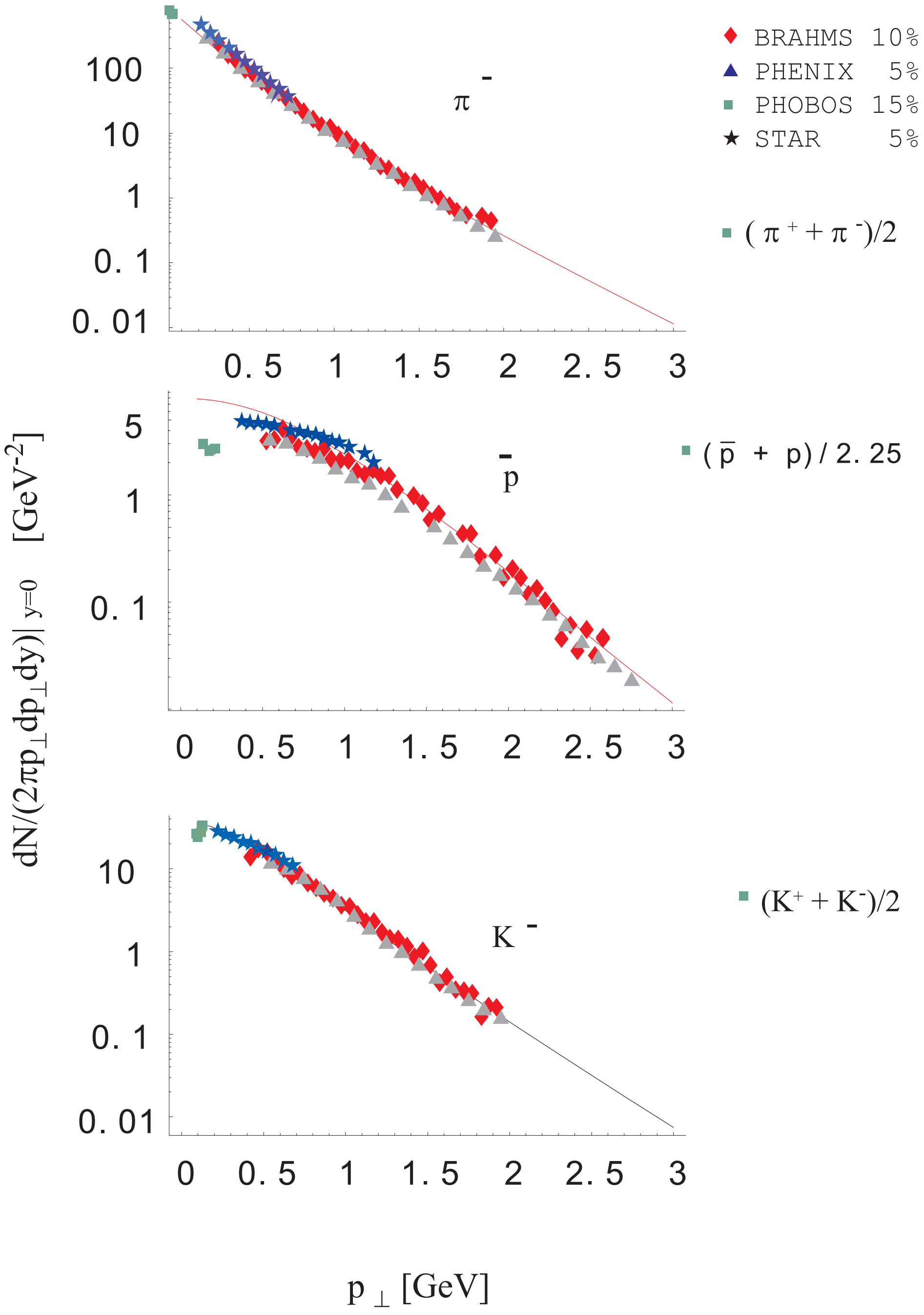

PHENIX @ 200
π⁻
π⁺
p̄
p
K⁻
K⁺
dN/(2πp⊥dp⊥dy)|y=0 [GeV -2]
p ⊥ [GeV]
100
1
0.01
10
0.1
0
0.5
1
1.5
2
2.5
3

(data at different centrality, or impact parameter)

Centrality c is defined as a percentage of the most central events. To a very good accuracy

$$c \simeq \frac{\pi b^2}{\sigma_{\rm inel}^{\rm tot}} \simeq \frac{b^2}{4R^2}$$

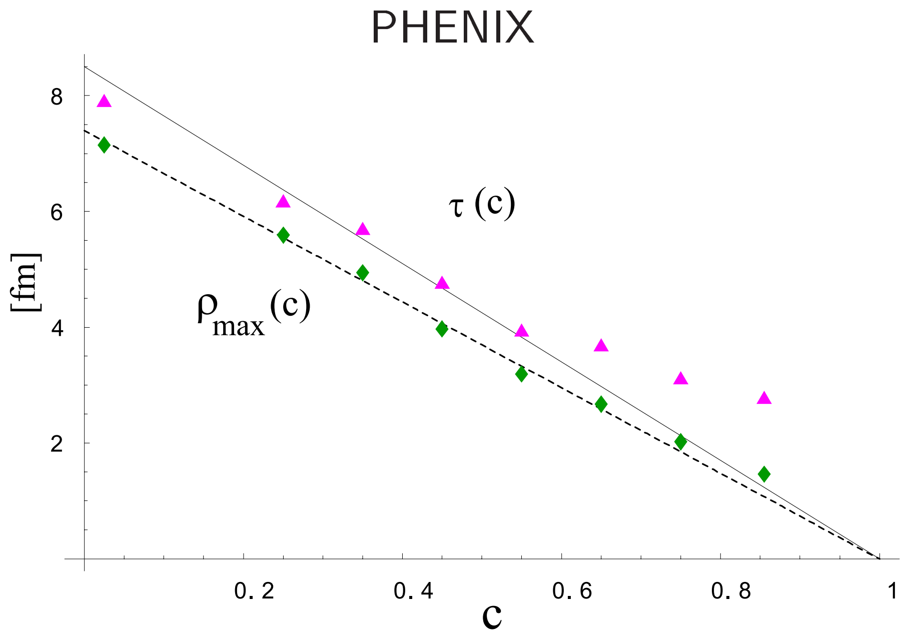
(WB+WF, PRC 65 (2002) 024905)

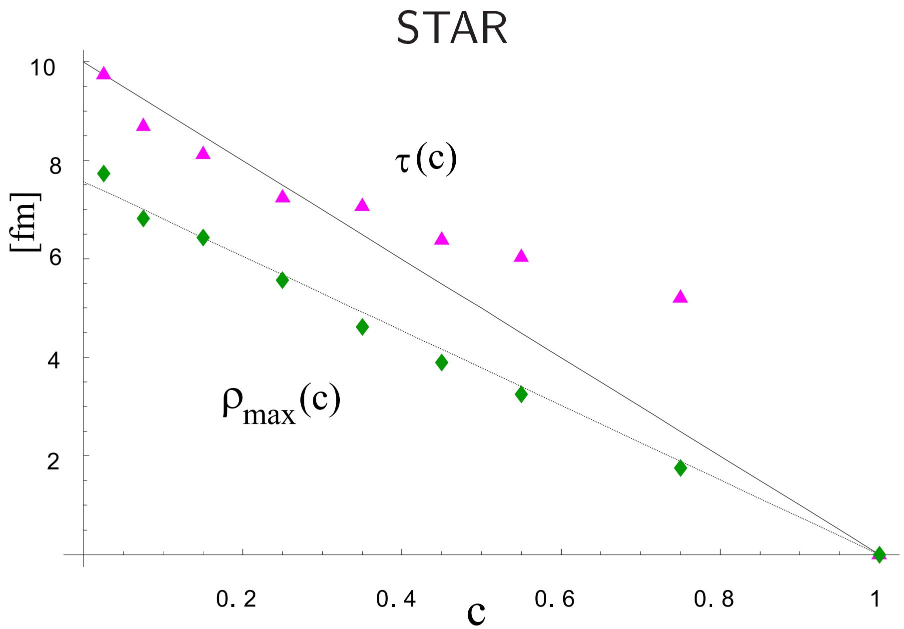


BRAHMS 10%
PHENIX 5%
PHOBOS 15%
STAR 5%
π^-
$(\pi^+ + \pi^-)/2$
$\bar{p}$
$(\bar{p} + p)/2.25$
K^-
$(K^+ + K^-)/2$
$dN/(2\pi p_\perp dp_\perp dy)|_{y=0}$ [GeV^{-2}]
$p_\perp$ [GeV]

Compilation of geometric parameters (by A. Baran)

	c [%]	τ [fm] (norm)	$\rho_{\max}$ [fm]	$\langle \beta_\perp \rangle$ (slope)
ALL	$0-5/10$	7.58 ± 0.32	7.27 ± 0.12	0.52 ± 0.02
BRAHMS	10	7.68 ± 0.19	7.46 ± 0.05	0.52 ± 0.01
STAR	$0-5$	9.74 ± 1.57	7.74 ± 0.68	0.45 ± 0.08
	$5-10$	8.69 ± 1.39	7.18 ± 0.64	0.47 ± 0.08
	$10-20$	8.12 ± 1.31	6.44 ± 0.57	0.45 ± 0.08
	$20-30$	7.24 ± 1.18	5.57 ± 0.50	0.44 ± 0.08
	$30-40$	7.07 ± 1.17	4.63 ± 0.39	0.39 ± 0.08
	$40-50$	6.38 ± 1.02	3.91 ± 0.33	0.37 ± 0.07
	$50-60$	6.19 ± 1.09	3.25 ± 0.28	0.32 ± 0.07
	$70-80$	5.48 ± 0.81	4.03 ± 0.10	0.43 ± 0.06
PHENIX	$0-5$	7.86 ± 0.38	7.15 ± 0.13	0.50 ± 0.02
	$20-30$	6.14 ± 0.32	5.62 ± 0.11	0.50 ± 0.02
	$30-40$	5.73 ± 0.16	4.95 ± 0.05	0.48 ± 0.01
	$40-50$	4.75 ± 0.28	3.96 ± 0.09	0.47 ± 0.03
	$50-60$	3.91 ± 0.23	3.12 ± 0.07	0.45 ± 0.03
	$60-70$	3.67 ± 0.12	2.67 ± 0.03	0.42 ± 0.01
	$70-80$	3.09 ± 0.11	2.02 ± 0.02	0.39 ± 0.01
	$80-91$	2.76 ± 0.20	1.43 ± 0.03	0.32 ± 0.03


PHENIX
τ (c)
ρmax(c)
[fm]
c
8
6
4
2
0. 2
0. 4
0. 6
0. 8
1

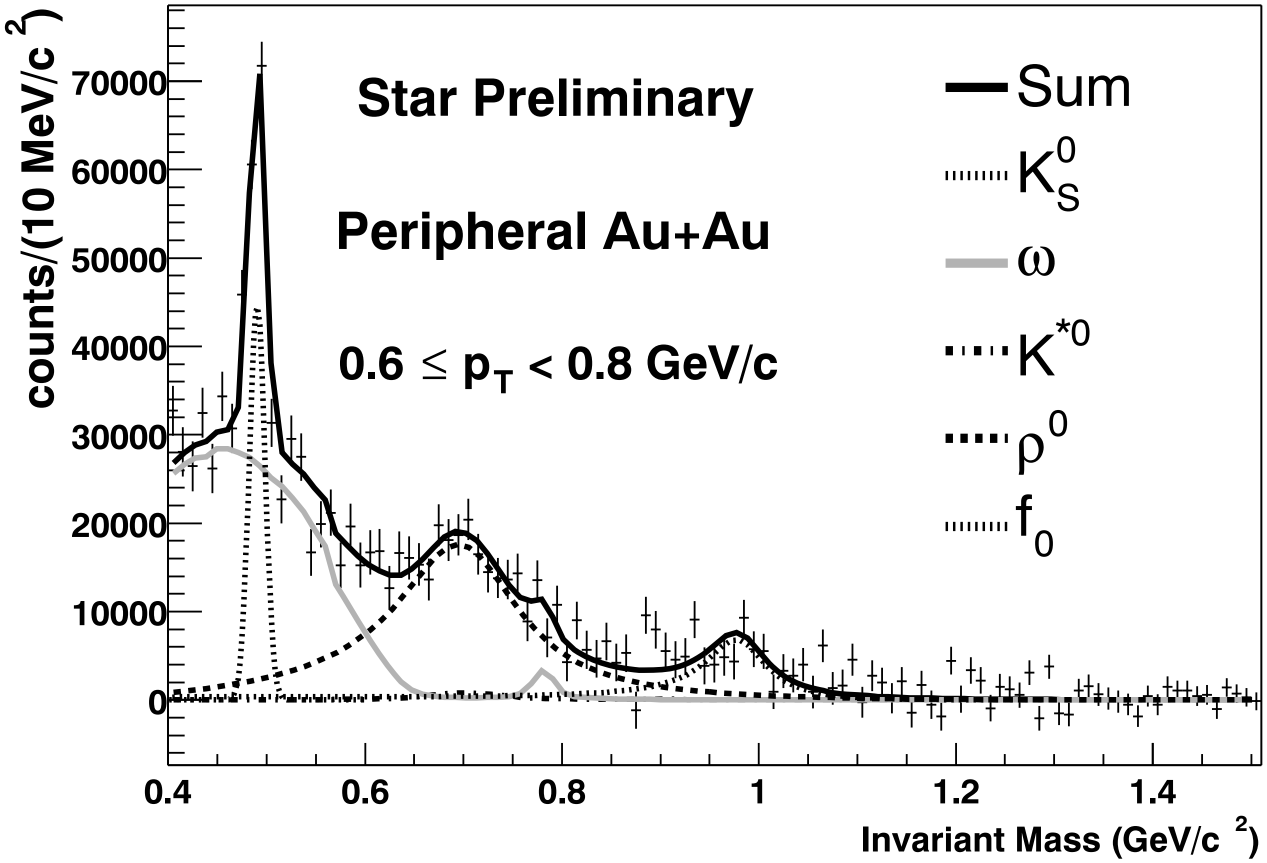

STAR
τ(c)
ρmax(c)
[fm]
c
10
8
6
4
2
0. 2
0. 4
0. 6
0. 8
1

Correlations of identified particles

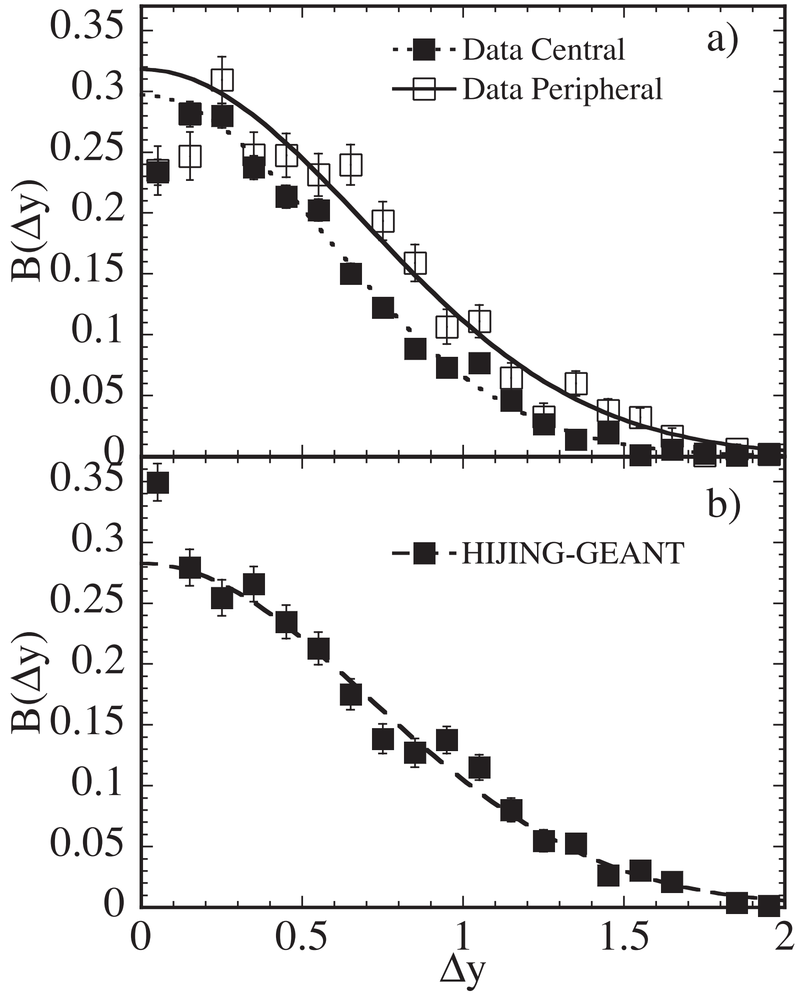
Two very clever techniques are used in order to subtract the background: mixed event ($K^*(892)$, $\Xi(1862)$) and like-sign subtraction (ρ)

- Invariant-mass spectra (K-π, π-π, to come out shortly: p-π)
- correlations in rapidity (balance functions)

$\pi^+\pi^-$ pairs from STAR



(from J. Adams et al., nucl-ex/0307023; P. Fachini, nucl-ex/0305034)



(from J. Adams et al., STAR Collaboration, Phys. Rev. Lett. **90** (2003) 172301)

Can we explain all this in the thermal model?

The phase-shift formula for the density of resonances

Beth,Uhlenbeck (1937); Dashen, Ma, Bernstein, Rajaraman (1974); **Weinhold (1998)**, Friman, Nörenberg; **WB, WF, B. Hiller**, PRC **68** (2003) 034911; Pratt, Bauer, nucl-th/0308087

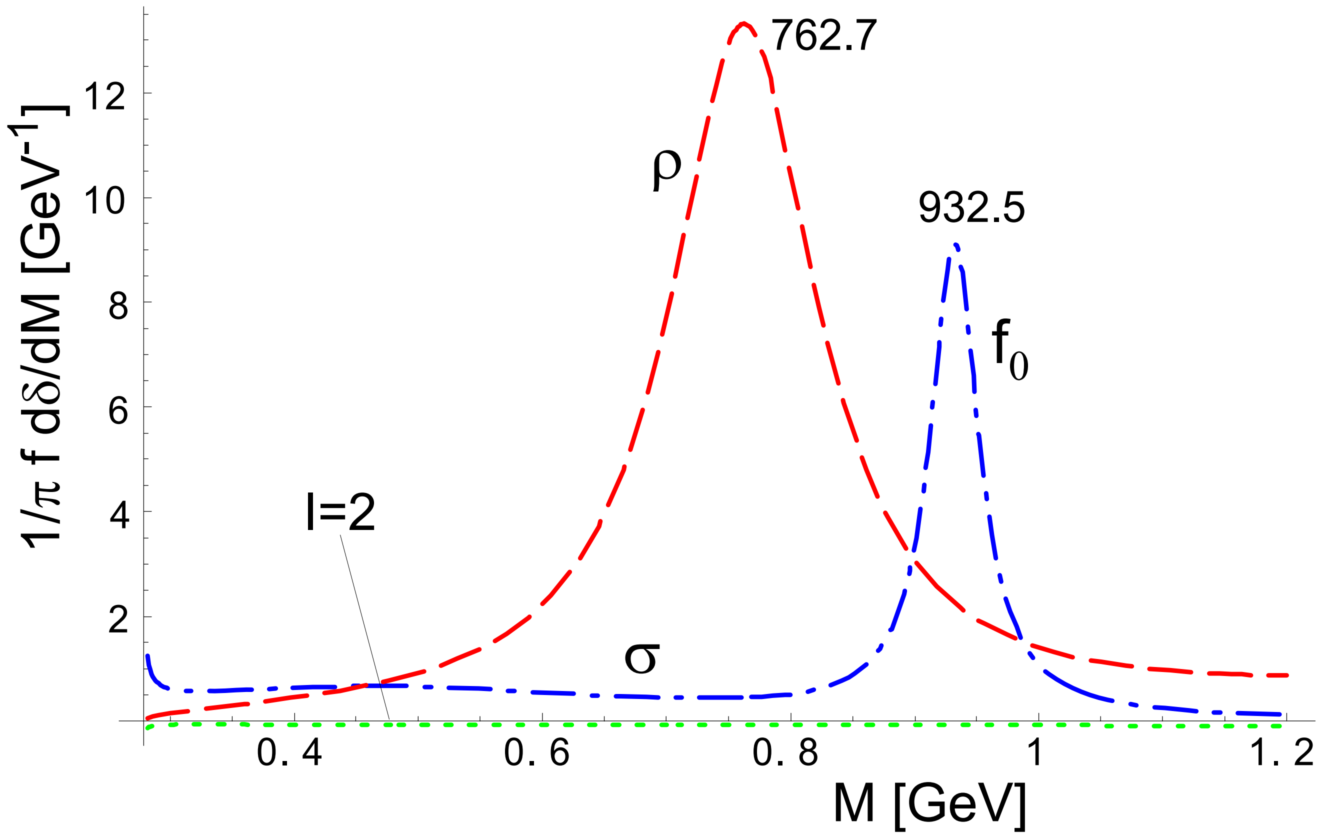
$$\frac{dn}{dM} = f \int \frac{d^3p}{(2\pi)^3} \frac{d\delta_{\pi\pi}(M)}{\pi dM} \frac{1}{\exp\left(\frac{\sqrt{M^2+\boldsymbol{p}^2}}{T}\right) \pm 1}$$

In some works the spectral function of the resonance is used *ad hoc* as the weight, instead of the derivative of the phase shift. For narrow resonances this does not make a difference, since then $d\delta(M)/dM \simeq \pi\delta(M - m_R)$, and similarly for the spectral function.

$$n^{\text{narrow}} = f \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{\sqrt{m_R^2+\boldsymbol{p}^2}}{T}\right) \pm 1}$$

For wide resonances, or for effects of tails, the difference between the correct formula and the one with the spectral function is signifcant

$d\delta_{\pi\pi}(M)/dM$ from experiment

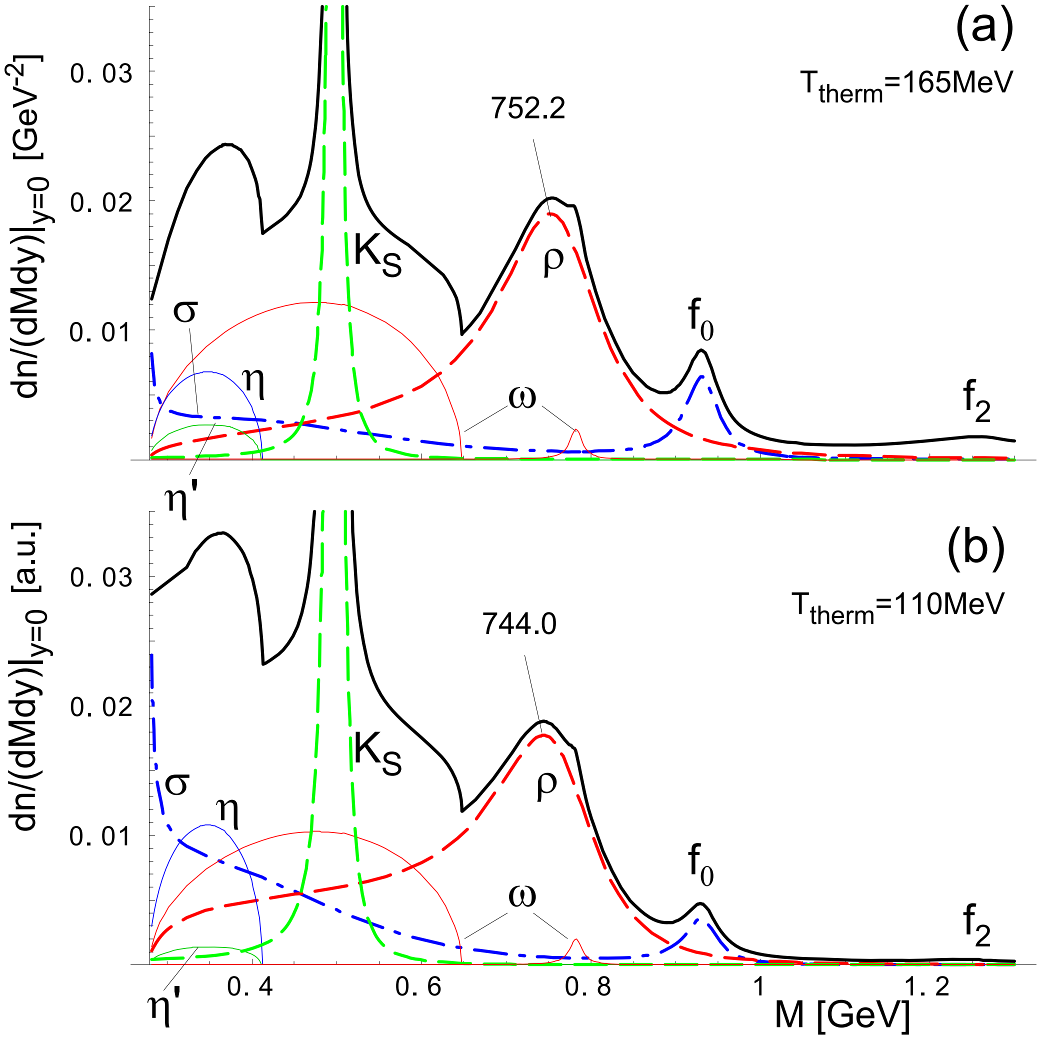


Small contribution from σ, negative and tiny contribution from $I = 2$, ρ-peak slightly shifted to lower M, $1/\sqrt{M - 4m_\pi^2}$ behavior for the σ

Warm-up calculation - static source

We compute the spectra at mid-rapidity, hence

$$\left.\frac{dn}{dM\,dy}\right|_{y=0} = \sum_i f_i \int_{0.2\mathrm{GeV}}^{2.2\mathrm{GeV}} \frac{p_\perp dp_\perp}{(2\pi)^2} \frac{d\delta_i(M)}{\pi dM} \frac{\sqrt{M^2+p_\perp^2}}{\exp\left(\frac{\sqrt{M^2+\boldsymbol{p}_\perp{}^2}}{T}\right)-1}$$

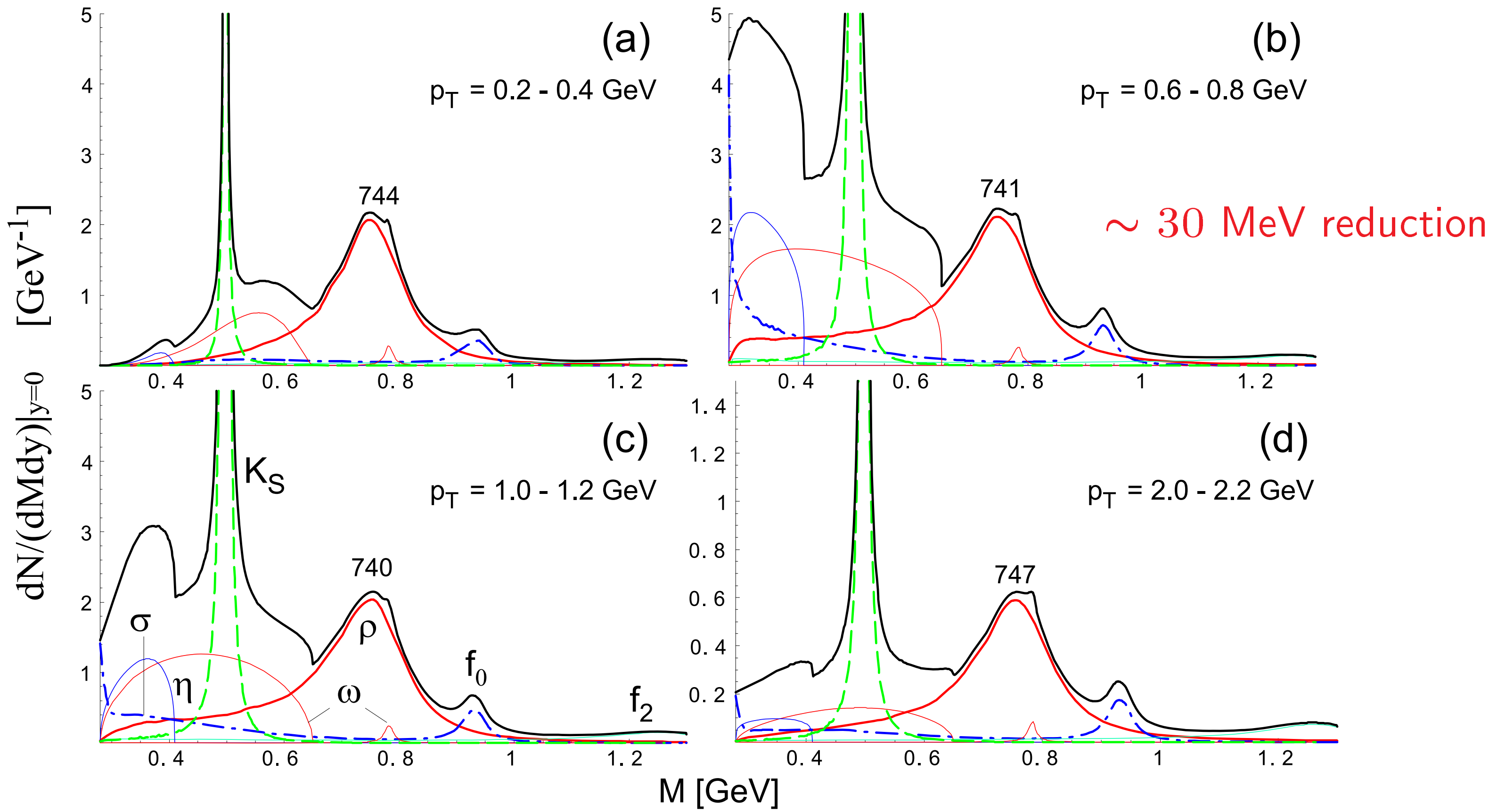


flat

steep

Cuts/flow + feeding from resonances

Flow has no effect on the invariant mass of a pair of particles produced in a resonance decay, since the quantity is Lorentz-invariant. Neverthelss, it affects the results since the kinematic cuts in an obvious manner break this invariance



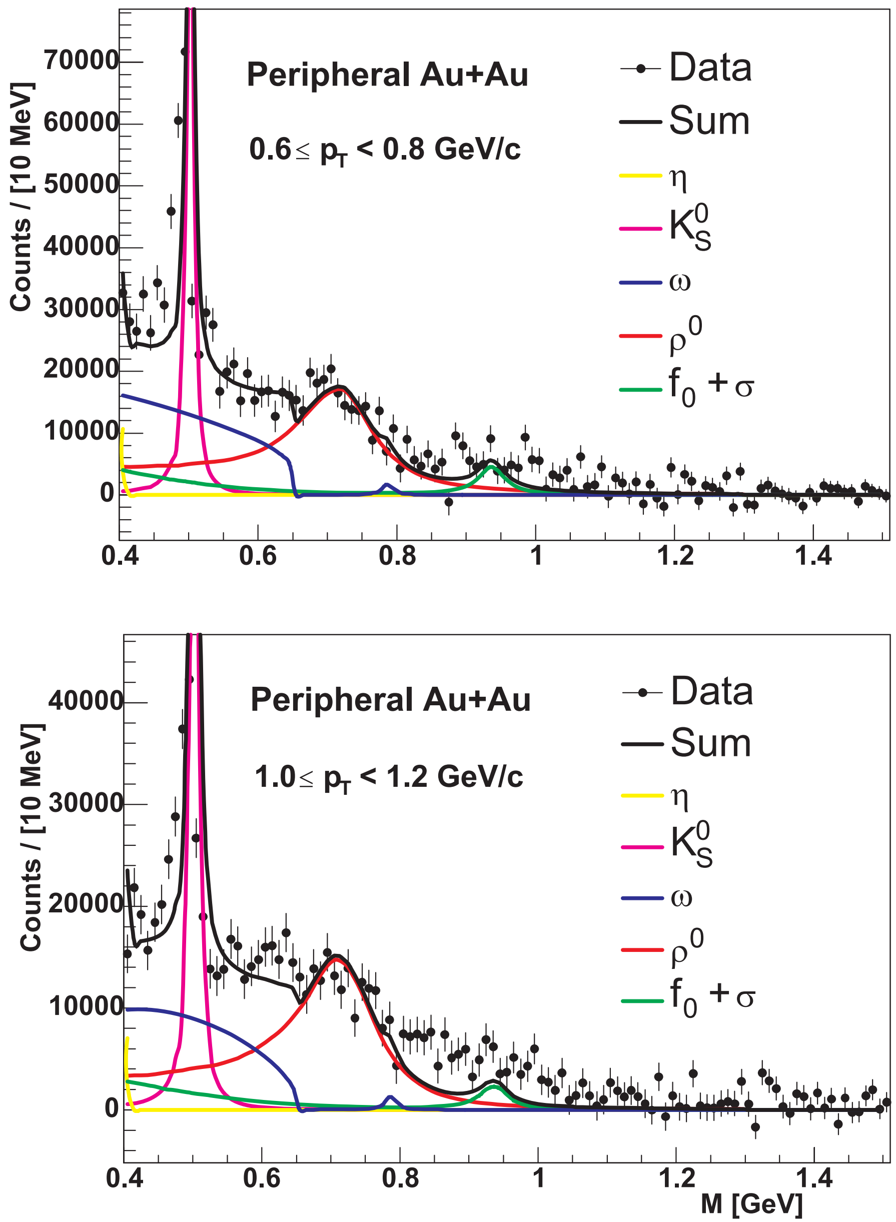
The invariant $\pi^+\pi^-$ mass spectra in the single-freeze-out model for four sample bins in the trasverse momentum of the pair, p_T, plotted as a function of M. η indicates $\eta + \eta'$. All kinematic cuts of the STAR experiment are incorporated

Resonanse decays

The higher-states decays lead to enhancement factors for low resonances: $K_S = 1.98$, $\eta = 1.74$, $\sigma = 1.13$, $\rho = 1.42$, $\omega = 1.43$, $\eta' = 1.08$, $f_0 = 1.01$, and $f_2 = 1.28$. Thus, the effects is strongest for light particles, K_S, η, ρ, and ω, while it is weaker for the heavier η' and scalar mesons.

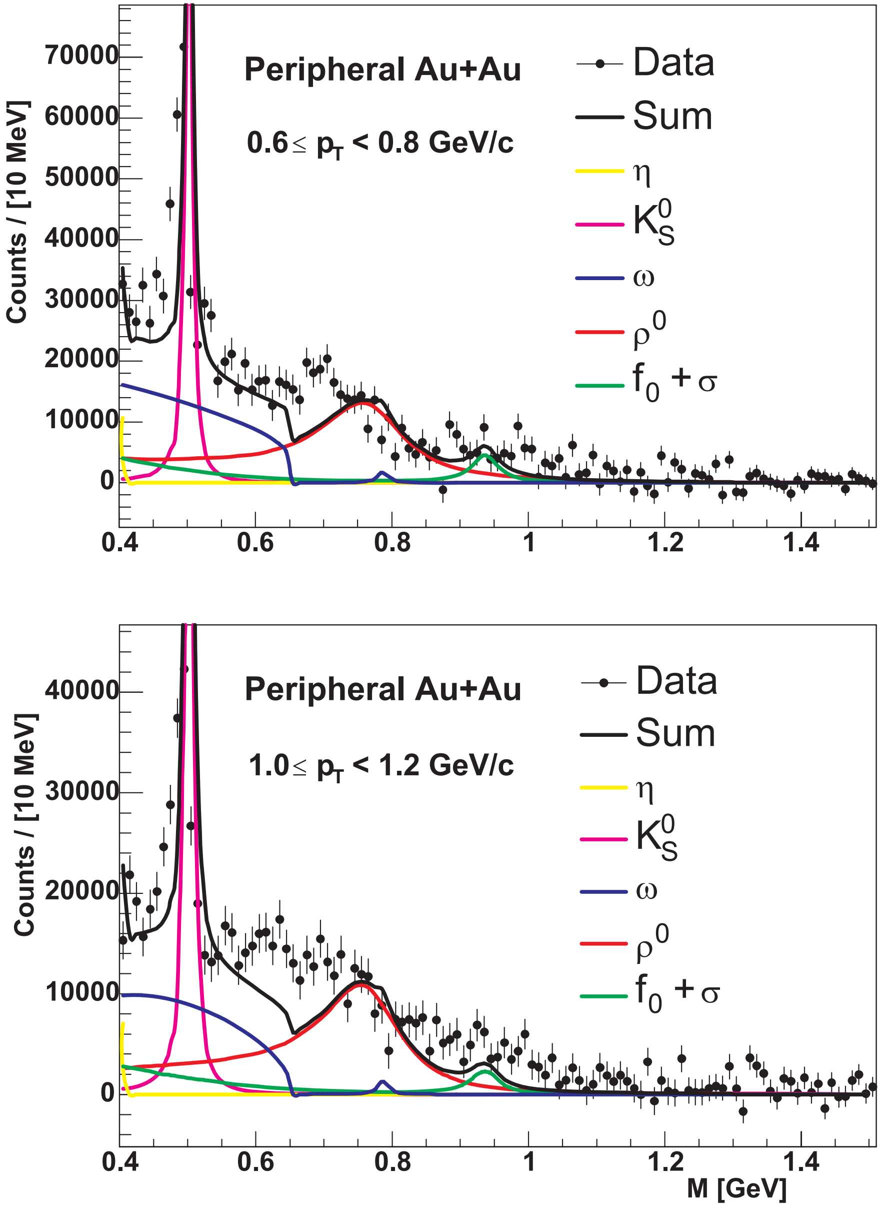
Full model, with feeding from higher resonances and flow/cuts at $T = 165$ MeV is similar to the naive model at $T = 110$ MeV !

STAR vs. thermal model, lowered ρ



(prepared by P. Fachini)

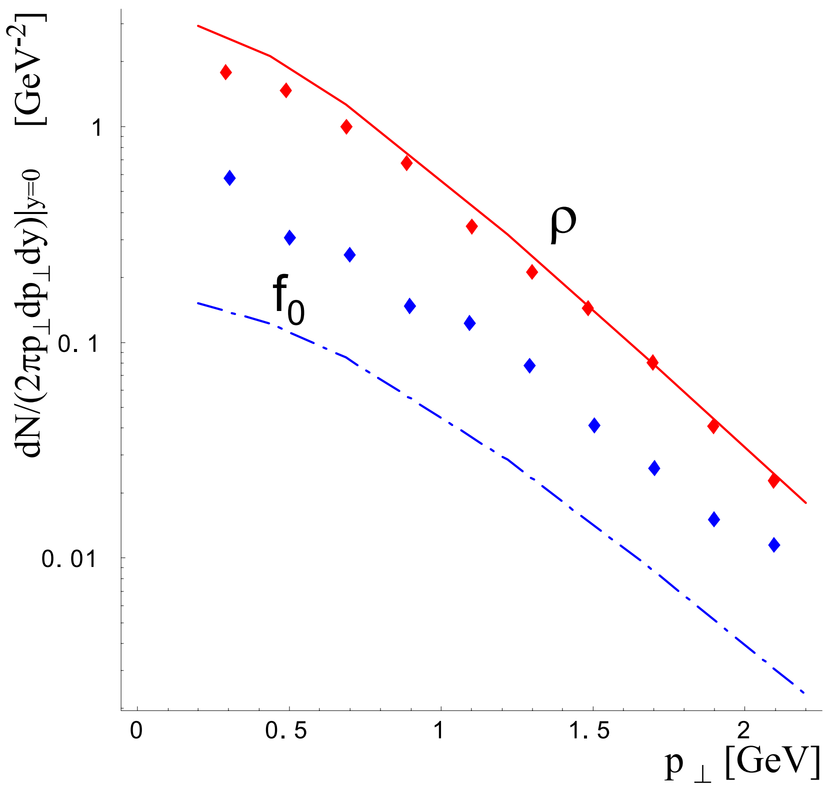
vacuum ρ



(worse agreement)

Is m_ρ lowered? (dileptons from CERES and HELIOS)

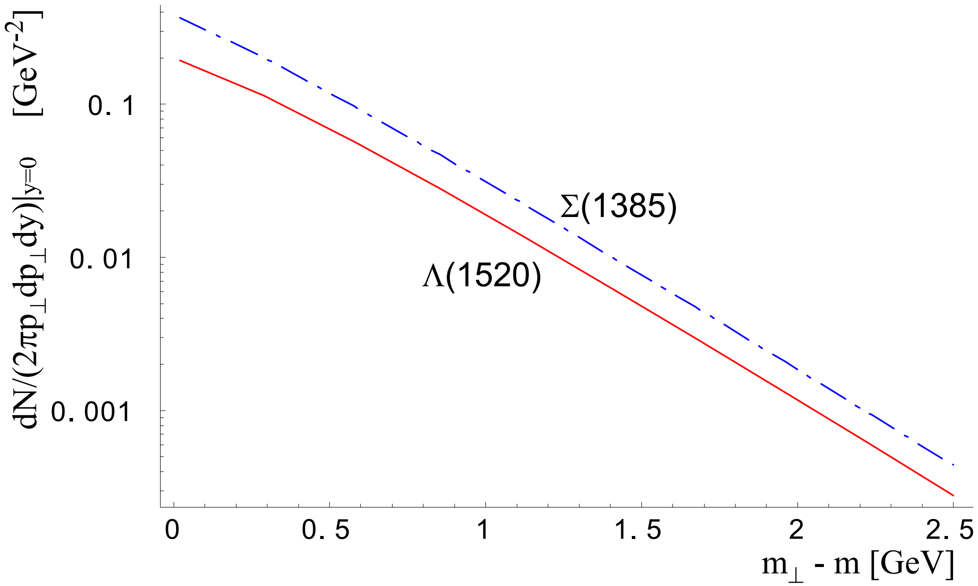
$p_\perp$ spectra of resonances

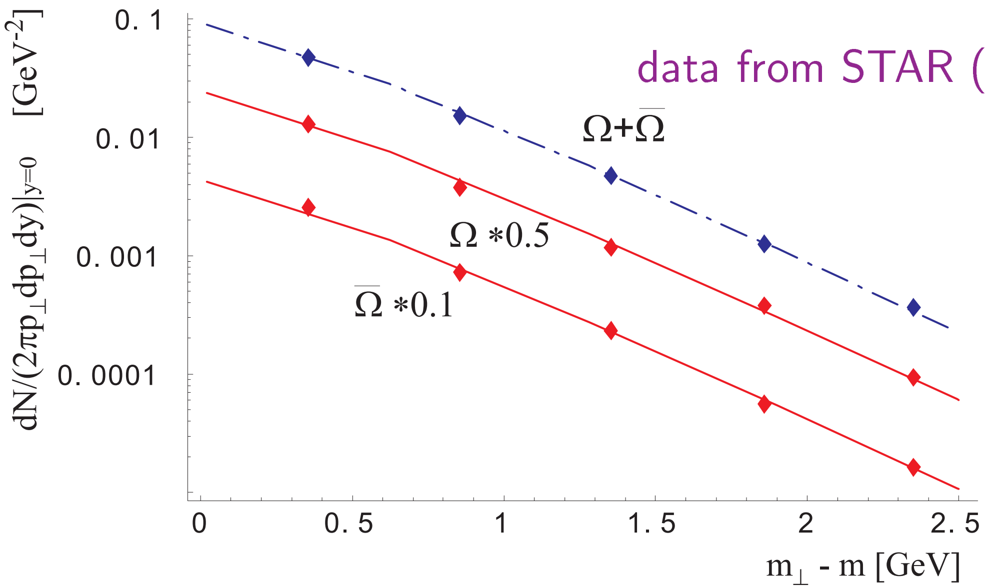


(model parameters, $\tau = 5$ fm and $\rho_{\max} = 4.2$ fm, correspond to centralities 40-80%)

For f_0 experiment $>$ thermal model!

Predictions





data from STAR (C. Suire, QM2002)

Balance functions

(based on: Piotr Bożek+WB+WF, *Balance functions in a thermal model with resonances*, `nucl-th/0310062`)

The balance functions analyzed by the STAR Collaboration at RHIC are defined as

$$B(\delta, Y) = \frac{1}{2}\left\{\frac{\langle N_{+-}(\delta)\rangle - \langle N_{++}(\delta)\rangle}{\langle N_+\rangle} + \frac{\langle N_{-+}(\delta)\rangle - \langle N_{--}(\delta)\rangle}{\langle N_-\rangle}\right\},$$

where $N_{+-}(\delta)$ counts the opposite-charge pairs when both members of the pair fall into the rapidity window Y. Their relative rapidity is $|y_2 - y_1| \equiv \delta$. N_+ is the number of positive particles in the interval Y.

For sufficiently large rapidity interval $Y \sim Y^{\max}$, the balance function of *all* charged hadrons is normalized to unity,

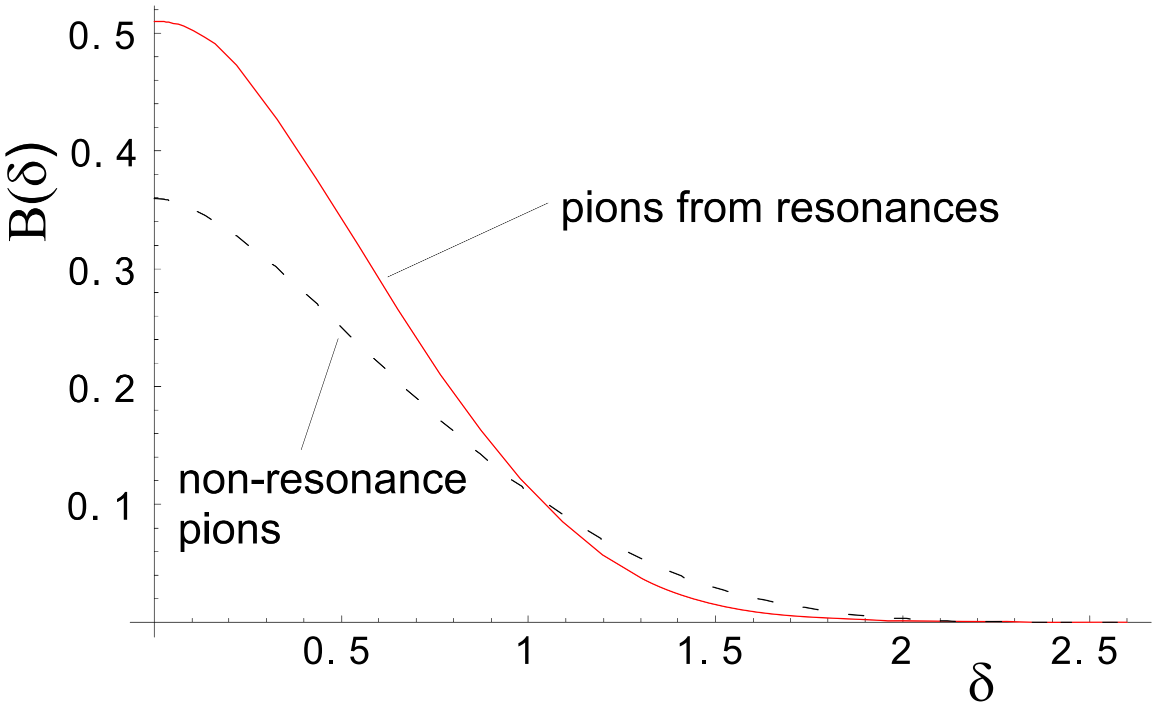
$$\int_0^{Y^{\max}} d\delta \, B(\delta, Y^{\max}) = 1,$$

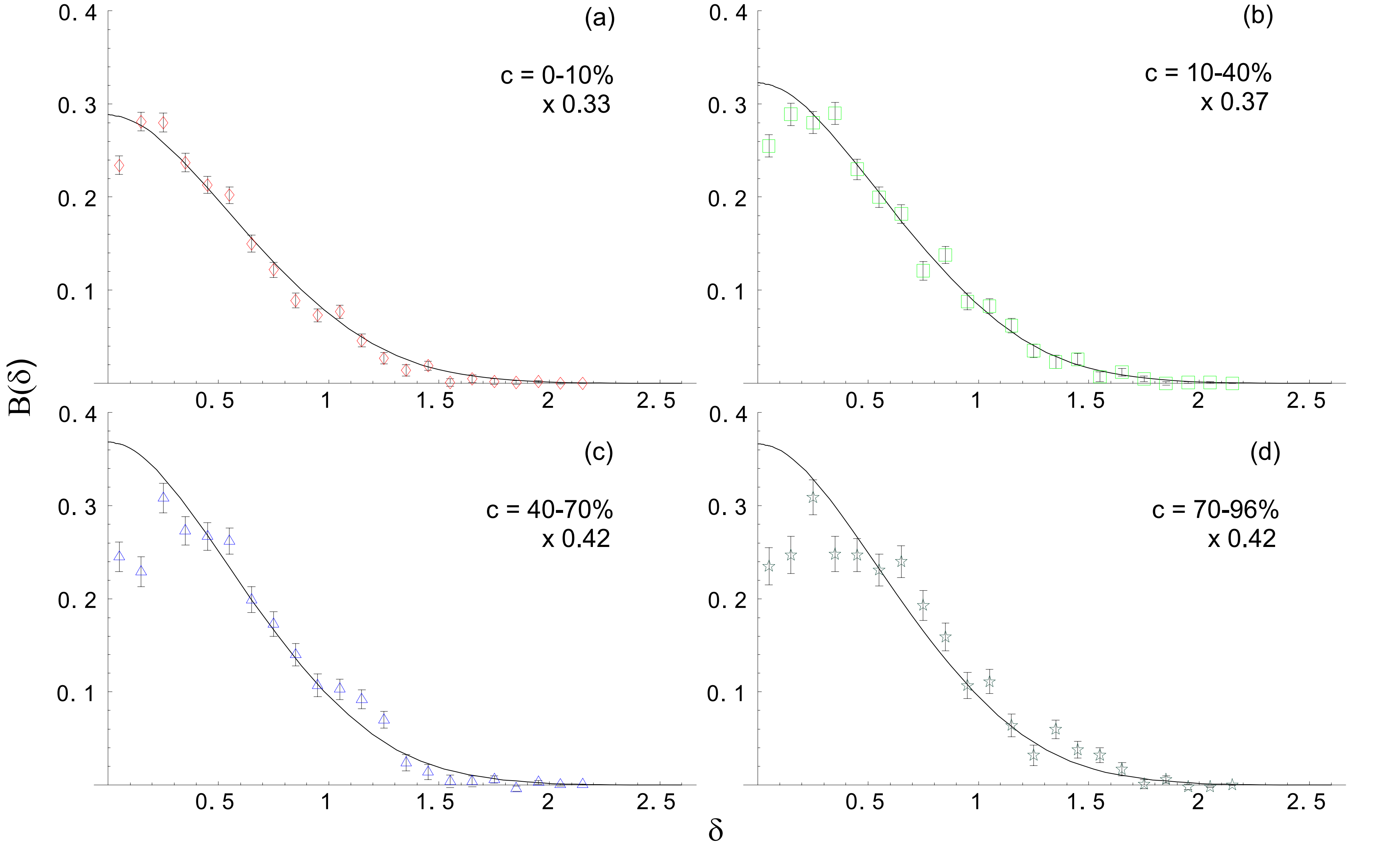
which is a condition reflecting the overall charge conservation.

Balance functions in the thermal model

Resonance and non-resonance contributions,

$$B(\delta, Y) = B_{\rm R}(\delta, Y) + B_{\rm NR}(\delta, Y)$$




(a)
c = 0-10%
x 0.33
(b)
c = 10-40%
x 0.37
(c)
c = 40-70%
x 0.42
(d)
c = 70-96%
x 0.42
B(δ)
δ
0. 4
0. 3
0. 2
0. 1
0. 5
1
1. 5
2
2. 5

The widths of the balance functions, $\langle\delta\rangle$, are obtained (as in experiment) for the range $0.2 < \delta < 2.6$

Model				
$\rho_{\max}/\tau$	$\langle\beta_\perp\rangle$	$\langle\delta\rangle_{\rm res}$	$\langle\delta\rangle_{\rm therm}$	$\langle\delta\rangle_{\rm tot}$
0.9	0.50	0.59	0.67	0.63
Experiment				
$c = 0 - 10\%$				0.594 ± 0.019
$c = 10 - 40\%$				0.622 ± 0.020
$c = 40 - 70\%$				0.633 ± 0.024
$c = 70 - 96\%$				0.664 ± 0.029

The dependence of the width on centrality cannot be reproduced by varying the transverse flow within limits consistent the the single-particle spectra.

Summary

1. Old story: success for abundances, $p_\perp$-spectra
2. Not covered: the model also works very reasonably for the HBT radii, in particular $R_{\text{out}}/R_{\text{side}} \sim 1$
3. ... and for the elliptic flow (A. Baran, in preparation)
4. New story: resonances are an important source of correlations between opposite-charge pions
5. Shape of the $\pi\pi$ "spectral line" - new thermometer
6. Derivative of phase shifts, not the spectral density as weight !
7. Full model gives similar results at 165MeV to the naive calculation at 110MeV (cooling via decays)
8. Kinematic cuts and flow important, higher resonance decays important
9. Not possible to place the ρ peak at the experimental value (medium effects - Brown-Rho scaling?, other effects?)
10. By summing up the resonance and non-resonance contributions we obtain the pion balance function with the shape similar to the data
11. The normalization of the model balance function is significantly larger than in the experiment (a factor of 2.5 - 3) because of the effect of a limited detector efficiency that we are not able to take into account

W. Czyż :

Things are so complicated that

they become simple again!

Back-up slides

The STAR cuts

The cuts in the STAR analysis of the $\pi^+\pi^-$ invariant-mass spectra have the following form (Fachini):

$$
\begin{aligned}
&|y_\pi| \leq 1, \\
&|\eta_\pi| \leq 0.8, \\
&0.2 \text{ GeV} \leq p_\pi^\perp \leq 2.2 \text{ GeV},
\end{aligned}
\tag{1}
$$

while the bins in $p_T \equiv |\boldsymbol{p}_\pi^\perp + \boldsymbol{p}_\pi^\perp|$ start from the range $0.2 - 0.4$ GeV, and step up by 0.2 GeV until $2 - 2.4$ GeV.

For two-body decays, the relevant formula for the number of pairs of particles 1 and 2 has the form

$$
\begin{aligned}
\frac{dN_{12}}{dM} = {} & \frac{d\delta_{12}}{dM}\frac{bm}{p_1^*}\int_{p^\perp_{1,\mathrm{low}}}^{p^\perp_{1,\mathrm{high}}} dp_1^\perp \int_{y_{1,\mathrm{low}}}^{y_{1,\mathrm{high}}} dy_1 \int_{p^\perp_{\mathrm{low}}}^{p^\perp_{\mathrm{high}}} dp^\perp \int_{y_{\mathrm{low}}}^{y_{\mathrm{high}}} dy \\
& \times C_2^0 C_1^\eta C_2^\eta \frac{\theta(1-\cos^2\gamma_0)}{|\sin\gamma_0|} S(p^\perp),
\end{aligned}
\tag{2}
$$

Lowering the ρ mass

In order to show how the medium modifications will show up in the $\pi^+\pi^-$ spectrum, we have scaled the $\pi\pi$ phase shift in the ρ channel, according to the simple law

$$\delta_1^1(M)_{\rm scaled} = \delta_1^1(s^{-1}M)_{\rm vacuum}, \tag{3}$$

Phase shift vs. spectral density