

# Photon and $\rho$ from chiral quarks

Wojciech Broniowski

Institute of Nuclear Physics PAN, Cracow & Jan Kochanowski  
University, Kielce

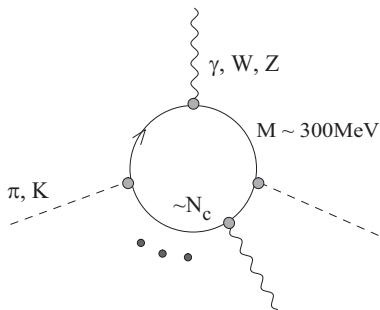
Photon 09, DESY, 11-15 May 2009

## More information:

- *Photon distribution amplitudes and light-cone wave functions in chiral quark models*, **Alexander E. Dorokhov**, WB, ERA, Phys. Rev. D74 (2006) 054023
- *Pion-photon Transition Distribution Amplitudes in the Spectral Quark Model*, WB, ERA, Phys. Lett. B649 (2007) 49
- *Generalized parton distributions of the pion in chiral quark models and their QCD evolution*, WB , ERA, **Krzysztof Golec-Biernat**, Phys. Rev. D77 (2008) 034023
- *Gravitational and higher-order form factors of the pion in chiral quark models*, WB, **Enrique Ruiz Arriola**, Phys. Rev. D78 (2008) 094011

Chiral quark models used in similar studies by the Bochum, Valencia, and Jagellonian groups

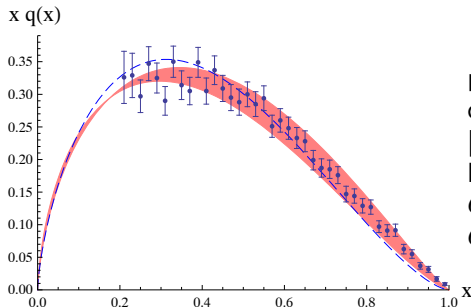
# Chiral quark models



- soft regime  $\rightarrow$  chiral sym. breaking
- NJL (Nobel 2008), instanton liquid, DSE
- relatively few parameters (traded for  $f_\pi, m_\pi, \dots$ )
- very many processes can be computed!
- no confinement - careful not to open the  $q\bar{q}$  threshold
- low quark-model scale  
 $\sim 320 - 500 \text{ MeV}$  - need for evolution
- local and non-local (instanton-based) models

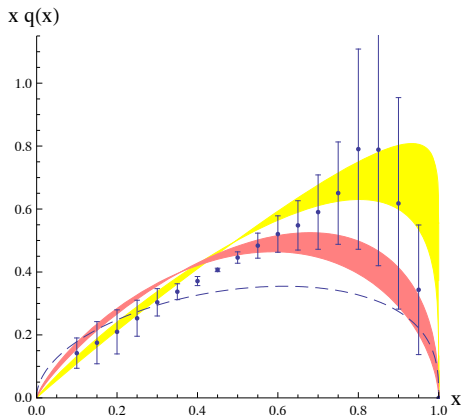
## PDF of the pion, QM vs. E615

LO DGLAP QCD evolution of the non-singlet part to the scale  $Q^2 = (4 \text{ GeV})^2$  of the E615 Fermilab experiment:



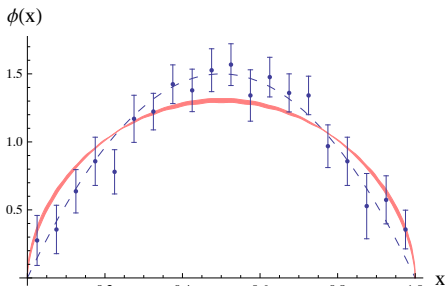
points: Drell-Yan from E615  
dashed: reanalysis of data  
[Wijesooriya et al., 2005]  
band: valence QM PDF evolved to  
 $Q = 4 \text{ GeV}$  from the QM scale  
 $Q_0 = 313_{-10}^{+20} \text{ MeV}$

# PDF of the pion, QM vs. lattice

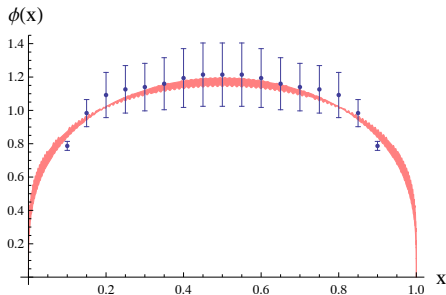


points: transverse lattice  
[Dalley, van de Sande, 2003]  
yellow: QM evolved to 0.35 GeV  
pink: QM evolved to 0.5 GeV  
dashed: GRS parameterization at  
0.5 GeV

## PDA of the pion, QM vs. E791 and lattice data

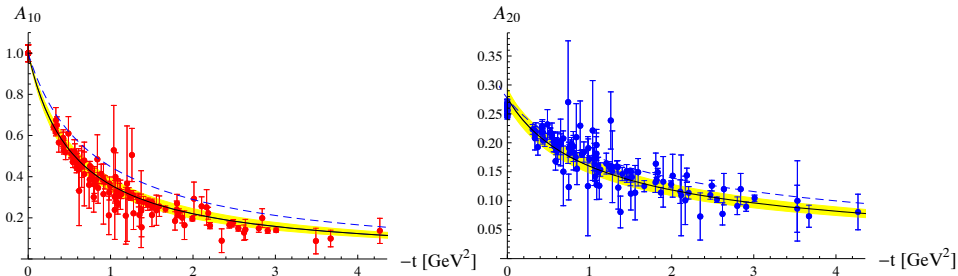


points: E791 data from di-jet  
production in  $\pi + A$   
band: QM at  $Q = 2$  GeV  
dashed line: asymptotic form  
( $Q \rightarrow \infty$ )



points: transverse lattice data  
[Dalley, van de Sande, 2003]  
band: QM at  $Q = 0.5$  GeV

# Pion form factors vs. lattice



The EM FF (left) and the quark part of the gravitational form factor  $\Theta_1$  (right) in SQM (solid line) and NJL (dashed line), compared to data from [Brömmel et al., 2005-7]

Quark-model relation:  $\langle r^2 \rangle_{\Theta} = \frac{1}{2} \langle r^2 \rangle_V$

Matter more concentrated than charge!

## Photon DA's: definitions

Twist-2 components of the photon Distribution Amplitude  
 [Ali+Braun 95, Ball+Braun 96- ]

$$\langle 0 | \bar{q}(z) \sigma_{\mu\nu} [z, -z] q(-z) | \gamma^\lambda(q) \rangle =$$

$$i e_q \langle \bar{q}q \rangle \chi_{\mathbf{m}} f_{\perp\gamma}^t(q^2) \left( \epsilon_{\perp\mu}^{(\lambda)} p_\nu - \epsilon_{\perp\nu}^{(\lambda)} p_\mu \right) \int_0^1 dx e^{i(2x-1)q \cdot z} \phi_{\perp\gamma}(x, q^2) + h.t.$$

$$\langle 0 | \bar{q}(z) \gamma_\mu [z, -z] q(-z) | \gamma^\lambda(q) \rangle =$$

$$e_q f_{3\gamma} f_{\parallel\gamma}^v(q^2) p_\mu \left( \epsilon^{(\lambda)} \cdot n \right) \int_0^1 dx e^{i(2x-1)q \cdot z} \phi_{\parallel\gamma}(x, q^2) + h.t.$$

$$\langle 0 | \bar{q}(z) \gamma_\mu \gamma_5 [z, -z] q(-z) | \gamma^\lambda(q) \rangle = h.t.$$

where

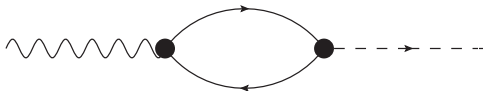
$$\epsilon^{(\lambda)} \cdot q = 0, \quad \epsilon^{(\lambda)} \cdot n = 0 \text{ for real photons}$$

$$p_\mu = q_\mu - \frac{q^2}{2} n_\mu, \quad n_\mu = \frac{z_\mu}{p \cdot z}, \quad e_\mu^{(\lambda)} = \left( e^{(\lambda)} \cdot n \right) p_\mu + \left( e^{(\lambda)} \cdot p \right) n_\mu + e_{\perp\mu}^{(\lambda)}$$



## Quark-model evaluation

Photon/ $\rho$ -to-current transition:



We have used

- Non-local quark model (instanton-based), with Gaussian regulator
- NJL model with PV subtraction
- Spectral Quark model (SQM), implementing the VMD

## Constants

QCD predicts the **scale dependence** for the quark condensate  $\langle 0 | \bar{q}q | 0 \rangle$ , its magnetic susceptibility  $\chi_m$ , and for  $f_{3\gamma}$ . At LO

$$\begin{aligned} \langle 0 | \bar{q}q | 0 \rangle |_{\mu} &= L^{-\gamma_{\bar{q}q}/b} \langle 0 | \bar{q}q | 0 \rangle |_{\mu_0} \\ \chi_m |_{\mu} &= L^{-(\gamma_0 - \gamma_{\bar{q}q})/b} \chi_m |_{\mu_0}, \quad f_{3\gamma} |_{\mu} = L^{-\gamma_f/b} f_{3\gamma} |_{\mu_0} \end{aligned}$$

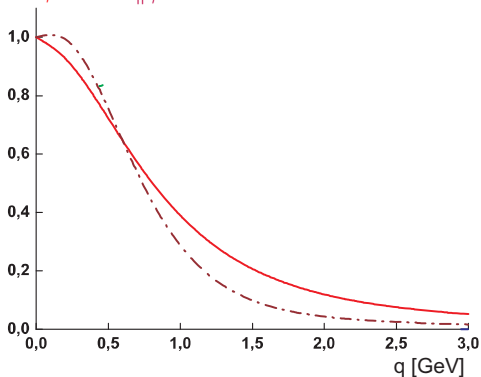
where  $r = \alpha_s(\mu^2) / \alpha_s(\mu_0^2)$ ,  $b = (11N_c - 2n_f) / 3$ ,  $\gamma_{\bar{q}q} = -3C_F$ ,  $\gamma_0 = C_F$ ,  $\gamma_f = 3C_A - C_F/3$ , with  $C_F = 4/3$  and  $C_A = 3$  for  $N_c = 3$ .

quantity at 1 GeV	non-local	SQM	QCD s.r.	VMD
$(-\langle 0   \bar{q}q   0 \rangle)^{1/3}$ [GeV]	0.24	0.24	$0.24 \pm 0.02$	-
$\chi_m$ [GeV <sup>2</sup> ]	2.73	1.37	$3.15 \pm 0.3$	3.37
$f_{3\gamma}$ [GeV <sup>-2</sup> ]	-0.0035	-0.0018	$-0.0039 \pm 0.0020$	-0.0046

SQM similar to NJL, local models have larger  $r$  than the non-local model  
 QCD s.r. and VDM from [Braun+Ball+Kivel 03]

## Form factors

$f_{\perp\gamma}^t(q^2)$ ,  $f_{\parallel\gamma}^v(q^2)$ , non-local model



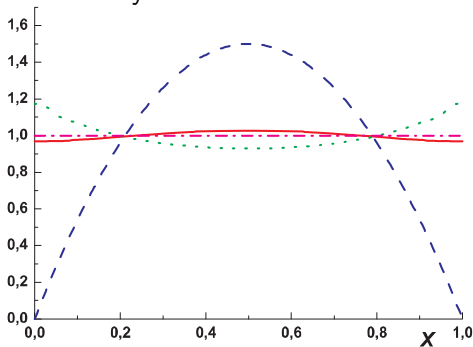
- Local models very similar
- Typical fall-off scale of  $\sim m_\rho$
- In SQM

$$f_{\perp\gamma}^{t,\text{SQM}}(q^2) = \frac{m_\rho^2}{m_\rho^2 + q^2}$$

# Photon DA's

evaluation at the **QM scale**

$\phi_{\perp\gamma}(x, q^2 = 0)$  – chirally-odd transversity DA



non-local    local    approx. to n.l. –  
 Bochum    asymptotic,  $6x(1-x)$

- $\phi_{\perp\gamma}(x, q^2 = 0) = 1$  in local models
- $\sim 1$  in non-local models
- For virtual photon SQM gives simple formulas:

$$\phi_{\parallel\gamma^*}(x, q^2) = \frac{1 + \frac{q^2}{m_\rho^2}}{\left(1 + \frac{4q^2}{m_\rho^2}x(1-x)\right)^{3/2}}$$

- In the limit  $q^2 \rightarrow -m_\rho^2$  it becomes  $\delta\left(x - \frac{1}{2}\right)$
- $\phi_{\perp\parallel}(x, q^2 = 0) = 1$  in local models

## Photon's LCWF

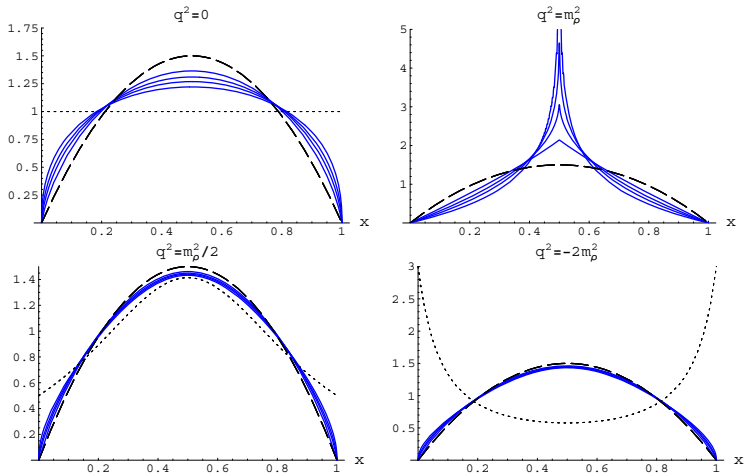
The light-cone wave function ( $k_{\perp}$ -unintegrated) has a simple form in SQM (at the QM scale):

$$\Phi_{\perp\gamma}(x, \mathbf{k}_{\perp}) = \frac{6}{m_{\rho}^2(1 + 4\mathbf{k}_{\perp}^2/m_{\rho}^2)^{5/2}}$$

Note the power-law fall-off at large transverse momenta,  $\Phi_{\perp\gamma}(x, \mathbf{k}_{\perp}) \sim 1/k_{\perp}^5$ . In cross section this leads to tails  $\sim 1/k_{\perp}^{10}$ .  
 For the virtual photon

$$\Phi_{\perp\gamma^*}(x, \mathbf{k}_{\perp}) = \frac{6 \left(1 + \frac{q^2}{m_{\rho}^2}\right)}{m_{\rho}^2 \left(1 + 4 \frac{\mathbf{k}_{\perp}^2 + q^2 x(1-x)}{m_{\rho}^2}\right)^{5/2}}$$

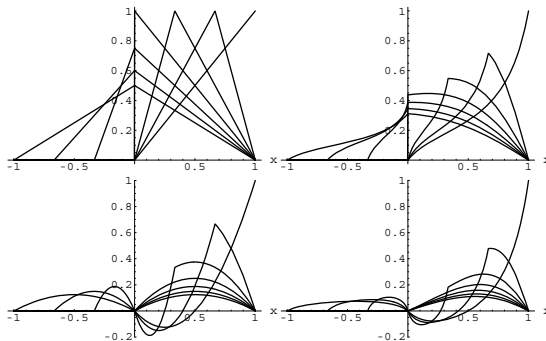
# LO ERBL of the tensor DA of the virtual photon



Initial conditions (dots) from SQM. Solid lines: evolution to scales 1, 2.4, 10, and 1000 GeV, asymptotic form  $6x(1-x)$ : dashed line

# Pion-photon Transition Distribution Amplitudes

[Pire+Szymanowski 05] – as GPD, but between the  $\pi$  and  $\gamma$  states



Top: vector TDA at  $q^2 = 0$  (left) and  $q^2 = 0.4 \text{ GeV}^2$  (right) several values of  $\zeta$ :  
 -1, -2/3, -1/3, 0, 1/3, 2/3, and 1. Bottom: same for the axial TDA (SQM  
 at the QM scale)

- 1 Chiral quark models provide a link between high- and low-energy analyses. They yield in a fully dynamical way the initial conditions for the QCD evolution, which is **essential** to bring the predictions to experimental/lattice scales
- 2 Numerous predictions for soft matrix element involving the Goldstone bosons and photons
- 3 Scale in chiral quark models is very low, 320-500 MeV, QCD evolution “fast”
- 4 Simple analytic formulas – useful to understand general properties, (e.g., no factorization of the  $t$ -dependence)
- 5 For the pion, with the LO QCD evolution the overall agreement with the available data and lattice simulations is **very reasonable** (PDF, DA, generalized form factors, GPD, TDA, ...)
- 6 [ Pire+Szymanowski 09 ]  $l\bar{l}$  photoproduction  $\rightarrow \phi_{\perp\gamma} \left( \frac{\alpha Q^2}{q^2 + Q_{\perp}^2} \right) h_1^q()$
- 7 Predictions can be further tested with future lattice simulations also for the photon/ $\rho$