Gravitational form factor of the pion

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research with **Enrique Ruiz Arriola**

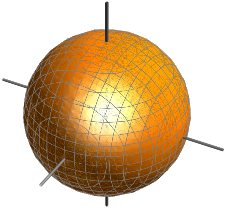
Problems in multi-quark states, Bled, 29 June - 6 July 2009

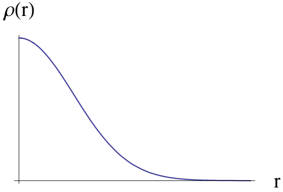
- *Gravitational and higher-order form factors of the pion in chiral quark models*, WB, **Enrique Ruiz Arriola**, Phys. Rev. D78 (2008) 094011
- *Generalized parton distributions of the pion in chiral quark models and their QCD evolution*, WB , ERA, **Krzysztof Golec-Biernat**, Phys. Rev. D77 (2008) 034023
- *Pion-photon Transition Distribution Amplitudes in the Spectral Quark Model*, WB, ERA, Phys. Lett. B649 (2007) 49
- *Photon distribution amplitudes and light-cone wave functions in chiral quark models*, **Alexander E. Dorokhov**, WB, ERA, Phys. Rev. D74 (2006) 054023

Other groups:

- Praszałowicz, Rostworowski, Bzdak, Kotko (Jagellonian)
- Noguera, Vento, Theussl, Courtoy (Valencia)
- Tiburzi, Miller (Seattle)
- Bochum, Tübingen, Wakamatsu (nucleon)

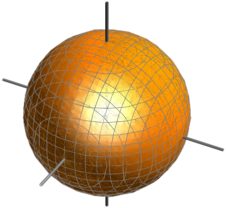
Distribution of charge

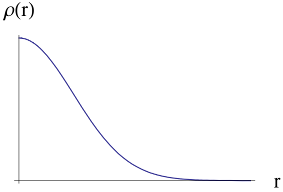




$$Q = \int d^3 r \rho(r), \qquad F(q^2) = \frac{1}{Q} \int d^3 r e^{-i\vec{q}\cdot\vec{r}} \rho(r)$$

Distribution of charge





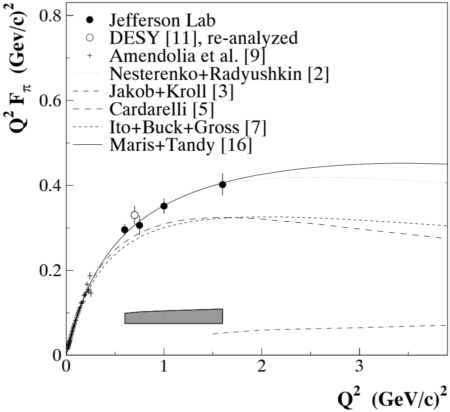
$$Q = \int d^3 r \rho(r), \qquad F(q^2) = \frac{1}{Q}\int d^3 r e^{-i\vec{q}\cdot\vec{r}}\rho(r)$$

$$= \frac{1}{Q}\int d^3 r\, \rho(r)[1 - i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + \dots] = 1 - \frac{q^2}{6Q}\int d^3 r\, r^2 \rho(r) + \dots$$

$$\langle r^2 \rangle = -6 \frac{d}{dq^2} F(q^2)$$

Electromagnetic form factor of the pion from TJLAB

[Volmer et al., 2001]



$$Q^2 = \vec{q}^2 - q_0^2 = -q^2 = -t$$

Covariant definition:

$$F_\pi(q^2)\, 2P^\mu\, \delta^{ab} = \langle \pi^a(p') \mid J^\mu_{em}(0) \mid \pi^b(p) \rangle$$

where $q = p' - p$, $P = \frac{1}{2}(p' + p)$

Gravitational form factor

Electromagnetic current:

$J_V^\mu = \sum_{q=u,d,\ldots} \bar{q}(x) \frac{\tau_a}{2} \gamma^\mu q(x)$

Gravitational form factor

Electromagnetic current:
$J_V^\mu = \sum_{q=u,d,\ldots} \bar{q}(x) \frac{\tau_a}{2} \gamma^\mu q(x)$

Energy-momentum tensor:
$\Theta^{\mu\nu} = \sum_{q=u,d,\ldots} \bar{q}(x) \frac{\mathrm{i}}{2} \left(\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu\right) q(x) + \text{gluons}$

For the pion two structures (form factors):

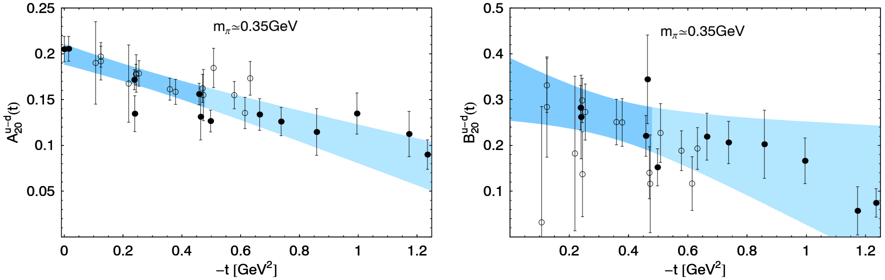
$$\langle \pi^b(p') \mid \Theta^{\mu\nu}(0) \mid \pi^a(p) \rangle = \frac{1}{2} \delta^{ab} \left[(g^{\mu\nu} q^2 - q^\mu q^\nu) \Theta_1(q^2) + 4 P^\mu P^\nu \Theta_2(q^2) \right]$$

(Θ_1 - spin-2, Θ_2 - spin-0)

How to determine Θ_1 and Θ_2?
Lattices, exclusive high-energy processes – no need to scatter gravitons!

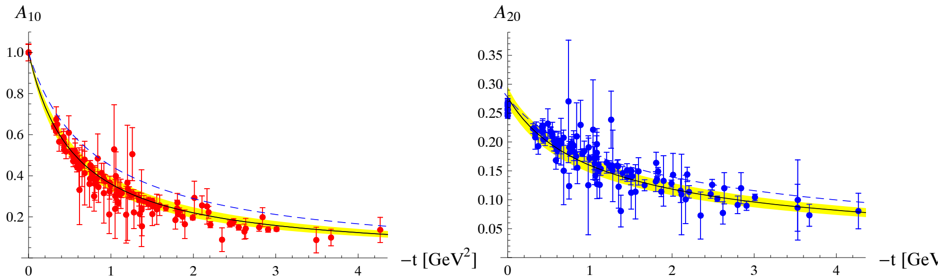
Nucleon

$$\langle N(p') \mid \Theta_{\mu\nu}(0) \mid N(p)\rangle = \bar{u}(p') \left[\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} A_{20}(t) \right.$$
$$\left. + \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\nu\rho}) q^\rho}{4M_N} B_{20}(t) + \frac{q_\mu q_\nu - q^2 g_{\mu\nu}}{M_N} C_{20}(t) \right] u(p)$$



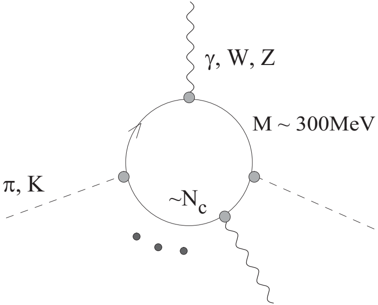
[Hagler et al., 2007] (can compute in QM, Bochum, Wakamatsu)

Full-QCD lattice results, pion



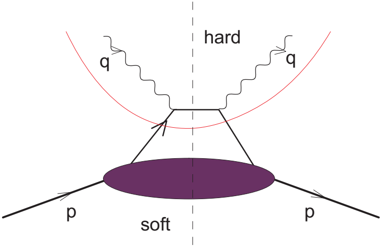
The electromagnetic form factor (left) and quark part of the spin-2 gravitational form factor (right) in SQM (solid line) and NJL model (dashed line) compared to the lattice data from [Brömmel 2005/7].

Chiral quark models



- soft regime → chiral sym. breaking
- NJL (Nobel 2008), instanton liquid, DSE
- relatively few parameters (traded for f_π, m_π, . . .)
- very many processes can be computed!
- no confinement - careful not to open the $q\overline{q}$ threshold

Example: Deep Inelastic Scattering



$Q^2 = -q^2, \;\; x = \frac{Q^2}{2p\cdot q}, \;\; Q^2 \to \infty$

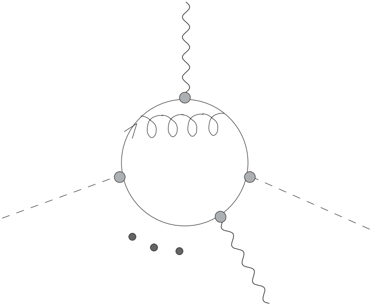
Factorization of soft and hard processes, Wilson's OPE, twist expansion

$$\langle J(q)J(-q)\rangle = \sum_i C_i(Q^2;\mu)\langle \mathcal{O}_i(\mu)\rangle, \;\; F(x,Q) = F_0(x,\alpha(Q)) + \frac{F_2(x,\alpha(Q))}{Q^2} + \ldots$$

The soft matrix element can be computed in low-energy models!

$$F_i(x,\alpha(Q_0))|_{\rm model} = F_i(x,\alpha(Q_0))|_{\rm QCD}\,, \quad Q_0 - \textbf{the matching scale}$$

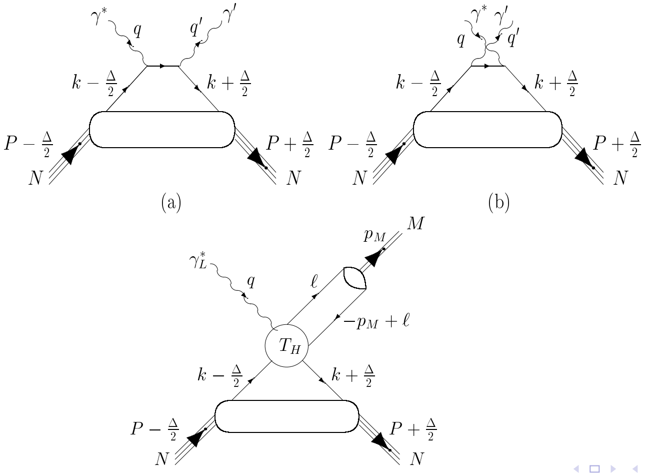
QCD evolution



inclusion of gluons

- Here: DGLAP (good for intermediate x)
- Chiral quark models provide **dynamically** the non-perturbative initial conditions for the QCD evolution
- Inclusive and exclusive high-energy processes and lattice calculations provide the relevant data to verify the scheme

Exclusive processes in QCD



Deeply
Virtual
Compton
Scattering

Hard
Meson
Production

Definition of Generalized Parton Distributions

Twist-2 even-parity GPDs of the pion

non-singlet:

$$\mathcal{H}^{q,I=1}(x,\zeta,t)=\int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle \pi^+(p+q)|\bar{\psi}(0)[0,z]\gamma^+\tau_3\psi(z)|\pi^+(p)\rangle\Big|_{z^+=0,z^\perp=0}$$

(similarly for singlet quarks and gluons)

$p^2 = m_\pi^2$, $q^2 = -2p\cdot q = t$, $\zeta = q^+/p^+$
ζ - momentum transfer along the light cone

($[0,z] = 1$ in the light-cone gauge)

Reviews:

- K. Goeke, M. V. Polyakov, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401, hep-ph/0106012
- M. Diehl, Phys. Rept. 388 (2003) 41, hep-ph/0307382
- A. V. Belitsky, A. V. Radushkin, Phys.Rept.418(2005)1, hep-ph/0504030

GPDs provide very rich information of the structure of hadrons, encoding form factors, PDFs, ... Data may come from such processes as $ep \to ep\gamma$, $\gamma p \to pl^+l^-$, $ep \to epl^+l^-$, or from lattices. Small cross sections of exclusive processes require very high accuracy experiments. First results for the nucleon are coming from HERMES and CLAS, also COMPASS, H1, ZEUS

Formal features

Symmetric notation: $\xi = \frac{\zeta}{2-\zeta}$, $X = \frac{x-\zeta/2}{1-\zeta/2}$, with $0 \le \xi \le 1$, $-1 \le X \le 1$

$H^{I=0}(X,\xi,t) = -H^{I=0}(-X,\xi,t)$, $H^{I=1}(X,\xi,t) = H^{I=1}(-X,\xi,t)$.

For $X \ge 0$ we have $\mathcal{H}^{I=0,1}(X,0,0) = q(X)$ - the usual PDF

The following **sum rules** hold:

$$\forall\xi: \quad \int_{-1}^{1} dX\, H^{I=1}(X,\xi,t) = 2F_V(t),$$

$$\int_{-1}^{1} dX\, X\, H^{I=0}(X,\xi,t) = 2\theta_2(t) - 2\xi^2\theta_1(t),$$

where $F_V(t)$ is the electromagnetic form factor, while $\theta_1(t)$ and $\theta_2(t)$ are the gravitational form factors (related to the charge conservation and the momentum sum rule in DIS)

The **polynomiality** conditions (Lorentz invariance, time reversal, and hermiticity):

$$\int_{-1}^{1} dX\, X^{2j}\, H^{I=1}(X,\xi,t) = 2\sum_{i=0}^{j} A_{2j+1,2i}(t)\xi^{2i},$$

(similarly for singlet)
A's – generalized form factors (GFFs)

Another way to look at GFFs:

$$\langle \pi^+(p') |\, \overline{u}(0)\, \gamma^{\{\mu}\, i\overleftrightarrow{D}^{\mu_1} i\overleftrightarrow{D}^{\mu_2} \dots i\overleftrightarrow{D}^{\mu_{n-1}\}}\, u(0)\, | \pi^+(p) \rangle =$$

$$2P^{\{\mu}P^{\mu_1}\dots P^{\mu_{n-1}\}}A_{n0}(t) + 2\sum_{\substack{k=2\\ \text{even}}}^{n} q^{\{\mu}q^{\mu_1}\dots q^{\mu_{k-1}}P^{\mu_k}\dots P^{\mu_{n-1}\}}\, 2^{-k} A_{nk}(t)$$

GPDs may be viewed as an infinite collection of GFFs

The **positivity bound**:

$$|H_q(X,\xi,t)| \leq \sqrt{q(x_{\rm in})q(x_{\rm out})}, \qquad \xi \leq X \leq 1.$$

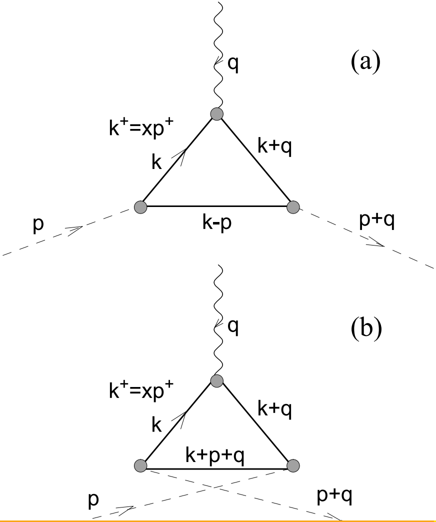
where $x_{\rm in} = (x+\xi)/(1+\xi)$, $x_{\rm out} = (x-\xi)/(1-\xi)$.

Finally, a **low-energy theorem** $H_{I=1}(2z-1,1,0) = \phi(z)$ holds, where ϕ is the pion distribution amplitude (DA)

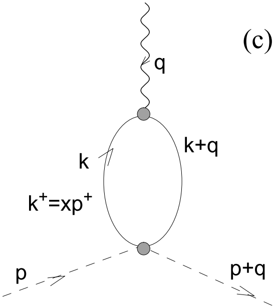
Above relations and bounds impose severe constraints on the form of the GPDs

All are satisfied in our quark-model calculation

QM evaluation of the GPDs



Large-N_c = one loop



Direct (a), crossed (b), and contact (c) contribution (D-term) to the GPD of the pion (wavy line: γ^+)

PDF, QM

With $\zeta = t = 0$, the GPD becomes the PDF. The Nambu–Jona-Lasinio model (Davidson, Arriola, 1995) gives

$$q(x) = 1$$

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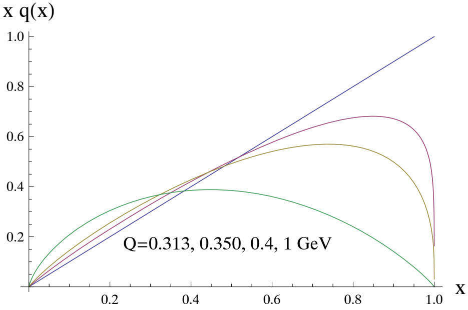
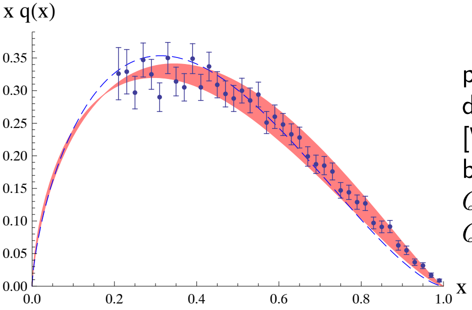
$$q(x) = 1$$

LO DGLAP QCD evolution (good at intermediate x) of the non-singlet part to growing scales



PDF, QM vs. E615

LO DGLAP QCD evolution of the non-singlet part to the scale $Q^2 = (4\ \mathrm{GeV})^2$ of the E615 Fermilab experiment:



points: Drell-Yan from E615
dashed: reanalysis of data [Wijesooriya et al., 2005]
band: valence QM PDF evolved to $Q = 4$ GeV from the QM scale $Q_0 = 313^{+20}_{-10}$ MeV

The quark-model scale Q_0

Various ways to fix: PDF, DA, moments

From experiment, the momentum fraction carried by the valence quarks is [SMRS 1992, GRS 1999]

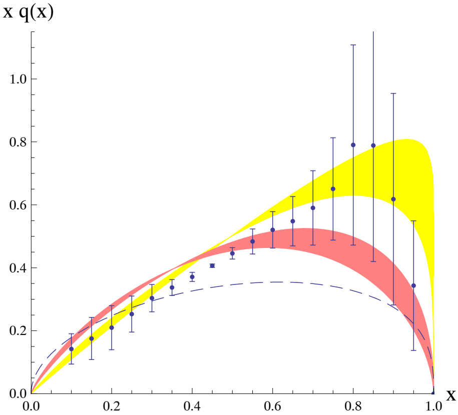
$$\langle x \rangle_v = 0.47(2) \quad \text{at} \quad Q^2 = 4 \text{ GeV}^2$$

QM scale = no gluons, may evolve backwards until $\langle x \rangle_v = 1$
$\rightarrow$ quark-model scale for NJL

$$Q_0 = 313^{+20}_{-10} \text{ MeV}$$

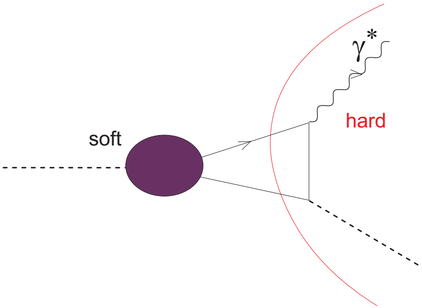
(here for the so called local model, for other QM Q_0 may vary)
At this scale $\alpha(Q_0^2)/(2\pi) = 0.34$, which makes the evolution very fast for the scales close to the initial value – calls for improvement!

PDF, QM vs. lattice



points: transverse lattice [Dalley, van de Sande, 2003]
yellow: QM evolved to 0.35 GeV
pink: QM evolved to 0.5 GeV
dashed: GRS parameterization at 0.5 GeV

Pion Distribution Amplitude



Definition (for π^+, leading twist):

$$\langle 0|\bar{d}(z)\gamma_\mu\gamma_5 u(-z)|\pi^+(q)\rangle =$$

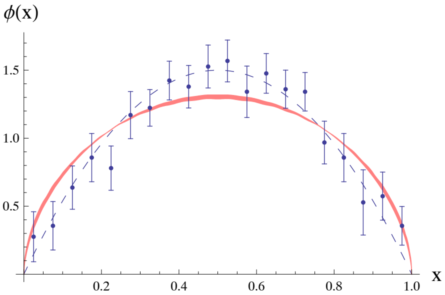
$$i\sqrt{2}f_\pi(q^2)q_\mu\int_0^1 dx e^{i(2x-1)q\cdot z}\phi(x)$$

Normalization $\int_0^1 dx\phi(x) = 1$, since $\langle 0|A_\mu^-(0)|\pi^+(q)\rangle = i f_\pi(q^2)q_\mu$

PDA is also relevant for the $\pi^0\gamma\gamma^*$ transition form factor measured by CLEO and CELLO

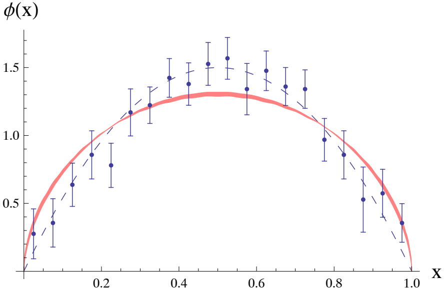
Similar studies in [Praszałowicz, Rostworowski, 2003]

PDA, QM vs. E791 and lattice data



points: E791 data from di-jet production in $\pi + A$
band: QM at $Q = 2$ GeV
dashed line: asymptotic form ($Q \to \infty$)

PDA, QM vs. E791 and lattice data



points: E791 data from di-jet production in $\pi + A$
band: QM at $Q = 2$ GeV
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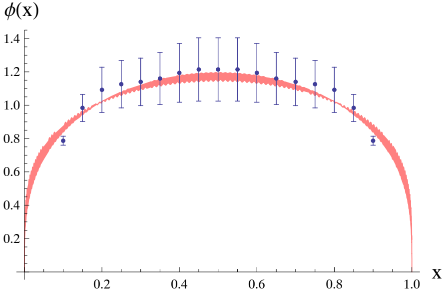


points: transverse lattice data [Dalley, van de Sande, 2003]
band: QM at $Q = 0.5$ GeV

Pion transition form factor

$$\Gamma^{\mu\nu}_{\pi^0\gamma\gamma}(p,q_1,q_2) = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha p^\beta F_{\pi\gamma\gamma}(p,q_1,q_2)$$

$$A = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2}, \qquad -1 \le A \le 1 \quad Q^2 = -(q_1^2 + q_2^2)$$

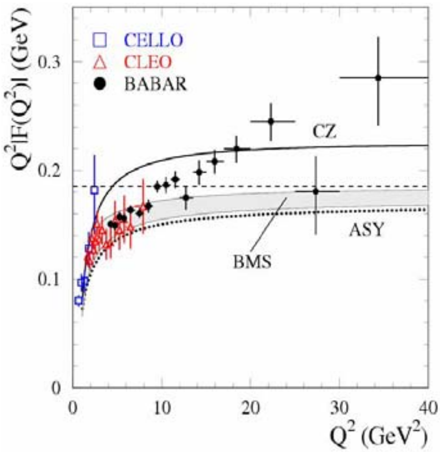
Brodsky-Lepage:

$$F_{\gamma^*\gamma^*\pi}(Q^2, A) = J^{(2)}(A)\frac{1}{Q^2} + J^{(4)}(A)\frac{1}{Q^4} + \dots$$

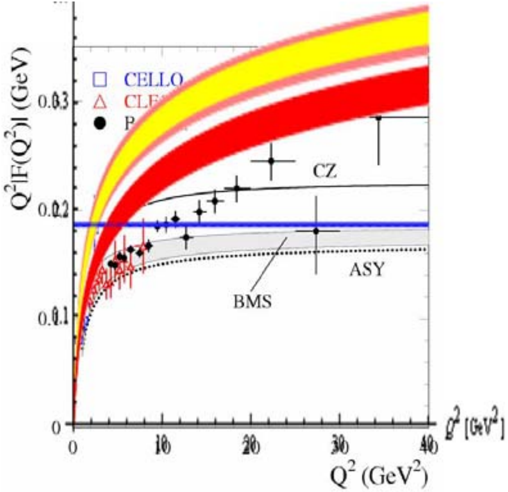
$$J^{(2)}(A) = \frac{4f_\pi}{N_c}\int_0^1 dx \frac{\varphi_\pi^{(2)}(x)}{1-(2x-1)^2A^2}$$

$$J^{(4)}(A) = \frac{8f_\pi\Delta^2}{N_c}\int_0^1 dx \frac{\varphi_\pi^{(4)}(x)[1+(2x-1)^2A^2]}{[1-(2x-1)^2A^2]^2}$$

New BaBar data



yellow: SQM, pink: Regge, $\sigma = 540$ MeV, red: Regge, $\sigma = 420$ MeV



GPD in chiral quark models

Analytic formulas derived, no factorization of the t-dependence
- sheds light on possible parameterizations.
Building block of the GPD in Spectral Quark Model (SQM):

$$J_{\rm SQM}(x,\zeta;t) = (\theta[x(\zeta-x)]\chi_1 + \theta[(1-x)(x-\zeta)]\chi_2)$$

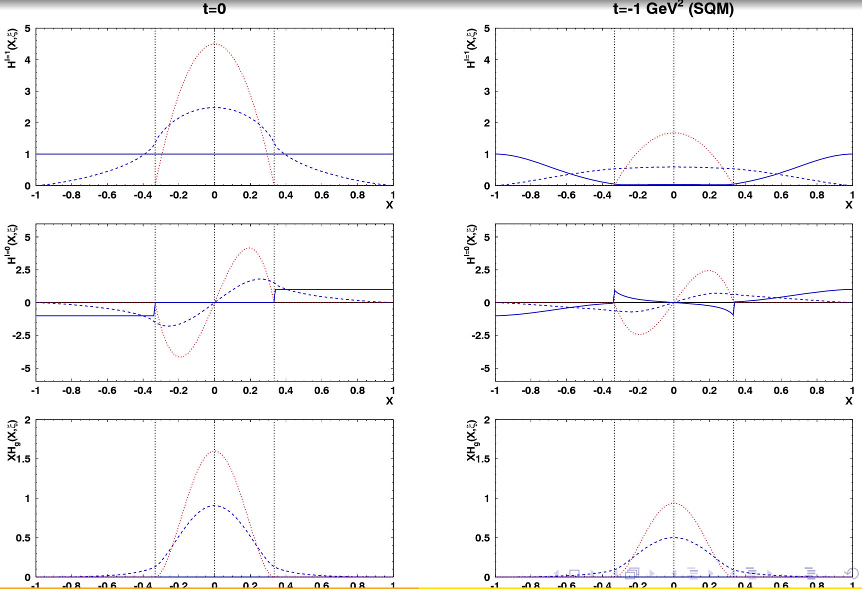
$$\chi_2 = \frac{2(x-1)\left[3(\zeta-1)M_V^2 + t(x-1)^2\right]}{\left[(\zeta-1)M_V^2 + t(x-1)^2\right]^2},$$

$$\chi_1 = \frac{(x(\zeta-2)+\zeta)\left(3M_V^2(\zeta-1)\zeta^2 + t\left(\left(\zeta^2+8\zeta-8\right)x^2 + 2(4-5\zeta)\zeta x + \zeta^2\right)\right)}{\left((\zeta-1)M_V^2 + t(x-1)^2\right)^2\left(\zeta^2 + \frac{4tx(x-\zeta)}{M_V^2}\right)^{3/2}} + \frac{1}{2}\chi_2$$

M_v – mass of the ρ meson

Similar studies in [Praszałowicz, Rostworowski, 2003] in a non-local model

Next slide:
LO DGLAP-ERBL evolution for SQM with $\xi = 1/3$. Solid - initial condition, dashed - evolved to $Q^2 = (4\mathrm{GeV})^2$, dotted - asymptotic form. Code: [Golec-Biernat, Martin, 1999]


t=0
t=-1 GeV² (SQM)
$H^{I=1}(X,\xi)$
$H^{I=0}(X,\xi)$
$XH_g(X,\xi)$
X

Gravitational form factors

Electromagnetic current:
$J_V^\mu = \sum_{q=u,d,\ldots} \bar{q}(x) \frac{\tau_a}{2} \gamma^\mu q(x)$

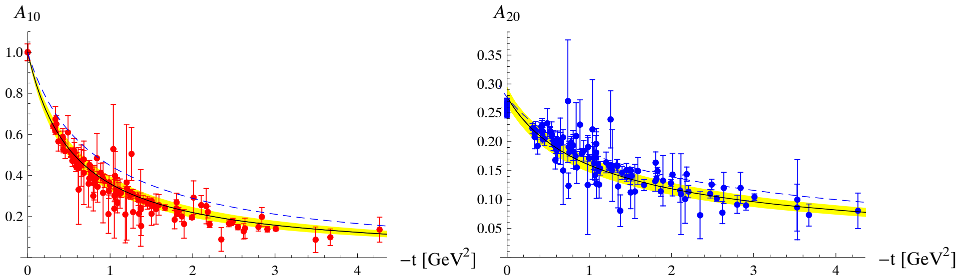
Energy-momentum tensor:
$\Theta^{\mu\nu} = \sum_{q=u,d,\ldots} \bar{q}(x) \frac{i}{2} \left(\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu\right) q(x) + \text{gluons}$

Two structures (form factors):

$$\langle \pi^b(p') \mid \Theta^{\mu\nu}(0) \mid \pi^a(p) \rangle = \frac{1}{2} \delta^{ab} \left[(g^{\mu\nu} q^2 - q^\mu q^\nu) \Theta_1(q^2) + 4 P^\mu P^\nu \Theta_2(q^2) \right]$$

traceless tensor – Θ_1 and scalar – Θ_2
Lattice, exclusive processes

Full-QCD Euclidean lattice results

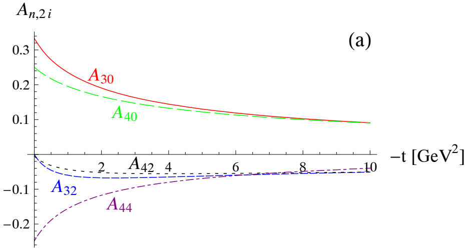


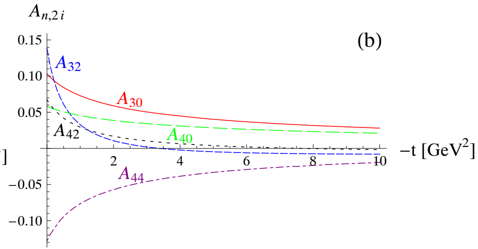
The EM FF (left) and the quark part of the gravitational form factor Θ_1 (right) in SQM (solid line) and NJL (dashed line), compared to data from [Brömmel et al., 2005-7]

Quark-model relation: $\langle r^2 \rangle_\Theta = \frac{1}{2} \langle r^2 \rangle_V$

Matter more concentrated than charge!

Higher-order form factors - predictions





The quark GFFs $A_{3,2i}$ and $A_{4,2i}$ at the quark-model scale $Q_0 \sim 320$ MeV (a) and at the lattice scale $Q = 2$ GeV (b)

Quark moments at $t = \xi = 0$

With the notation $\langle x^n \rangle = A_{n+1,0}(0)$, one finds at the lattice scale of $Q = 2$ GeV [Brömmel et al., 2007]

$$\langle x \rangle = 0.271 \pm 0.016$$
$$\langle x^2 \rangle = 0.128 \pm 0.018$$
$$\langle x^3 \rangle = 0.074 \pm 0.027$$
$$\text{(lattice)}$$

while in QM after the LO DGLAP evolution to the lattice scale

$$\langle x \rangle = 0.28 \pm 0.02$$
$$\langle x^2 \rangle = 0.10 \pm 0.02$$
$$\langle x^3 \rangle = 0.06 \pm 0.01$$
$$\text{(chiral quark models)}$$

Agreement within uncertainties

Other quantities

- Photon DAs (with A. E. Dorokhov)
- Transition Distribution Amplitudes (TDA) [Pire, Szymanowski, 2005] (as the GPD, but between the π and γ states)
- b-representation of GPDs and transverse lattices

1. Link between high- and low-energy analyses: quark models provide initial conditions for the QCD evolution
2. Analytic formulas – useful for general properties, (*e.g.*, no factorization of the t-dependence
3. Q_0 low, as follows form the momentum sum rule
4. With LO DGLAP-ERBL evolution the agreement with the data and lattices very reasonable (PDF, DA, GFFs, GPD, photon DA, TDA)
5. The pion transition form factor in qualitative agreement with the new BaBar data
6. The pion gravitational form factor agrees with the lattice data, the mean squared EM radius is twice the gravitational one
7. Predictions can be further tested with future lattice simulations for higher-order form factors
8. GPDs of the **nucleon** - see the talk by **Wakamatsu**: more challenging (Bochum, Tübingen – soliton) but data exist