# Limits on hadron spectrum from bulk medium properties

**Wojciech Broniowski**

Inst. of Nuclear Physics PAN, Cracow, and
Jan Kochanowski U., Kielce

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# Outline

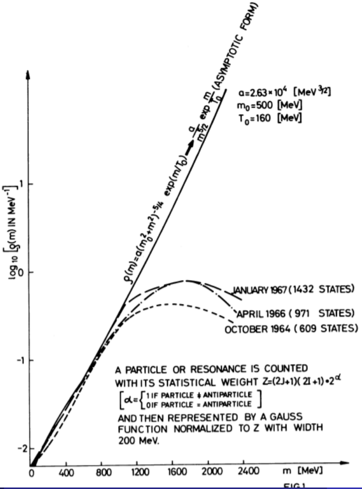
- A look at the PDG hadron spectra
- Hagedorn hypothesis
- Hadron resonance gas vs LQCD
- Contribution from the high-lying QCD spectrum to thermodynamics
- Treatment of the $\sigma$ meson
- Repulsive channels
- Contribution from the high-lying QCD spectrum

# A look at the PDG spectra of light (uds) hadrons

# Hagedorn growth $\rho(m) \sim \exp\left(\frac{m}{T_H}\right)$

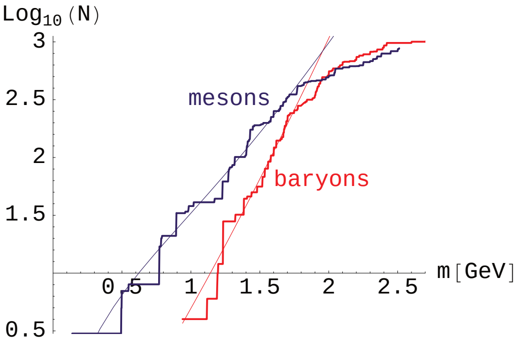
## Density of states

Hagedorn & Ranft 1967



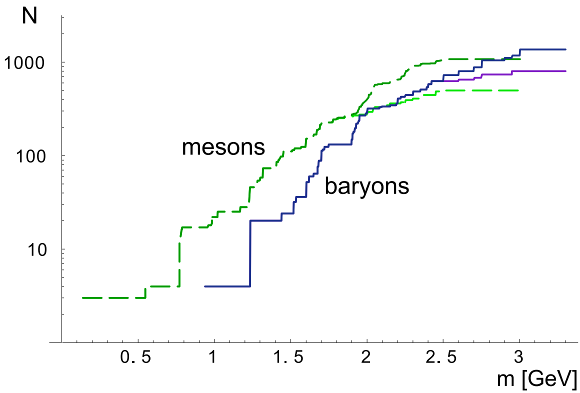
# Hagedorn growth $\rho(m) \sim \exp\left(\frac{m}{T_H}\right)$

Number of states below $M$   WB & Florkowski 2000, also Bled Proc. 2000



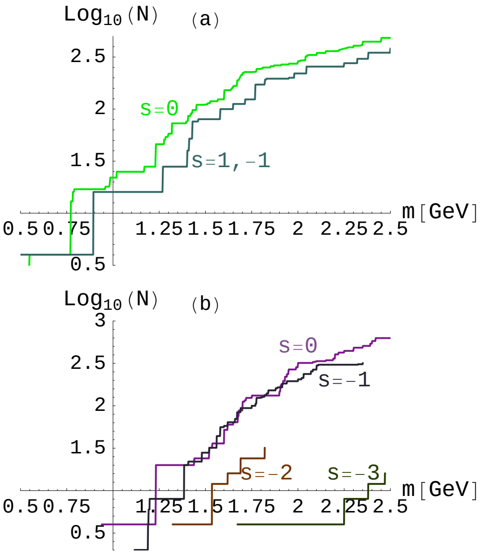
# Hagedorn growth $\rho(m) \sim \exp\left(\frac{m}{T_H}\right)$

Number of states below $M$ WB, WF & Glozman 2004




extra unconfirmed states + symmetry

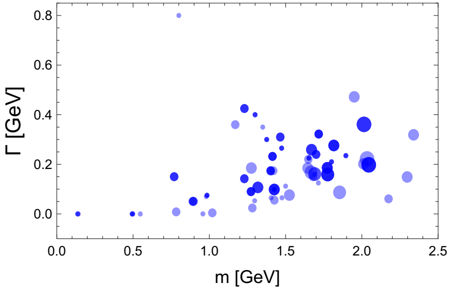
# Flavor independence



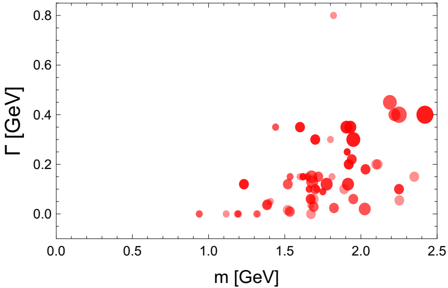

(a) – mesons, (b) – baryons

# Width

mesons



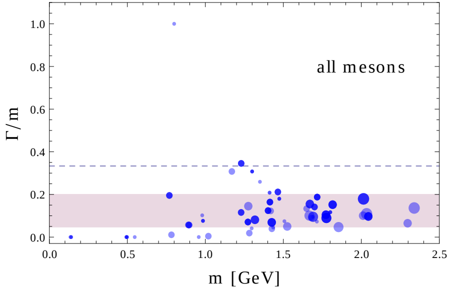

baryons



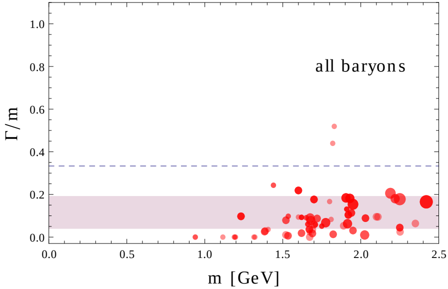

[ size $\sim (2J+1)$, intensity $\sim (2I+1)$ ]

Mesonic strings: $l \sim m/\sigma$, the decay rate of a string per unit time, $\Gamma$, is proportional to $l$, which yields constant $\Gamma/m$

mesons



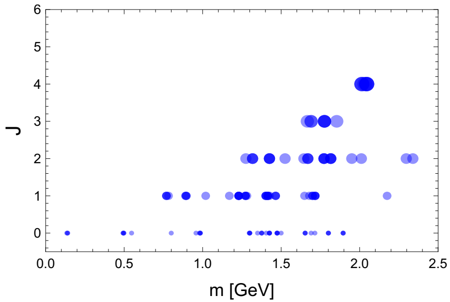

baryons



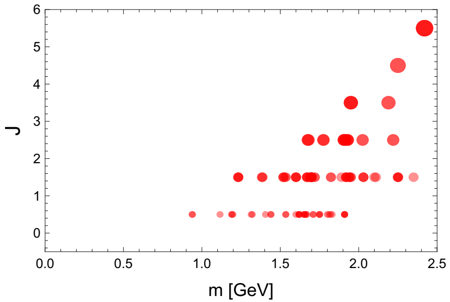

complies to $1/N_c$ argument

# Spin vs. mass

mesons



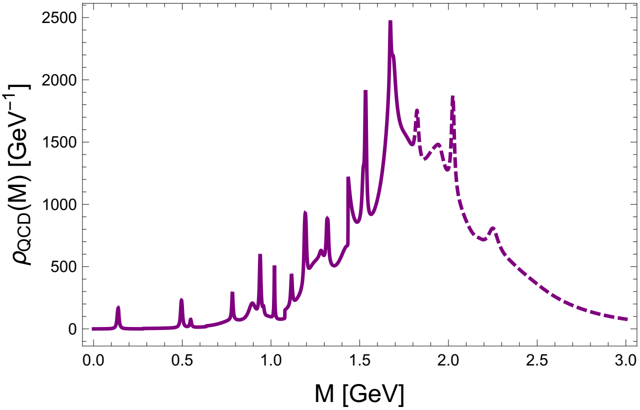

baryons




(Regge trajectories)

# Particle Data Tables

All hadrons from PDG Tables (Breit-Wigner functions with thresholds)



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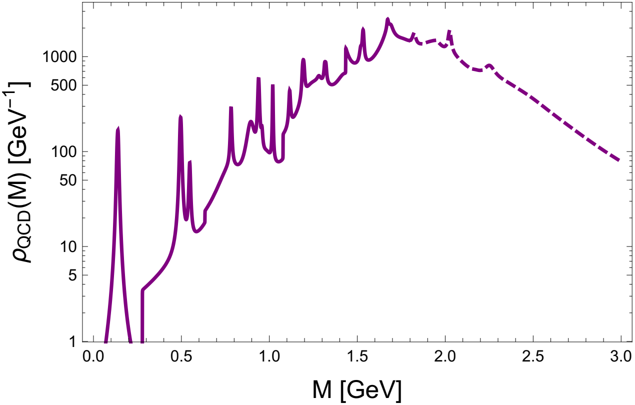

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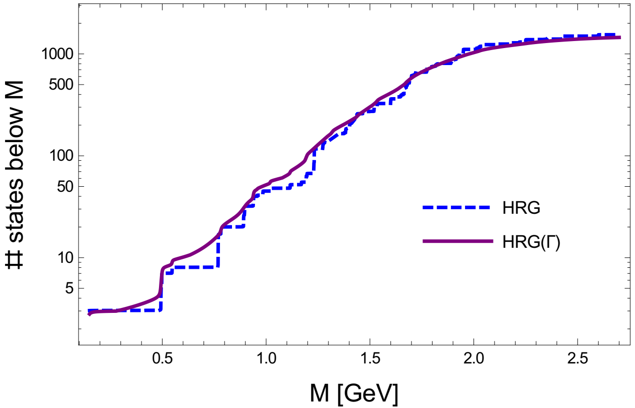
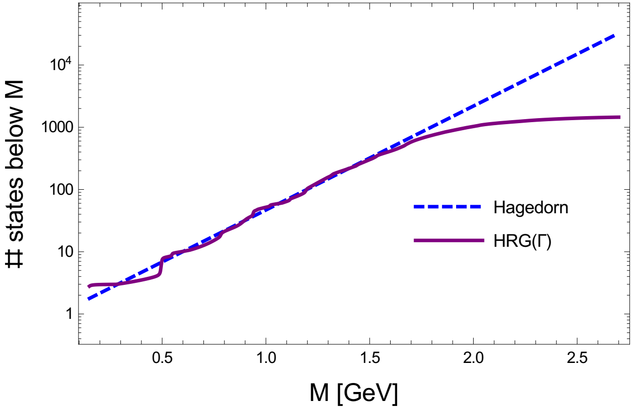


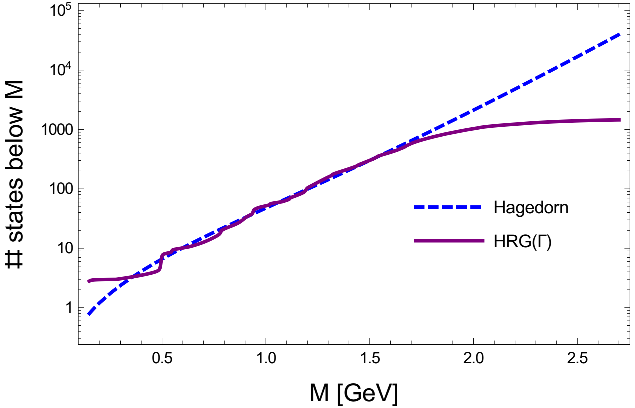
# Hagedorn hypothesis

$$\rho(m) = \mathrm{const}\,\exp\left(\frac{m}{260\mathrm{MeV}}\right)$$



# Hagedorn hypothesis

$$\rho(m) = \frac{\text{const}}{[(500\text{MeV})^2 + m^2]^{5/4}} \exp\left(\frac{m}{190\text{MeV}}\right)$$



# Hadron resonance gas

# Hadron resonance gas

Virial expansion (Kamerlingh Onnes): $p/T = \rho + B_2(T)\rho^2 + B_3(T)\rho^3 + \ldots$
Generalization:

$$\ln Z = \ln Z^{(1)} + \ln Z^{(2)} + \ldots$$

($1 \to 1$, $2 \to 2$, etc., processes)

Sum over stable particles:

$$\ln Z^{(1)} = \sum_k \ln Z_k^{\rm stable} = \sum_k f_k V \int \frac{d^3p}{(2\pi)^3} \ln \left[1 \pm e^{-E_p/T}\right]^{\pm 1}$$

Second-order virial coefficient – sum over pairs of stable particles:

$$\ln Z^{(2)} = \sum_K f_K V \int_0^\infty d_K(M)\, dM \int \frac{d^3P}{(2\pi)^3} \ln \left[1 \pm e^{-E_P/T}\right]^{\pm 1}$$

where $d_K(M) = \frac{d\delta_K(M)}{\pi dM}$

# Quantum-mechanical derivation

$$\psi_l(r) \propto \sin[kr - l\pi/2 + \delta_l]$$

If we confine our system into a sphere of radius $R$, the condition

$$kR - l\pi/2 + \delta_l = n\pi$$

$n = 0, 1, 2, \ldots$, must be met, since $\psi_l(r)$ has to vanish at the boundary. For a free system $kR - l\pi/2 = n_{\rm free}\pi$. In the limit of $R \to \infty$ (to have large $n$), upon subtraction,

$$\frac{\delta_l}{\pi} = n - n_{\rm free}.$$

Differentiation with respect to $M$ yields $d\delta_l/(\pi dM)$ as the difference of the density of states in $M$ of the interacting and free systems

# Remarks

QM arguments extended to field theory by [Dashen, Ma & Bernstein 1969]
(see also [Weinhold, Friman & Noerenberg 1996, WB, Florkowski & Hiller 2004])

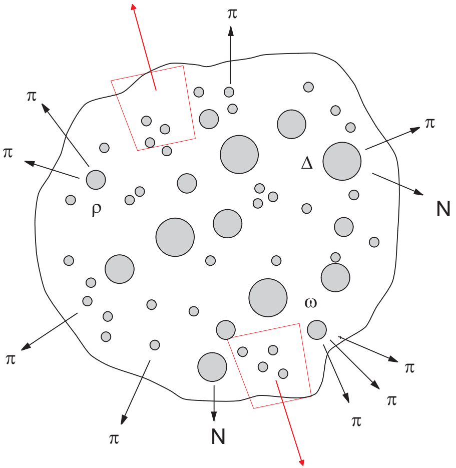
For sharp resonances $d_k(M) \simeq \delta(M - m_k) \rightarrow$ can be treated as stable particles (essence of the treatment of the hadron resonance gas)

For broad resonances use physical phase shifts!

For higher terms in the virial expansion no practical methods

Concept of hadron resonance gas

# Hadronic soup




$T_c \simeq 165$ MeV

# Hadron resonance gas vs. LQCD

# Trace anomaly

For ideal gas

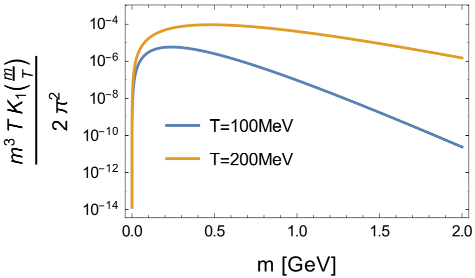
$$
\begin{aligned}
T^\mu_\mu &= \epsilon - 3P = \int_0^\infty dm\, \rho_{\mathrm{QCD}}(m) \int \frac{d^3p}{(2\pi)^3 E_p} m^2 \frac{1}{e^{E_p/T} \pm 1} \\
&= \int_0^\infty dm \sum_{n=1}^\infty \left[ \frac{1}{2} \left( (-1)^{2j+1} + 1 \right) (-1)^{n+1} + \frac{1}{2} \left( (-1)^{2j} + 1 \right) \right] \\
\times &\quad \rho_{\mathrm{QCD}}(m) \frac{m^3 T K_1\left(\frac{mn}{T}\right)}{2\pi^2 n}
\end{aligned}
$$

with $E_p = \sqrt{m^2 + p^2}$, $K_n(z) \simeq \sqrt{\frac{\pi}{2}} e^{-z} \sqrt{\frac{1}{z}}$

(damped weighting, singularity at $T \to T_H$)

# Weighting functions

For $T^\mu_\mu$

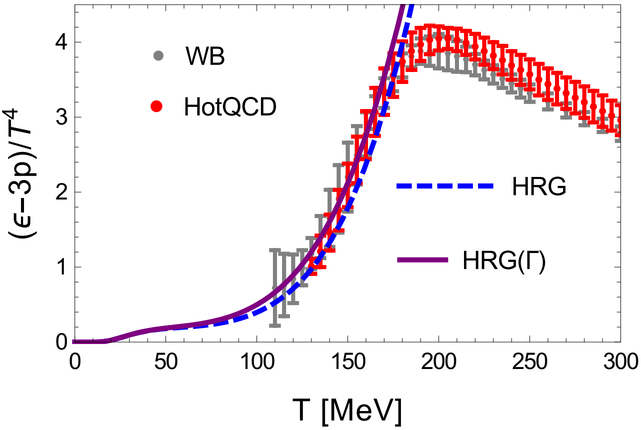



At higher temperatures sensitivity to high-lying mass spectrum (but cannot go too high)

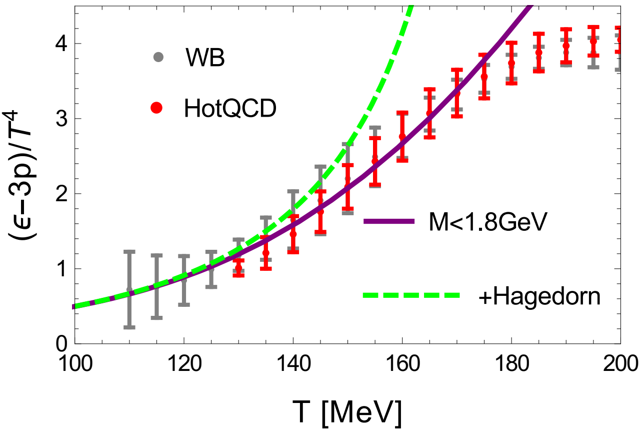
Many previous studies with HRG: Karsch, Redlich, Tawfik, Andronic, Braun-Munzinger, Stachel, Winn, Huovinen, Petreczky, Megias, Arriola, Salcedo ...

Lattice data (physical quark masses and continuum limit):
WB = Wuppertal-Budapest 2013, HotQCD 2014

# Trace anomaly

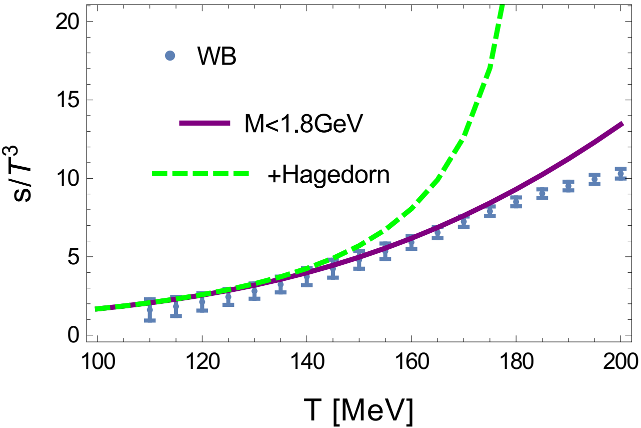


# Trace anomaly

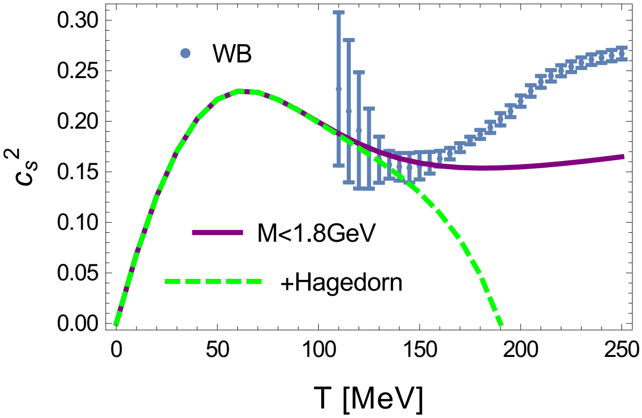



solid – PDG with $M < 1.8$ GeV
dashed – this + Hagedorn extrapolation with $T_H = 190$ MeV

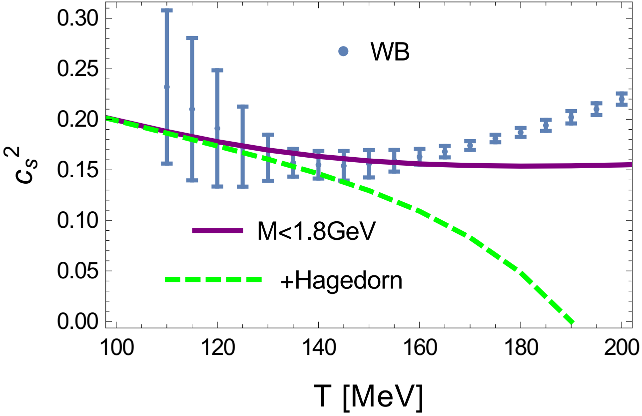
# Entropy density



# Sound velocity



# Sound velocity



# Remarks

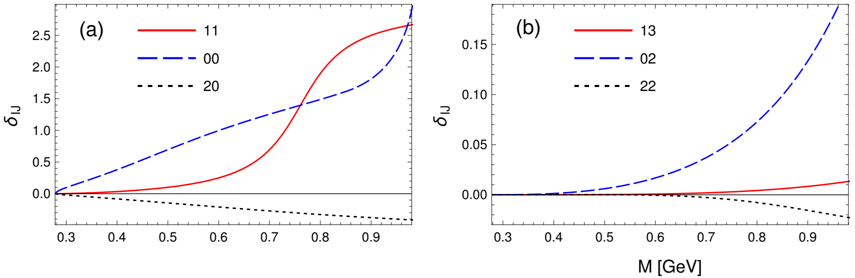
- Agreement remarkable
- Recall the very simple treatment (sharp-resonance approximation)

# The sigma meson

[WB, Giacoca & Begun 2015]

# Experimental $\pi - \pi$ phase shifts

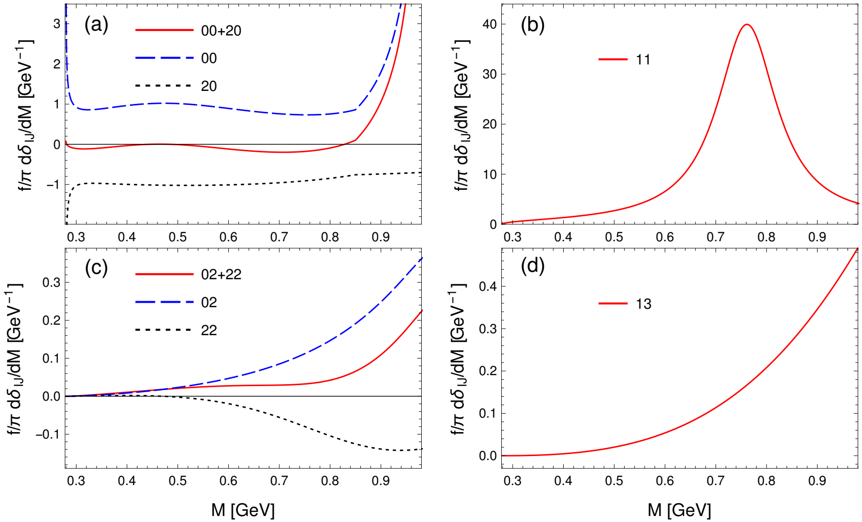
Parameterizations of physical data:



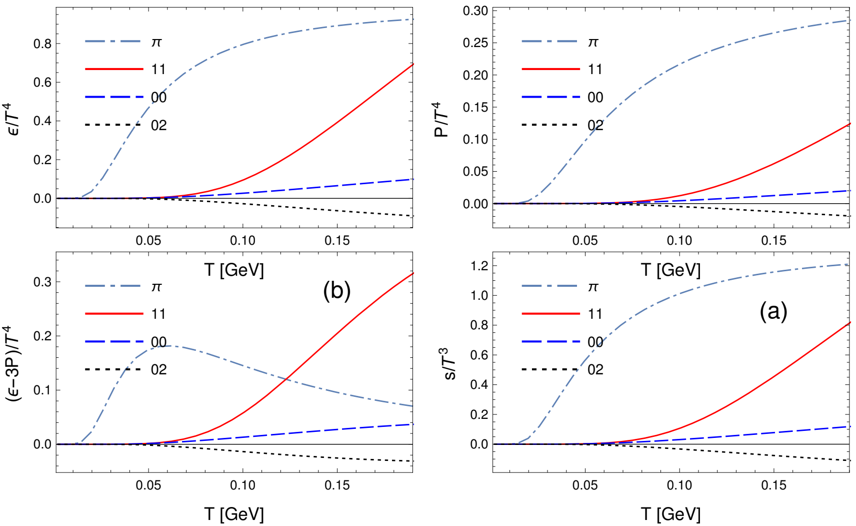

Recall the formula $d_{IJ} = \frac{d\delta_{IJ}}{\pi dM}$

# Derivatives of phase shifts

$$f = (2I+1)(2J+1)$$



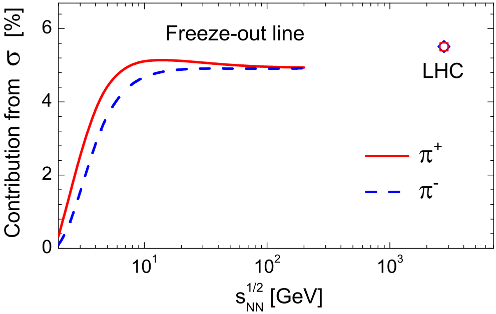
# Anatomy of bulk properties




Almost exact cancellation of the $\sigma$ contribution

# Abundance of pions

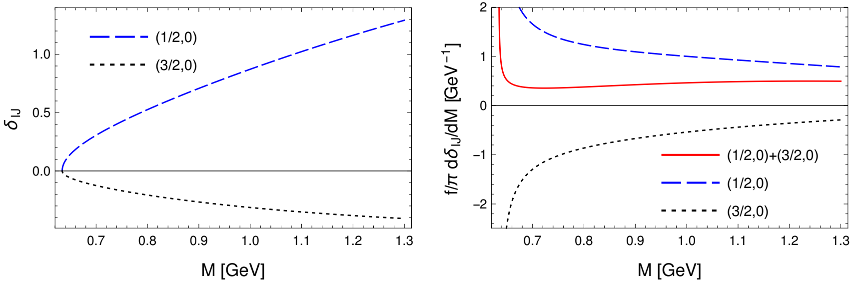
The naive contribution:




... in reality is not there due to the cancellation

(5% effect from one state)

$K - \pi$ phase shifts:




partial cancellation

# Remarks on repulsion

- There is also repulsion!
- Levinson's theorem – some states must bring in repulsion, as the phase shift must go down to 0
- Attempts to model repulsion with excluded volume
- $n_1 \to n_2$ processes (higher virial coefficients) ?
- Higher states may be incorporated only when accompanied with some repulsion (opposite effects)

# Conclusions

# Conclusions

- Can we find limits on the high-lying light-quark hadron spectrum?
- LQCD offers probes
- Overall agreement with HRG remarkable
- More states via Hagedorn hypothesis – significant effects in bulk properties close to the transition point
- Effects of repulsion may be important (see the example of the $\sigma$ meson) and work in the opposite direction
- Higher-order terms in the virial expansion