Gluonowy kondensat $\langle A^2 \rangle$

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[Enrique Ruiz Arriola, Patrick Bowman, WB, "Landau-gauge condensates from the quark propagator on the lattice", PRD **70** (2004) 097505]

Outline:

• condensates and OPE • Narison-Zakharov and new leading power corrections • Lavelle propagators • Landau-gauge condensates from lattice measurements • interpretation and significance of the results: gauge-independence, Gribov copies, topological structure of the vacuum and confinement

Condensates in QCD

A correlator of two currents,

$$\Pi_{AB}(q) \equiv i \int d^4x e^{iq\cdot x} \langle 0 \mid T\{J_A(x), J_B^\dagger(0)\} \mid 0\rangle$$

can be expanded at large Euclidean momenta with the help of the Wilson expansion. For vector currents, $J_{\mu,a}^{V\pm A} = \bar{q}(1 \pm \gamma_5)\frac{\tau_a}{2}q$, one gets explicitly $\Pi_{\mu\nu,ab}^{V\pm A} = (q_\mu q_\nu - g_{\mu\nu}q^2)\delta_{ab}\Pi^{V\pm A}$ with

$$\begin{aligned}
\Pi^{V+A} &= -\frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\log(Q^2/\mu^2) + \frac{1}{12}\frac{\langle\frac{\alpha_S}{\pi}(G^a_{\mu\nu})^2\rangle}{Q^4} + \frac{64\pi}{81}\frac{\alpha_S\langle\bar{q}q\rangle^2}{Q^6} + \ldots \\
\Pi^{V-A} &= \frac{2m_c\langle\bar{q}q\rangle}{Q^4} - \frac{32\pi}{9}\frac{\alpha_S\langle\bar{q}q\rangle^2}{Q^6} + \ldots
\end{aligned}$$

(for other channels similar expressions)

Parameterization of non-perturbative physics in terms of condensates:
$m_c\langle\bar{q}q\rangle = -\frac{1}{2}m_\pi^2 f_\pi^2 = -0.8\times 10^{-4}\mathrm{GeV}^4$, $\frac{\alpha_S}{\pi}\langle G^2\rangle = (0.31^{+0.05}_{-0.10}\mathrm{GeV})^4$, ...

Vast applications in hadronic physics: QCD sum rules, lattice, . . .

Structures selected for condensates were dictated by gauge-invariance. K. G. Chetyrkin, S. Narison, and V. I. Zakharov (1999) append the standard OPE with the $1/Q^2$ power correction, introducing a gluon-mass-correction term. Subsequently, Narison and Zakharov (2001) demonstrated, to a big surprise, that some lattice data are much better reproduced with this term. Explicitly,

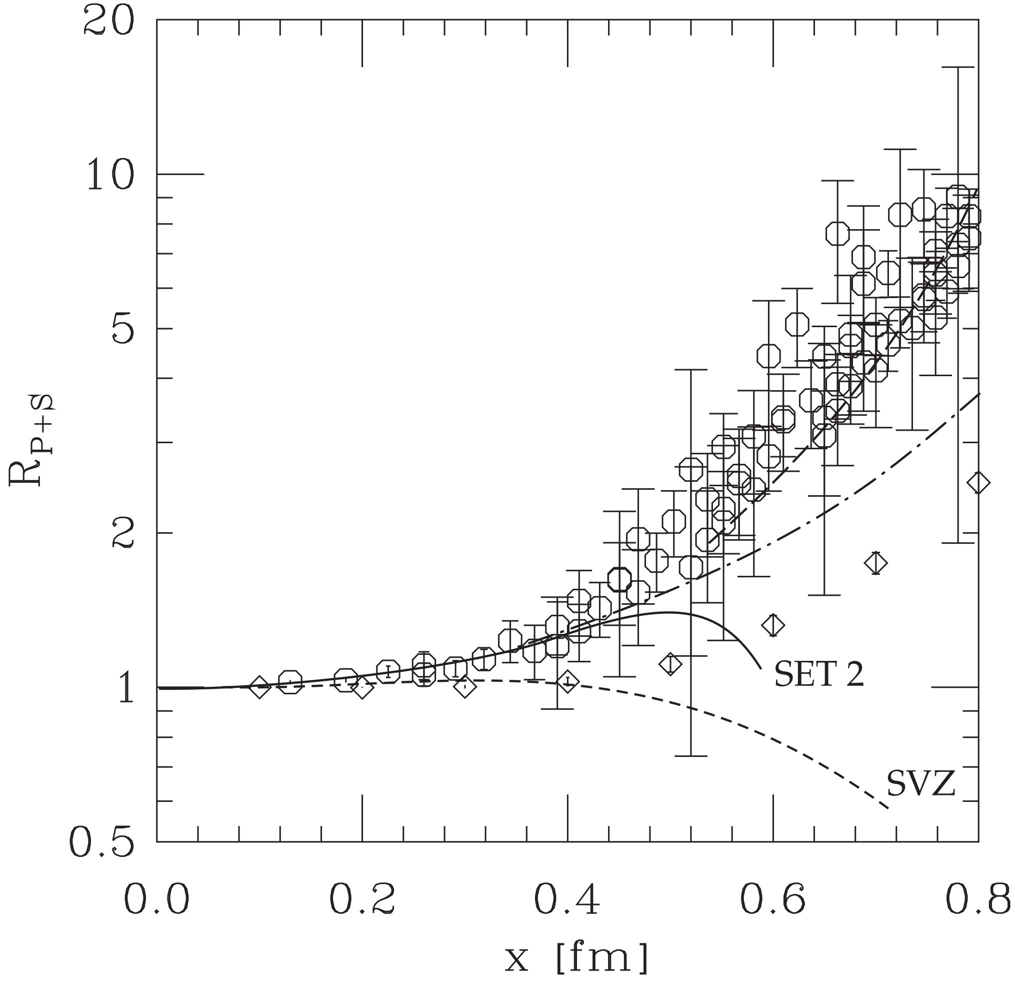
$$\Pi^{V+A} = -\frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\log(Q^2/\mu^2) - \frac{\alpha_S}{4\pi^3}\frac{\lambda^2}{Q^2} + \frac{1}{12}\frac{\langle\frac{\alpha_S}{\pi}G^2\rangle}{Q^4} + \ldots$$

$$\Pi^{V-A} = \frac{2m_c\langle\bar{q}q\rangle}{Q^4} - \frac{32\pi}{9}\frac{\alpha_S\langle\bar{q}q\rangle^2}{Q^6} + \ldots$$

In coordinate space this leads to quadratic terms

$$\frac{\Pi^{V+A}}{\Pi^{V+A}_{\rm pert}} \to 1 - \frac{\alpha_S}{4\pi}\lambda^2 x^2 - \frac{\pi^2}{48}\langle\frac{\alpha_S}{\pi}G^2\rangle x^4\log(x^2) + \ldots$$

$$R_{P+S} \equiv \frac{1}{2}\left(\frac{\Pi^P}{\Pi^P_{\rm pert}} + \frac{\Pi^S}{\Pi^S_{\rm pert}}\right) \to 1 - \frac{\alpha_S}{2\pi}\lambda^2 x^2 - \frac{\pi^2}{96}\langle\frac{\alpha_S}{\pi}G^2\rangle x^4 + \ldots$$



Narison and Zakharov
Phys. Lett. B 522 (2001)266

Fit: $\frac{\alpha_S}{\pi}\lambda^2 = -0.12\mathrm{GeV}^2$

"Comparison of the lattice data with the OPE predictions for two Sets of QCD condensate values. The dot-dashed curve is the prediction for SET 3 where the contribution of the x^2-term has been added to SET 2. The bold dashed curve is SET 3 + a fitted value of the $D = 8$ condensate contributions. The diamond curve is the prediction from the instanton liquid model of Shuryak."

Far reaching consequences:

- Standard OPE might be improved with the leading power corrections

- Interpretation via gluon mass, or, equivalently, via the gluon condensate $\langle A_\mu^a A_a^\mu \rangle$ (comes next)

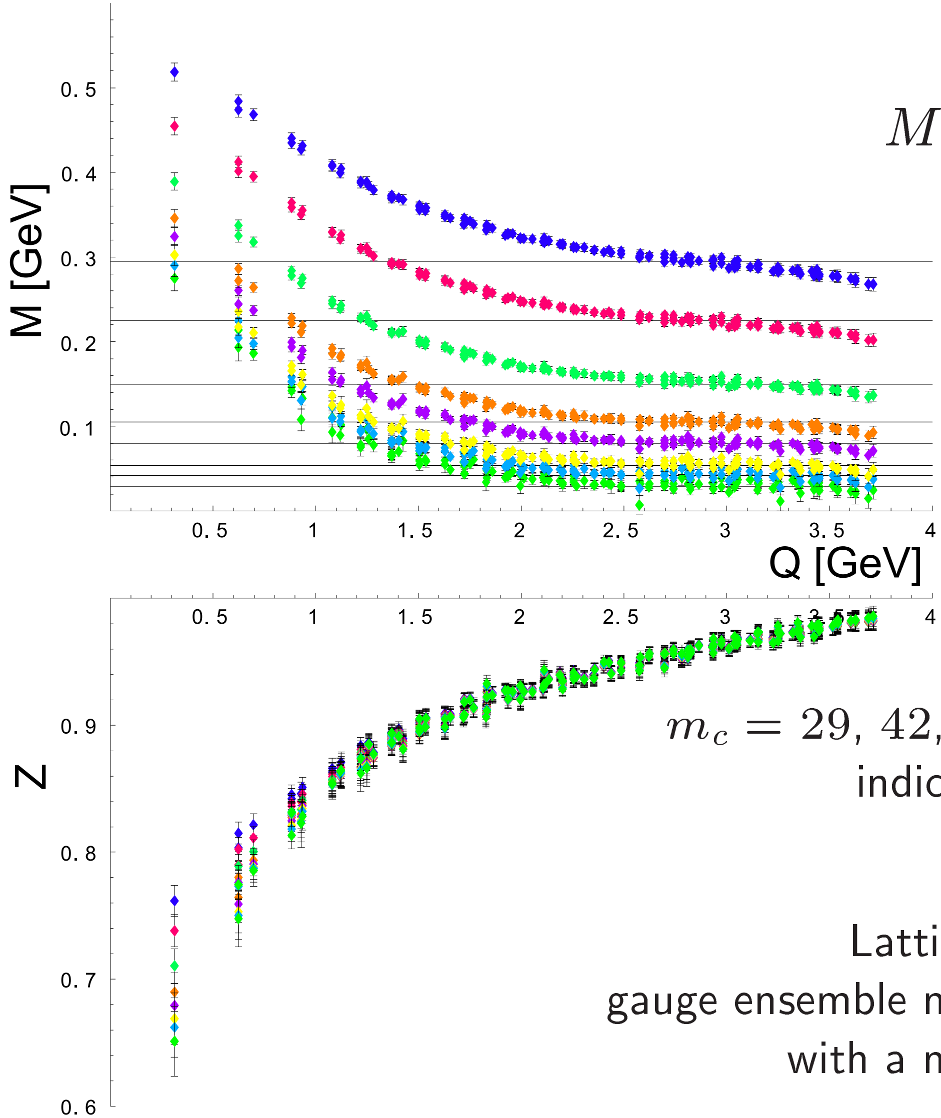
- This apparently gauge-dependent condensate may be written in terms of a gauge-independent expression! (last part of the talk)

... now we go on the lattice!

Quark propagator from the lattice

$S(p) = \frac{Z(p)}{\not{p} - M(p)}$, Landau gauge $\partial \cdot A = 0$

Bowman *et al.* (2002)



$$M \to -\frac{4\pi^2 d_M \langle \bar{q}q \rangle_\mu [\log(Q^2/\Lambda^2_{\rm QCD})]^{d_M - 1}}{3Q^2 [\log(\mu^2/\Lambda^2_{\rm QCD})]^{d_M}}$$

$$+\frac{m(\mu^2)[\log(\mu^2/\Lambda^2_{\rm QCD})]^{d_M}}{[\log(Q^2/\Lambda^2_{\rm QCD})]^{d_M}}$$

$$d_M = 12/(33 - 2N_f)$$
$$N_f = 0,\ \mu = 0$$

Various sets of points correspond to $m_c = 29, 42, 54, 80, 105, 150, 225$, and 295 MeV, indicated by horizontal lines in the top panel

Lattice: "Asqtad" improved staggered action, gauge ensemble made of 100 quenched, $16^3 \times 32$ lattices with a nominal lattice spacing of $a = 0.124$ fm set from the static quark potential

Lavelle propagator

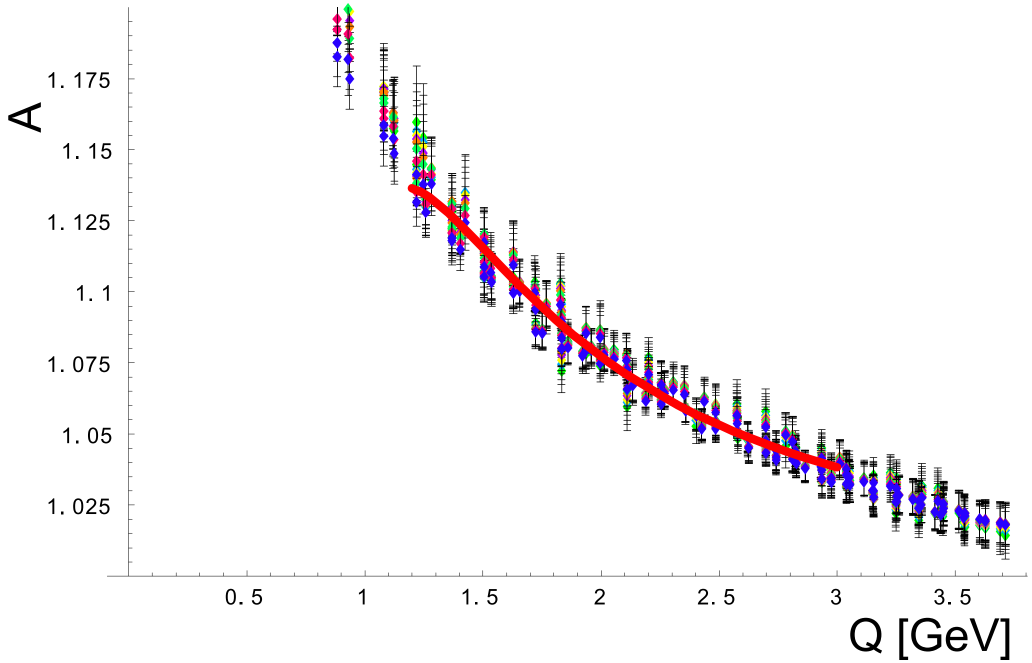
Lavelle and Schaden (1988) and Lavelle and Oleszczuk (1992) worked out the quark propagator in presence of condensates in the general covariant gauge. In the Landau gauge $\partial \cdot A^a = 0$ this propagator yields

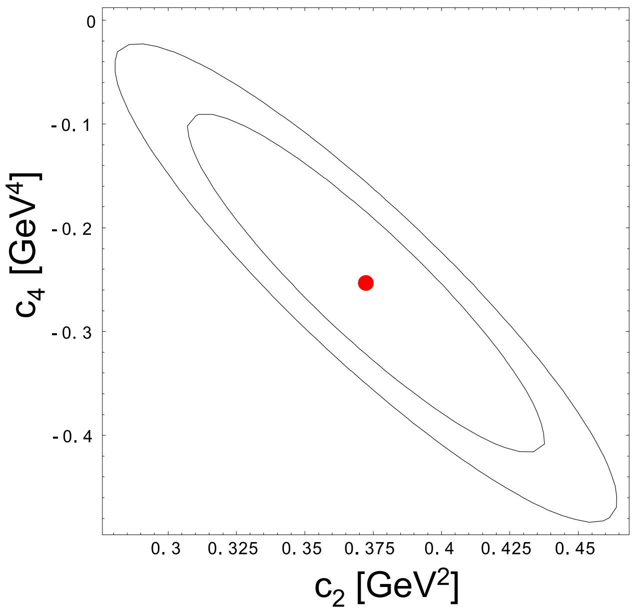
$$\begin{aligned} A(Q) \equiv Z^{-1}(Q) \quad &\rightarrow \quad 1 + \frac{\pi \alpha_s \langle A^2 \rangle}{N_c Q^2} - \frac{\pi \alpha_s \langle G^2 \rangle}{3 N_c Q^4} + \frac{3\pi \alpha_s \langle \bar{q} g_s \not{A} q \rangle}{4 Q^4} + \dots \\ &= \quad 1 + \frac{c_2}{Q^2} + \frac{c_4}{Q^4} + \dots \end{aligned}$$

... now we have the lattice data to compare to!

Matching to lattice

[Ruiz Arriola, Bowman, WB, PRD **70** (2004) 097505]





The optimum values for c_2 and c_4 yield

$$\alpha_s \langle A^2 \rangle = (0.36 \pm 0.04)\ \mathrm{GeV}^2 \ \text{ or } \ g_s^2 \langle A^2 \rangle = (2.1 \pm 0.1\ \mathrm{GeV})^2$$

and

$$\alpha_s \langle \bar{q} g_s \not{A} q \rangle - \frac{4\pi}{27} \langle \frac{\alpha_s}{\pi} G^2 \rangle = (-0.11 \pm 0.03)\ \mathrm{GeV}^4$$

Since $\langle \frac{\alpha_s}{\pi} G^2 \rangle \simeq 0.01 \text{ GeV}^4$ its contribution is negligible compared to the mixed-condensate term. Thus

$$\alpha_s \langle \bar{q} g_s \not{A} q \rangle = (-0.11 \pm 0.03) \text{ GeV}^4$$

(first estimate of this quantity).

The errors are statistical. In addition, there are certain systematic errors originating from the choice of the fitted function $A(Q)$ and from the choice of the "fiducial" region in Q. Quantities quoted in physical units are also subject to the uncertainty in the scale that goes with quenched lattice simulations.

Running of $\langle A^2 \rangle$

Operators in QCD run with appropriate anomalous dimesions. For instance, the quark condensate runs at LO as

$$\langle \bar{q}q \rangle_{\mu_1} = \left(\frac{\alpha_s(\mu_1^2)}{\alpha_s(\mu_2^2)} \right)^{\gamma_{\bar{q}q}/\beta_0} \langle \bar{q}q \rangle_{\mu_2}, \;\; \gamma_{\bar{q}q} = 4, \; \beta_0 = (11 - \frac{2}{3} N_f)$$

The anomalous dimension for the A^2 condensate has been worked out by Gracey and Boucaud *et al.* with the result

$$\alpha_s(\mu^2) \langle A^2 \rangle_\mu \sim \alpha_s(\mu^2)^{1 - \gamma_{A^2}/\beta_0},$$

where $\gamma_{A^2} = 35/4$ and $\beta_0 = 11$ for $N_f = 0$, hence $1 - \gamma_{A^2}/\beta_0 = 9/44$ and the evolution is very slow. For instance, the change of μ^2 from $1 \; \mathrm{GeV}^2$ up to $10 \; \mathrm{GeV}^2$ results in a reduction of $\alpha_s \langle A^2 \rangle$ by 10% only (we use $\alpha_s(\mu^2) = 4\pi/(9 \log[\mu^2/\Lambda^2])$, with $\Lambda = 226 \; \mathrm{MeV}$ for the LO evolution.)

Gluon mass

Many estimates in the literature refer to the gluon mass ($\rightarrow$ Lavelle propagator for the gluon)

$$m_A^2 = \frac{3}{32} g_s^2 \langle A^2 \rangle$$

Our estimate for $\langle A^2 \rangle$, when evolved from 2 GeV^2 (assumed lattice scale) to 10 GeV^2 (physical scale), yields

$$m_A = (625 \pm 33) \text{ MeV} \;\; (\text{at } 10 \text{ GeV}^2)$$

Evolution from 1 to 10 GeV^2 gives $m_A = (611 \pm 32)$ MeV, while evolution from 4 to 10 GeV^2 produces $m_A = (635 \pm 34)$ MeV.

Comparison to other approaches, compilation of Field (2002) + mine

Author	Year	Estimation Method	Gluon Mass
Parisi, Petronzio	1980	$\mathrm{J}/\psi \to \gamma \mathrm{X}$	800 MeV
Cornwall	1982	Various	500 $\pm$ 200 MeV
Donnachie, Landshoff	1989	Pomeron parameters	687-985 MeV
Hancock, Ross	1993	Pomeron slope	800 MeV
Nikolaev *et al.*	1994	Pomeron parameters	750 MeV
Spiridonov, Chetyrkin	1988	$\Pi^{em}_{\mu\nu}$, $\langle \mathrm{Tr} G^2_{\mu\nu} \rangle$	750 MeV
Lavelle	1991	$qq \to qq$, $\langle \mathrm{Tr} G^2_{\mu\nu} \rangle$	640 MeV$^2/Q(\mathrm{MeV})$
Kogan, Kovner	1995	QCD vacuum energy, $\langle \mathrm{Tr} G^2_{\mu\nu} \rangle$	1.46 GeV
Field	1994	pQCD at low scales (various)	$1.5^{+1.2}_{-0.6}$ GeV
Liu, Wetzel	1996	$\Pi^{em}_{\mu\nu}$, $\langle \mathrm{Tr} G^2_{\mu\nu} \rangle$ Glue ball current, $\langle \mathrm{Tr} G^2_{\mu\nu} \rangle$	570 MeV 470 MeV
Ynduráin	1995	QCD potential	10^{-10}-20 MeV
Leinweber *et al.*	1999	Lattice Gauge	1.02 $\pm$ 0.10 GeV
Field	2002	$\mathrm{J}/\psi \to \gamma \mathrm{X}$ $\Upsilon \to \gamma X$	$0.721^{+0.016}_{-0.068}$ GeV $1.18^{+0.09}_{-0.29}$ GeV
Celenza, Shakin	1986	Ginzburg-Landau	649 MeV
Boucaud *et al.*	2000	gluon propagator, lattice	710 MeV
Boucaud *et al.*	2000	gluon vertex, lattice	1.33 GeV
Narison, Zakharov	2001	$S + P$ correlator, lattice	0.7-1 GeV ($\times i$?)
our value	2004	quark propagator, lattice	600-650 MeV

Gauge-independent meaning of gauge-dependent operators

[Gubarev, Stodolsky, Zakharov, PRL **86** (2001) 2220]

Example from magnetostatics:

" ... since there is a nonzero magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$, we know $some$ nonzero $\mathbf{A}$ must be present; $\mathbf{A}$ cannot be zero everywhere. Now consider $\int \mathbf{A}^2 d^3x$. It is a positive quantity and cannot be zero. It must then have some minimum value. Therefore of all the possible $\mathbf{A}$ configurations which yield the given $\mathbf{B}$ the one (or the ones) with the smallest integral of $\mathbf{A}^2$ has in a sense an invariant significance ($\langle A^2_{\rm min}\rangle$). Suppose ... that $\int \mathbf{A}^2 d^3x$ is at its minimum value; then under a gauge transformation it is stationary. Considering $\mathbf{A} \to \mathbf{A} + \nabla\phi$ for infinitesimal ϕ we have $\int \mathbf{A} \cdot \nabla\phi \, d^3x = 0$ and integrating by parts

$$\int \phi \nabla \cdot \mathbf{A} \, d^3x + \text{surface terms} = 0.$$

Since ϕ is arbitrary ... the "minimum A^2" condition is equivalent to the familiar gauge condition $\nabla \cdot \mathbf{A} \;=\; 0$ "

There is the vector relation (from $(\mathbf{k}\times\mathbf{A})^2 = k^2A^2 - (\mathbf{k}\cdot\mathbf{A})^2$)

$$\int \mathbf{A}^2(x)d^3x = \frac{1}{4\pi}\int d^3x d^3x' \left(\frac{[\nabla\times\mathbf{A}(x)]\cdot[\nabla\times\mathbf{A}(x')]}{|\mathbf{x}-\mathbf{x}'|} + \frac{[\nabla\cdot\mathbf{A}(x)][\nabla\cdot\mathbf{A}(x')]}{|\mathbf{x}-\mathbf{x}'|}\right) + \text{surface terms}$$

Hence

$$A^2_{\rm min} = \frac{1}{4\pi V}\int d^3x d^3x' \frac{\mathbf{B}(\mathbf{x})\cdot\mathbf{B}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} + \text{surface terms}$$

Locality traded for manifest gauge invariance!

In 4 dimensions $\frac{1}{2}(\epsilon_{\mu\nu\lambda\rho}k_\lambda A_\rho)^2 = k^2A^2 - (k\cdot A)^2$, minimization equivalent to the Landau (*i.e.* Lorentz) condition

$$\int A^2(x)d^4x = \frac{1}{2\pi^2}\int d^4x d^4x' \frac{[F_{\mu\nu}(x)][F^{\mu\nu}(x')]}{(x-x')^2} + \frac{1}{2\pi^2}\int d^4x d^4x' \frac{[\partial_\mu A_\mu(x)][\partial_\nu A_\nu(x')]}{(x-x')^2} + \text{surface terms}$$

"... The logical situation concerning $A^2_{\rm min}$ resembles somewhat that of the question of the energy of a particle in relativity. The energy of a particle is of course a frame dependent quantity. However the minimum energy, which is the energy in the rest frame, has an invariant meaning, namely the mass. In going to the rest frame of the particle we do make a certain choice of frame, but nevertheless the mass is an undeniably meaningful quantity... "

Non-abelian case:

$$A^2_{\rm min} = \frac{1}{VT} {\rm min}_g \int d^4x \left(gA_\mu g^\dagger - \frac{i}{g_s} g\partial_\mu g^\dagger \right)^2 .$$

Here g is the group element and g_s denotes the coupling constant. The minimization gives the function $g_{\rm min}(x; A)$. We can always compensate the gauge transformation $A_\mu \to g' A_\mu g'^\dagger + g' \partial_\mu g'^\dagger$ with $g_{\rm min} \to g_{\rm min} g'^{-1}$, and $A^2_{\rm min}$ remains invariant.

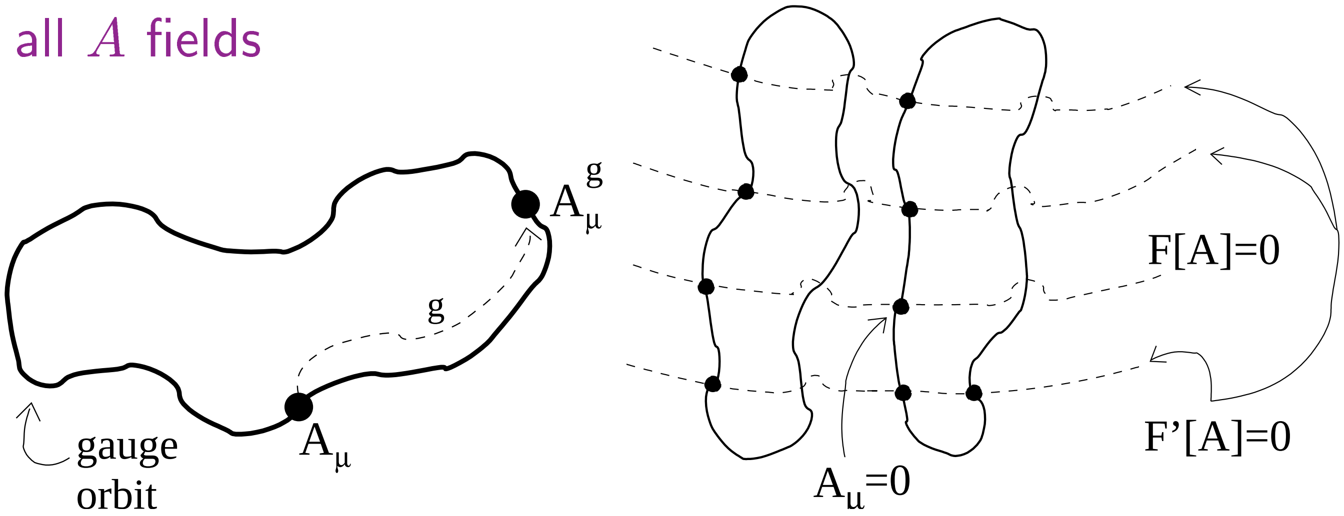
Gribov copies

[Gribov (1978)]

Abelian theory - no problems, perturbation theory - no problems.

Non-abelian and non-perturbative: no local gauge fixing is free of Gribov copies, *i.e.* configuration of fields from the same gauge orbit entering the path integration.

space of all A fields

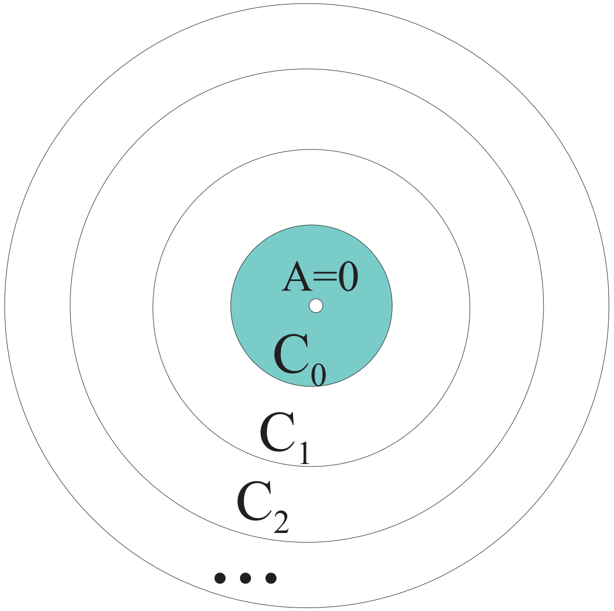


from Williams, 2003

F - complete gauge fixing
F' - incomplete gauge fixing, *e.g.* $F'(A(x)) = \partial \cdot A(x)$

Divide the gauge-fixed space of A into regions containing n negative values of the Fadeev-Popov operator for the Landau gauge, $-\partial^\mu(\partial_\mu \cdot + [A_\mu, \cdot])$

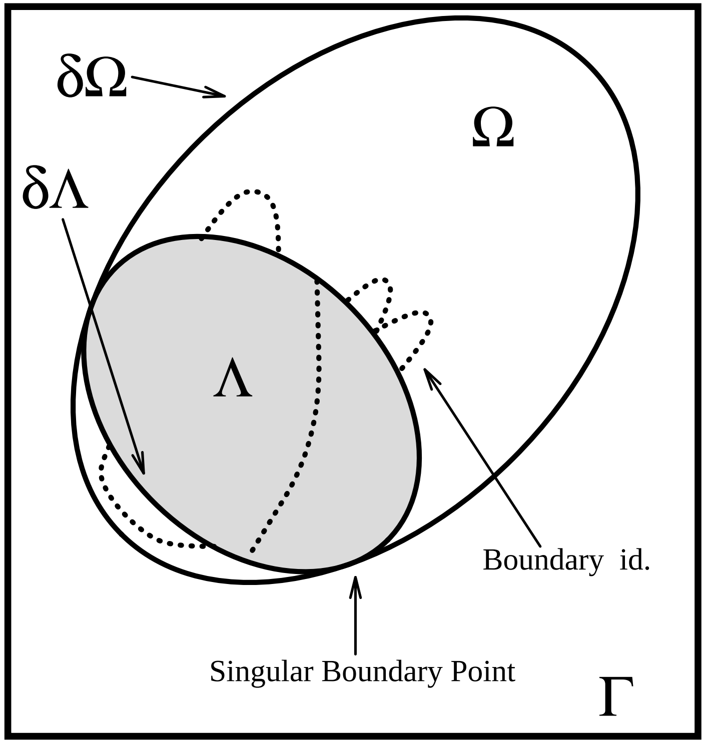
space of $A : F'(A) = 0$



Boundaries (where det of the FP operator vanishes) are called the Gribov horizons. Gribov showed that neighboring regions $C_n - C_{n-1}, \ldots, C_2 - C_1$ contain copies. He suspected that C_0 is free of copies. However, Zwanziger (1989) showed that this conjecture is not true.

- $A = 0$ belongs to C_0
- C_0 is convex
- Every gauge orbit passes throught C_0

Γ - space of all A subject to the gauge-fixing constraint, $\Omega = C_0$ - first Gribov region, $\delta\Omega$ - first Gribov horizon, Λ - fundamental modular region (FMR), dashed lines - group orbits connecting Gribov copies



[from Stodolsky, van Baal, Zakharov (2002)]

FMR is free of Gribov copies! Furthermore, Zwanziger showed that the first Gribov region (Ω) corresponds to all mimima of A^2 with respect to g (local and absolute), while FMR (Λ) corresponds to the absolute minima of A^2! Further Gribov regions correspond to extrema, with 1, 2, ... negative eigenvalues.

Significance for calculations of propagators on the lattice

Modification of the IR behavior occur when restrictions to Ω or Λ regions are made.

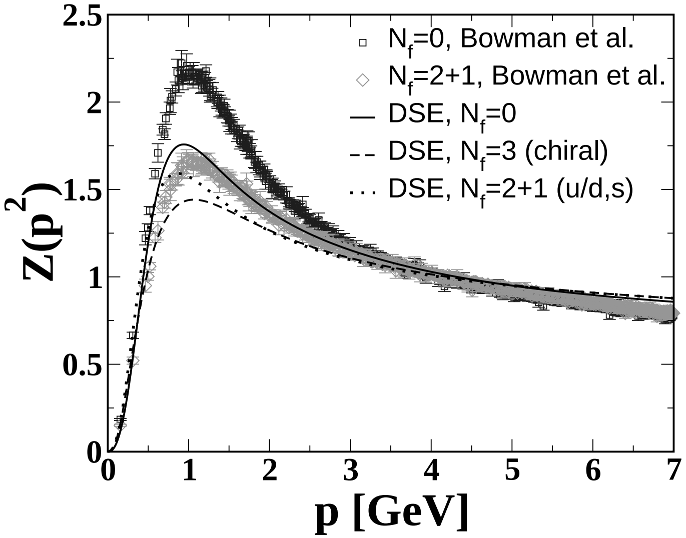
$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}, \quad D_G(p) = -\frac{G(p^2)}{p^2}$$

Dyson-Schwinger equations in the Landau gauge for the ghost and gluon dressing functions, $G(p^2)$ and $Z(p^2)$ yield simple power laws

$$Z(p^2) \sim (p^2)^{2\kappa}, \quad G(p^2) \sim (p^2)^{-\kappa},$$

with $\kappa \simeq 0.6$.

Ghost IR enhanced, gluon IR vanishing!



Kugo-Ojima (1979) confinement criterion (in Landau gauge):
Ghost propagator more singular than the pole $\equiv$ confinement. Seen on the lattice!

Restriction to FMR is not trivial to accomplish. Silva and Oliveira (2004) showed that the difference for the gluon propagator when restricting from Ω further to Λ is at the level of a few percent at soft momenta.

Other recent developments

• Boucaud *et al.* (2000), Gubarev et al. (2001) - topological structure of the vacuum, relevance for confinement

"The minimal value of the potential squared, $\langle A^2_{\rm min} \rangle$, encodes information on the topological defects in gauge theories"

• Kondo (2001) - gauge-covariant redefinition of the gluon field and discovery of a BRST invariant genaralization of the A^2 condensate:

$$\frac{1}{VT}\langle \int d^4x \left(\frac{1}{2} A_\mu(x) A^\mu(x) - i\alpha c(x) \bar{c}(x) \right) \rangle$$

(α - gauge-fixing parameter, c, $\bar{c}$- ghosts). In Landau gauge $\alpha = 0$.

• Slavnov (2004) - proof of gauge invariance of the *expectation value* of A^2 (uses non-commutative geometry)

"The gauge invariance of the condensate follows from the hidden symmetry of Yang-Mills theory, which becomes explicit if one considers it as a limit of the noncommutative gauge model"

Conclusion

- One can obtain Landau-gauge condensates from the lattice propagators

- Our value of $\langle A^2 \rangle$ compatible with other estimates (lattice gluon propagator, gluon mass). We find from the quark propagator $m_A = 600 - 650$ MeV

- First estimate of the mixed condensate $\langle \bar{q} \not{A} q \rangle$

- Saving the gauge invariance, or interpreting the apparently gauge-variant quantity in a gauge-independent way: minimum over g, Kondo, Slavnov

- Deeper meaning of the A^2 condensate: possible modification of OPE, Gribov copies, topological structure of the vacuum and confinement, lattice calculations, Dyson-Schwinger equations, ...

Interesting and hot topic!

Backup slides

Lattice

Asqtad action - improved Kogut-Susskind action, with errors of the order $\mathcal{O}(a^4, a^2 g^2)$, with tadpoles summed up

Enforcing Landau gauge - minimizing $\int d^4x \mathrm{Tr}[A^g_\mu(x) A^g_\mu(x)]$ over the group g is equivalent to maximizing

$$\sum_{x,\mu} \mathrm{Re}\left(\mathrm{Tr}[g(x) U_\mu(x) g^\dagger(x+\hat{\mu})]\right) .$$

The algorithm may produce global as well as many local minima. That way one may test the numerical significance of restricting to FMR. Typically, it is a few percent effect for the gluon propagator in the soft region [Silva and Oliveira, 2004].

Fadeev-Popov quantization

[from A. G. Williams, Prog. Theor. Phys. Supp. **151** (2003) 154]

Let us denote for each gauge orbit the gauge transformation, $\tilde{g} \equiv \tilde{g}[A^0]$, as the transformation which takes us from the origin of that orbit, A^0_μ, to the corresponding configuration on the FMR. Then

$$\mathcal{D}A = \quad \mathcal{D}A^0 \quad \mathcal{D}g = \quad \mathcal{D}A^{\mathrm{FMR}} \quad \mathcal{D}(g - \tilde{g}).$$

The *inverse Faddeev-Popov determinant* is

$$\Delta_F^{-1}[A^{\mathrm{FMR}}] \equiv \quad \mathcal{D}g\ \delta[F[A]] = \quad \mathcal{D}g\ \delta(g - \tilde{g}) \left|\det\left(M_F([A]; x, y)^{ab}\right)\right|^{-1}$$

with $M_F([A]; x, y)^{ab} \equiv \delta F^a([A]; x)/\delta g^b(y)$. We have
$1 = \int \mathcal{D}g\ \Delta_F[A]\ \delta[F[A]] = \int \mathcal{D}(g - \tilde{g})\ \Delta_F[A]\ \delta[F[A]]$ by definition and hence

$$\mathcal{D}A^{\mathrm{FMR}} \equiv \quad \mathcal{D}A^{\mathrm{FMR}} \quad \mathcal{D}(g - \tilde{g})\ \Delta_F[A]\delta[F[A]] = \quad \mathcal{D}A\ \Delta_F[A]\delta[F[A]]$$

Since for an ideal gauge-fixing there is one and only one $\tilde{g}$ per gauge orbit, such that $F([A]; x)|_{\tilde{g}} = 0$, then $|\det M_F[A]|$ is non-zero on the FMR. It follows that since there is at least one smooth path between any two configurations in the FMR and since the determinant cannot be zero on the FMR, then it cannot change sign on the FMR. The *first Gribov horizon* is defined to be those configurations with $\det M_F[A] = 0$ which lie closest to the FMR. By definition the determinant can change sign on or outside this horizon.

Assume that we have a family of *ideal* gauge fixings $F([A]; x) = f([A]; x) - c(x)$ for any Lorentz scalar $c(x)$ and for $f([A]; x)$ being some Lorentz scalar function, (e.g., $\partial^\mu A_\mu(x)$ or $n^\mu A_\mu(x)$). Therefore, using the fact that we remain in the FMR and can drop the modulus on the determinant, we have $\int \mathcal{D}A^{\mathrm{FMR}} = \int \mathcal{D}A \; \det M_F[A] \; \delta[f[A] - c]$. Since $c(x)$ is an arbitrary function, we can define a new "gauge" as the Gaussian weighted average over $c(x)$, i.e.,

$$
\begin{aligned}
\mathcal{D}A^{\mathrm{FMR}} \quad &\propto \quad \mathcal{D}c \, \exp\left\{-\frac{i}{2\xi} \quad d^4x c(x)^2\right\} \quad \mathcal{D}A \; \det M_F[A] \; \delta[f[A] - c] \\
&\propto \quad \mathcal{D}A \; \det M_F[A] \exp\left\{-\frac{i}{2\xi} \quad d^4x f([A]; x)^2\right\} \\
&\propto \quad \mathcal{D}A \mathcal{D}\chi \mathcal{D}\bar{\chi} \;\; \exp\left\{-i \quad d^4x d^4y \, \bar{\chi}(x) M_F([A]; x, y) \chi(y)\right\} \\
&\qquad \times \exp\left\{-\frac{i}{2\xi} \quad d^4x f([A]; x)^2\right\},
\end{aligned}
$$

where we have introduced the anti-commuting ghost fields χ and $\bar{\chi}$. Note that this kind of ideal gauge fixing does not choose just one gauge configuration on the gauge orbit, but rather is some Gaussian weighted average over gauge fields on the gauge orbit. We then obtain

$$\langle\Omega|\,T(\hat{O}[\ldots])\,|\Omega\rangle = \frac{\int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}A\mathcal{D}\chi\mathcal{D}\bar{\chi}\; O[\ldots]\; e^{iS_\xi[\ldots]}}{\int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}A\mathcal{D}\chi\mathcal{D}\bar{\chi}\; e^{iS_\xi[\ldots]}},$$

where

$$\begin{aligned} S_\xi[q,\bar{q},A,\chi,\bar{\chi}] \;\; = & \quad d^4x \left[-\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{2\xi}\left(f([A];x)\right)^2 + \sum_f \bar{q}_f(i\not{D} - m_f)q_f \right] \\ + & \quad d^4x d^4y\; \bar{\chi}(x) M_F([A];x,y)\chi(y)\,. \end{aligned}$$

We can now recover standard gauge fixing schemes as special cases of this generalized form. First consider standard covariant gauge, which we obtain by taking $f([A];x) = \partial_\mu A^\mu(x)$ and by *neglecting* the fact that this leads to Gribov copies. We need to evaluate $M_F[A]$ in the vicinity of the gauge-fixing surface (specified by $\tilde{g}$):

$$M_F([A];x,y)^{ab} = \frac{\delta F^a([A];x)}{\delta g^b(y)} = \frac{\delta[\partial_\mu A^{a\mu}(x) - c(x)]}{\delta g^b(y)} = \partial^x_\mu \frac{\delta A^{a\mu}(x)}{\delta g^b(y)]}.$$

Under an infinitesimal gauge transformation about the FMR, $\delta g \equiv g - \tilde{g}$, we have

$(A^{\tilde{g}})_\mu \to (A^{\tilde{g}+\delta g})_\mu$, where

$$(A^{\tilde{g}+\delta g})^a_\mu(x) = (A^{\tilde{g}})^a_\mu(x) + g_s f^{abc}\omega^b(x)A^c_\mu(x) - \partial_\mu\omega^a(x) + \mathcal{O}(\omega^2)$$

and hence near the gauge fixing surface (i.e., for small fluctations along the orbit around $A^{\rm FMR}_\mu$) using $M_F([A];x,y)^{ab} \equiv \partial^x_\mu[\delta A^{a\mu}(x)/\delta(\delta\omega^b(y)])|_{\omega=0}$ we find

$$M_F([A];x,y)^{ab} = \partial^x_\mu\left([-\partial^{x\mu}\delta^{ab} + g_s f^{abc}A^{c\mu}(x)]\delta^{(4)}(x-y)\right) .$$

We then recover the standard covariant gauge-fixed form of the QCD action

$$\begin{aligned} S_\xi[q,\bar{q},A,\chi,\bar{\chi}] \quad = \quad & d^4x\left[-\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \sum_f \bar{q}_f(i\not{D} - m_f)q_f\right] \\ & +(\partial_\mu\bar{\chi}_a)(\partial^\mu\delta^{ab} - gf_{abc}A^\mu_c)\chi_b. \end{aligned}$$

However, this gauge fixing has not removed the Gribov copies and so the formal manipulations which lead to this action are not valid. This Lorentz covariant set of naive gauges corresponds to a Gaussian weighted average over generalized Lorentz gauges, where the gauge parameter ξ is the width of the Gaussian distribution over the configurations on the gauge orbit. Setting $\xi = 0$ we see that the width vanishes and we obtain Landau gauge (equivalent to Lorentz gauge, $\partial^\mu A_\mu(x) = 0$). Choosing $\xi = 1$ is referred to as "Feynman gauge" and so on. We can similarly derive the QCD action for axial gauge.