Production of light nuclei at colliders
& my adventures with Paweł

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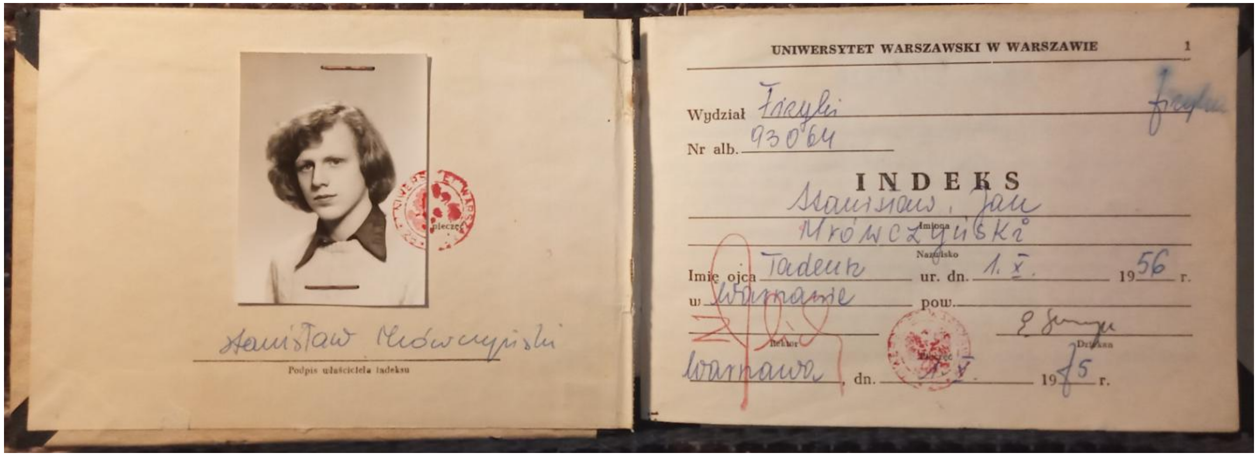
Celebrating the career of Paweł Danielewicz

New Orleans 2008

Warsaw University 1978/1979

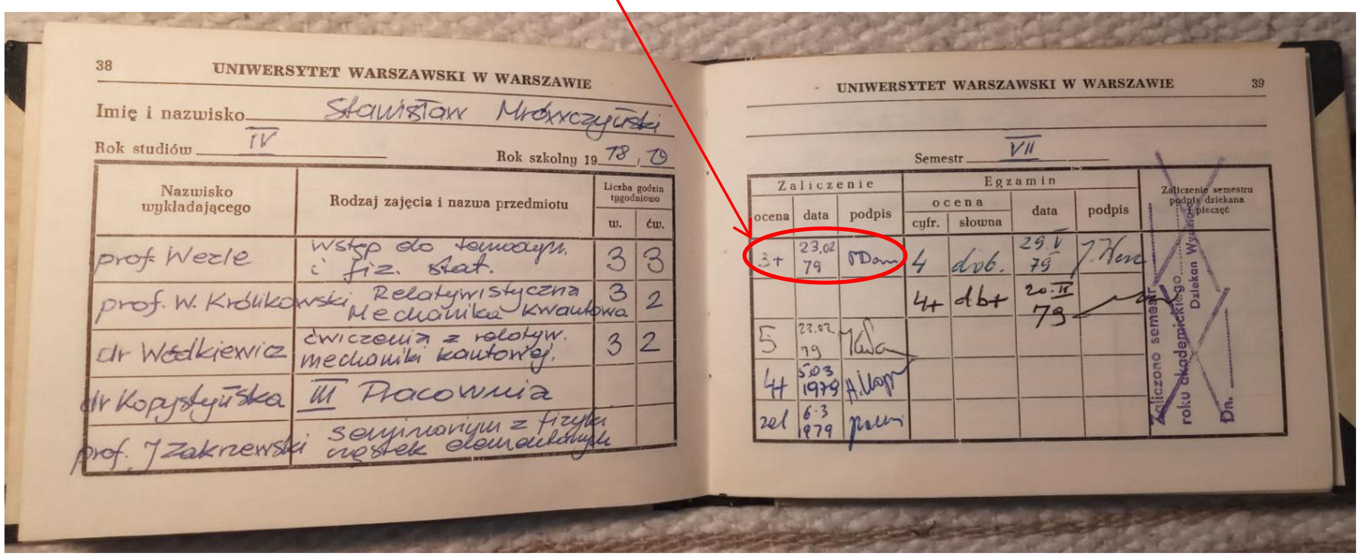
Faculty of Physics

Warsaw University 1978/1979



My student book

Paweł – first-year assistant professor



Dubna 1982-1986

How to derive transport equations
of quark-gluon plasma?

Nikolay Nikolayevich Bogolyubov (1909 – 1992)

Start with QCD – underlying
dynamical theory

From QFT to Transport Theory

ANNALS OF PHYSICS 152, 239–304 (1984)

Quantum Theory of Nonequilibrium Processes, I*

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Received December 28, 1982; revised July 5, 1983

ANNALS OF PHYSICS 152, 305–326 (1984)

Quantum Theory of Nonequilibrium Processes II. Application to Nuclear Collisions*

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Received December 28, 1982; revised July 5, 1983

In the high-energy ($E_{lab} \geqslant 200$ MeV/nucl) heavy ion-collisions, the quantum uncertainty of nucleon energies, given by the collision frequency, is of the order of (50–100) MeV. At hundreds MeV/nucl beam energies, the uncertainty is comparable with nucleon energies in the equal ion-velocity frame, indicating a quantum character of the dynamics. The quantum dynamics of a collision process is examined using nonequilibrium Green's function methods. Numerical calculations of collisions in an interpenetrating nuclear matter model, at the energy $E_{lab} = 400$ MeV/nucl, are performed. Comparison of the quantum dynamics, with the classical Markovian dynamics from the Boltzmann equation, reveals effects of the ill-defined nucleon energies in the nucleon momentum distribution. It is shown that the quantum dynamics proceeds twice as slow as Boltzmann dynamics, but the off-shell kinematics compensates for this somewhat.

1. INTRODUCTION

With recent availability of high-energy beams, the physics of high-energy heavy-ion collisions has undergone a rapid development. The theory of the collisions has concentrated on the explanation of basic reaction mechanisms and on the possible occurrence of exotic phenomena in the reactions (see the reviews [1, 2]). In principle, a full theoretical description of the collisions would necessitate a complete relativistic quantum field theory of strong interactions. At sufficiently low density and low excitation energies of a system, it is, however, believed that nucleons may be

Nuclear Physics B342 (1990) 345–380
North-Holland

GREEN FUNCTION APPROACH TO TRANSPORT THEORY OF SCALAR FIELDS

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Received 24 October 1989
(Revised 28 December 1989)

The contour Green function technique is used to derive the relativistic transport equations of neutral and charged scalar fields with the interaction lagrangian densities proportional to ϕ^3 and $(\phi\phi^*)^2$, respectively. The mean field and the collision terms in the equations are discussed in detail.

1. Introduction

Transport theory based on kinetic Boltzmann-type equations offers a natural framework to study nonequilibrium phenomena, whereas quantum field theory

From QFT to Transport Theory

ANNALS OF PHYSICS 229, 1–54 (1994)

Towards Relativistic Transport Theory of Nuclear Matter

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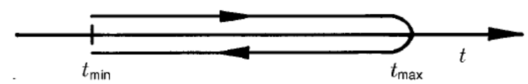
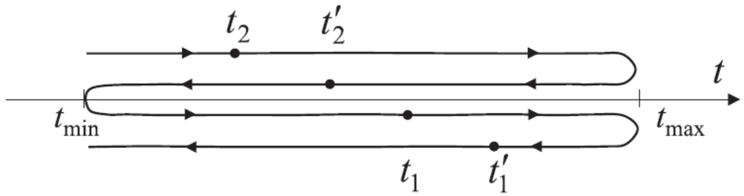
AND

ULRICH HEINZ





PHYSICAL REVIEW D VOLUME 56, NUMBER 4 15 AUGUST 1997

Transport theory of massless fields

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(Received 3 February 1997)

PHYSICAL REVIEW D 71, 065007 (2005)

Transport theory beyond binary collisions

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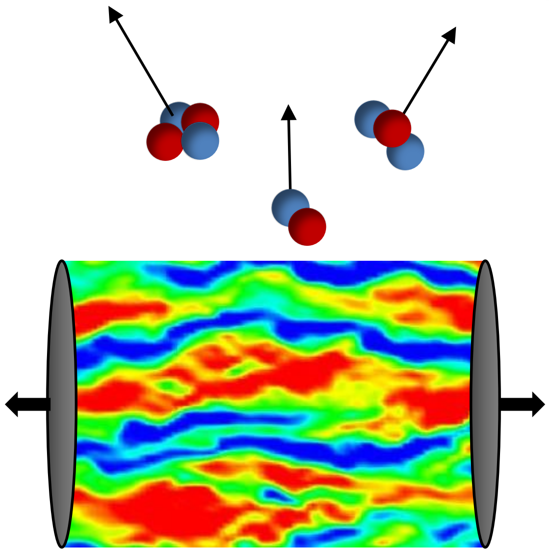
(Received 30 August 2004; revised manuscript received 11 January 2005; published 21 March 2005)

Using the Schwinger-Keldysh technique, we derive the transport equations for a system of quantum scalar fields. We first discuss the general structure of the equations and then their collision terms. Taking into account up to three-loop diagrams in ϕ^3 model and up to four-loop diagrams in ϕ^4 model, we obtain transport equations which include the contributions of multiparticle collisions and particle production processes, in addition to mean-field effects and binary interactions.

DOI: 10.1103/PhysRevD.71.065007 PACS numbers: 05.20.Dd, 11.10.Wx

Production of light nuclei at RHIC & LHC



baryonless matter

$^{2}\mathrm{H},\ ^{2}\overline{\mathrm{H}},\ ^{3}\mathrm{H},\ ^{3}\overline{\mathrm{H}},\ ^{3}\mathrm{He},\ ^{3}\overline{\mathrm{He}},\ ^{4}\mathrm{He},\ ^{4}\overline{\mathrm{He}},\ ^{3}_{\Lambda}\mathrm{H},\ ^{3}_{\bar{\Lambda}}\overline{\mathrm{H}}$

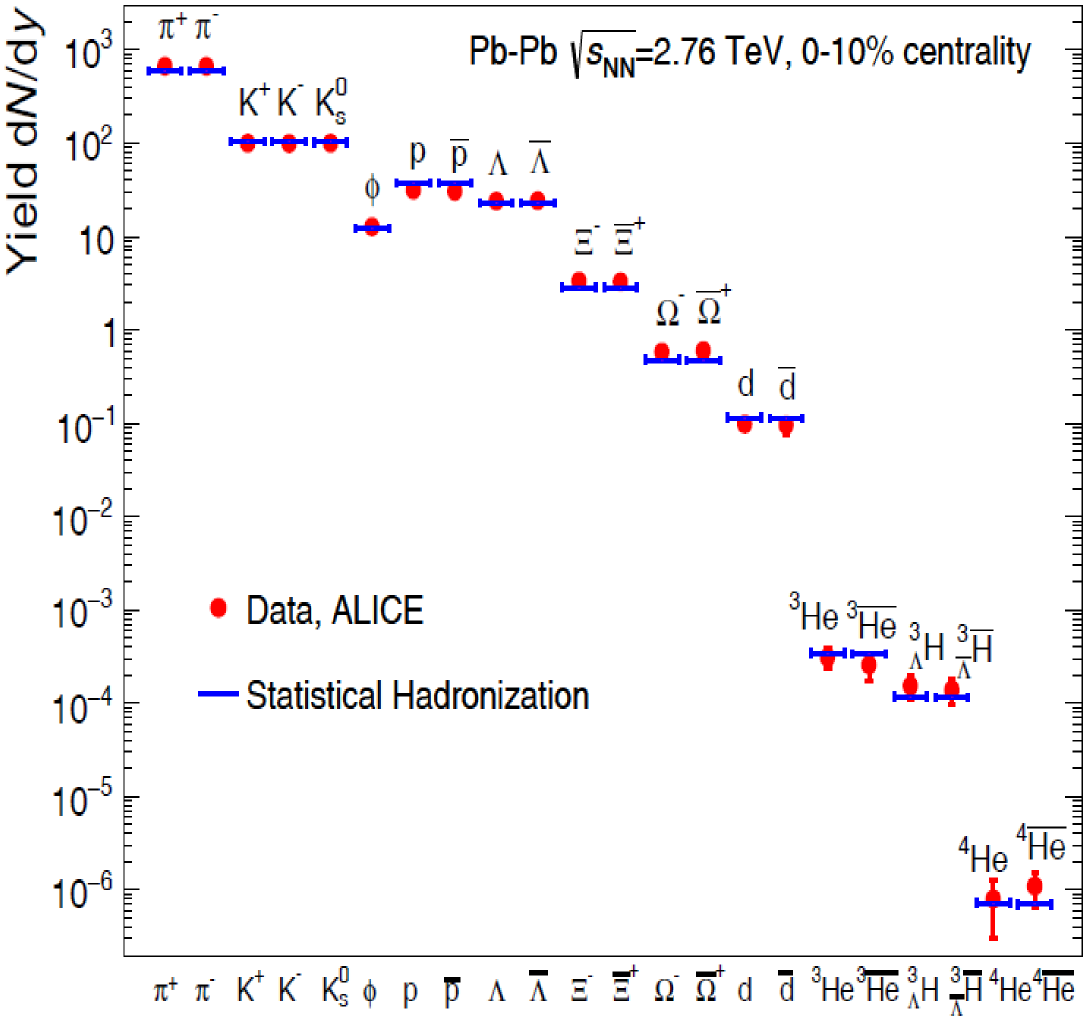
Genuine production!

Matter-antimatter symmetry!

Two approaches to production of light nuclei

- Coalescence model – final state interactions of nucleons
- Thermal model – direct production from thermalized hadron matter

Thermal model prediction



baryonless fireball

$$\text{Yield} \sim g\, e^{-\frac{m}{T}}$$

$$T = 156 \text{ MeV}$$

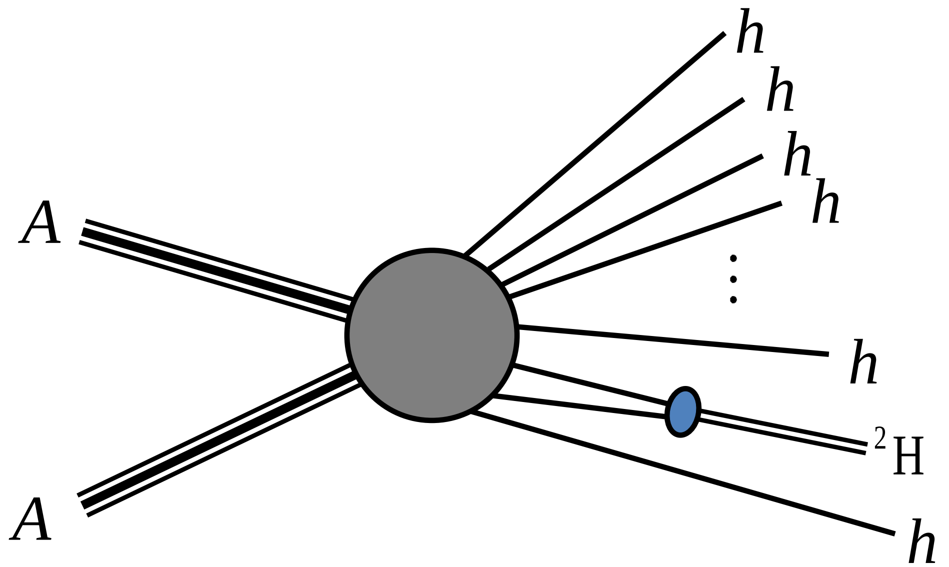
A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature **561**, 321 (2018)

Can light nuclei exist in a fireball?

- Interparticle spacing in a hadron gas is about 1.5 fm at T = 156 MeV.
- Root mean square radius of a deuteron is 2.0 fm.
- Binding energy of a deuteron is ε_B = 2.2 MeV.
- A characteristic time of deuteron formation t is longer than 2 fm/c.
- A hadron gas at T = 156 MeV is essentially a classical system.

Snowflakes in hell ?
or
Snowflakes from hell ?

Final state interaction – conventional approach to production of light nuclei



energy scale of particle production - particle's mass >> energy scale of formation of a nucleus - binding energy

fast process — slow process

S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963)
A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)

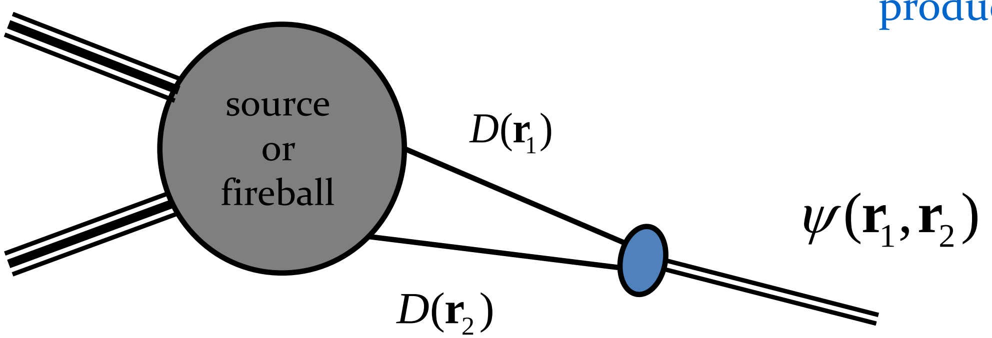
Factorization of production of nucleons and formation of a nucleus

Deuteron production cross section

$$\frac{d\sigma^D}{d^3\mathbf{P}_D} = W \frac{d\sigma^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p} \qquad \frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

deuteron formation

production of *np* pair



spin factor

$$W = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2\, D(\mathbf{r}_1) D(\mathbf{r}_2) \left|\psi(\mathbf{r}_1, \mathbf{r}_2)\right|^2$$

H. Sato and K. Yazaki, Phys. Lett. B **98**, 153 (1981)

Deuteron formation rate vs. n-p correlation

$$\psi(\mathbf{r}_1,\mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}}\varphi(\mathbf{r})$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2), \qquad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$W = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}\, D_r(\mathbf{r}) \left|\varphi(\mathbf{r})\right|^2$$

$$D_r(\mathbf{r}) \equiv \int d^3\mathbf{R}\, D\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) D\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$

n-*p* – correlation function

$$C(\mathbf{q}) = \int d^3\mathbf{r}\, D_r(\mathbf{r}) \left|\varphi_{\mathbf{q}}(\mathbf{r})\right|^2$$

$\varphi(\mathbf{r})$ – wave function of a bound state

$\varphi_{\mathbf{q}}(\mathbf{r})$ – wave function of a scattering state

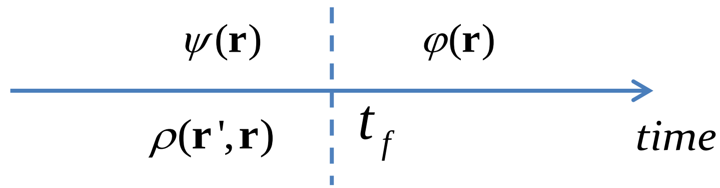
If emission time included

$$R_s \rightarrow \sqrt{R_s^2 + v^2\tau^2}$$

St. Mrówczyński, Phys. Lett. B **277**, 43 (1992)

Quantum-mechanical meaning of the formation rate formula

Sudden approximation



Transition matrix element

$$W = \left| \int d^3\mathbf{r}\, \psi^*(\mathbf{r})\, \varphi(\mathbf{r}) \right|^2 = \int d^3\mathbf{r}\, d^3\mathbf{r}'\, \varphi^*(\mathbf{r}')\, \underbrace{\psi(\mathbf{r}')\psi^*(\mathbf{r})}_{\rho(\mathbf{r}',\mathbf{r})}\, \varphi(\mathbf{r})$$

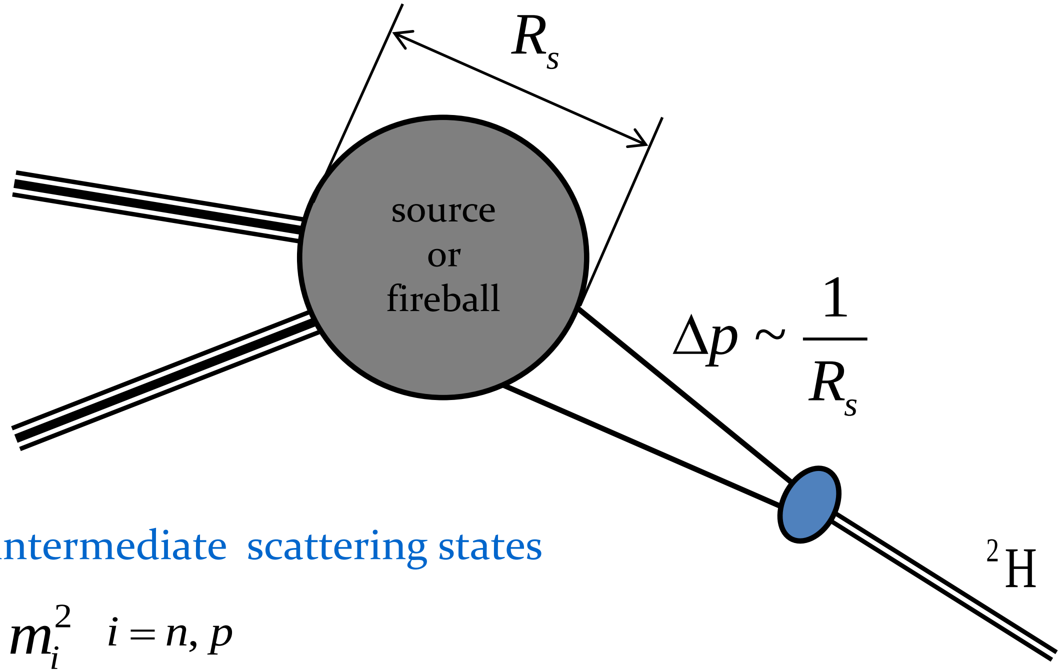
density matrix

$$W = \int d^3\mathbf{r}\, d^3\mathbf{r}'\, \varphi^*(\mathbf{r}')\, \rho(\mathbf{r}',\mathbf{r})\, \varphi(\mathbf{r})$$

If density matrix is diagonal (random phase approximation)

$$\rho(\mathbf{r}',\mathbf{r}) = D(\mathbf{r})\, \delta^{(3)}(\mathbf{r}'-\mathbf{r}) \qquad \Rightarrow \qquad \boxed{W = \int d^3\mathbf{r}\, D(\mathbf{r}) \left|\varphi(\mathbf{r})\right|^2}$$

Energy-momentum conservation



Nucleons are intermediate scattering states

$$E_i^2 - \mathbf{p}_i^2 \neq m_i^2 \quad i = n, p$$

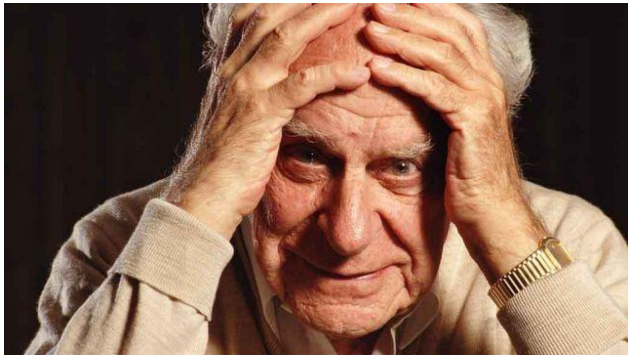
Energy-momentum conservation

$$\begin{cases} \mathbf{p}_p + \mathbf{p}_n = \mathbf{p}_D \\ E_p + E_n = E_D \end{cases}$$

St. Mrówczyński, J. Phys. G **11**, 1087 (1987)

Thermal vs. coalescence model

- The two models usually give quantitatively similar predictions.
- How to falsify one of the models experimentally?



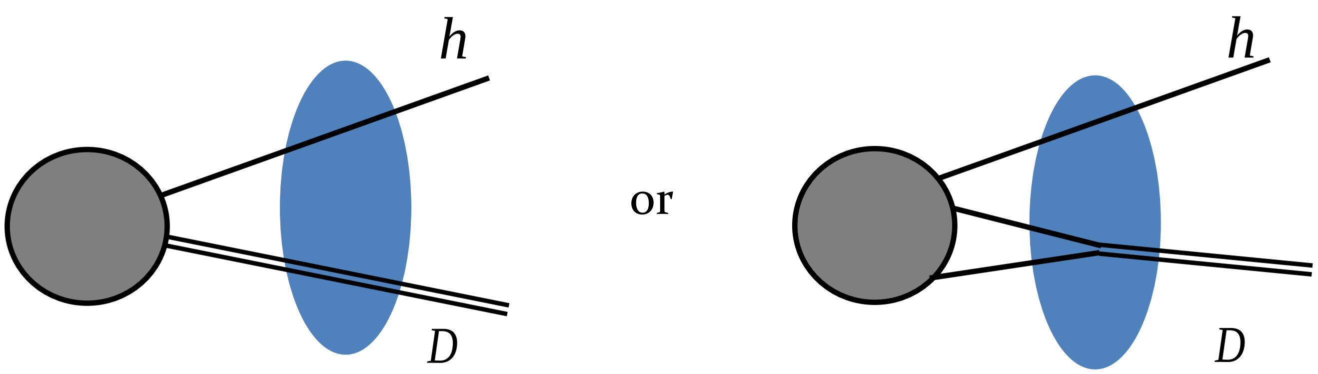
Karl Popper 1902-1994

St. Mrówczyński, Acta Phys. Pol. B **48**, 707 (2017)

The first idea: *h*-*D* & *D*-*D* correlations

- Hadron-deuteron correlations carry information about a source of deuterons.

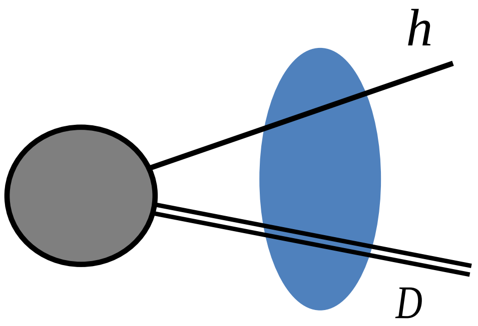
- A measurement of *p*-*D* & *p*-*p* correlation functions can falsify the thermal or coalescence model.



St. Mrówczyński & P. Słoń, Acta Physica Polonica B **51**, 1739 (2020)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle



Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h\, d\mathbf{p}_D} = R(\mathbf{p}_h, \mathbf{p}_D)\, \frac{dN_h}{d\mathbf{p}_h}\, \frac{dN_D}{d\mathbf{p}_D}$$

Theoretical formula

$$R(\mathbf{p}_h, \mathbf{p}_D) = \int d^3 r_h\, d^3 r_D\, D(\mathbf{r}_h)\, D(\mathbf{r}_D) \left|\psi(\mathbf{r}_h, \mathbf{r}_D)\right|^2$$

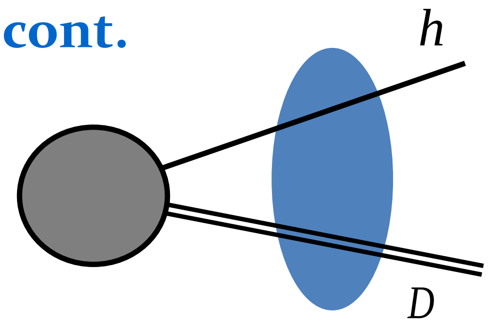
distribution of emission points

h-*D* wave function

S.E. Koonin, Phys. Lett. B **70**, 43 (1977)
R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle cont.



Separation of CM and relative motion

$$\begin{cases} \mathbf{R} \equiv \dfrac{m_D \mathbf{r}_D + m_h \mathbf{r}_h}{m_D + m_h} \\ \mathbf{r} \equiv \mathbf{r}_D - \mathbf{r}_h \end{cases}$$

$$\psi(\mathbf{r}_h, \mathbf{r}_D) = e^{i\mathbf{PR}} \phi_{\mathbf{q}}(\mathbf{r})$$

$$R(\mathbf{q}) = \int d^3 r \; D_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$$

„Relative" source function

$$D_r(\mathbf{r}) \equiv \int d^3 R \; D\left(\mathbf{R} - \frac{m_D}{m_D + m_h} \mathbf{r} \right) D\left(\mathbf{R} + \frac{m_h}{m_D + m_h} \mathbf{r} \right) = \left(\frac{1}{4\pi R_s^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_s^2} \right)$$

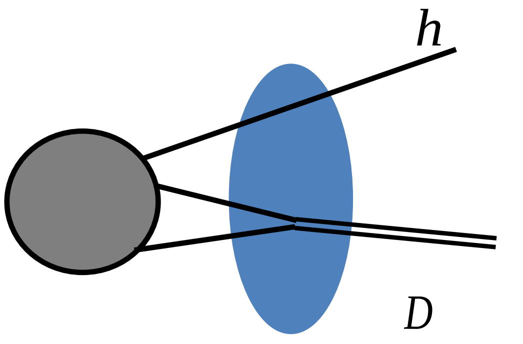
$$D(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_s^2} \right)$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = R(\mathbf{p}_h, \mathbf{p}_D)\, W_D \frac{dN_h}{d\mathbf{p}_h} \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$



Theoretical formula

$$R(\mathbf{p}_h, \mathbf{p}_D) W_D = \int d^3r_h\, d^3r_n\, d^3r_p\, D(\mathbf{r}_h)\, D(\mathbf{r}_n)\, D(\mathbf{r}_p) \left|\psi_{hD}(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p)\right|^2$$

Deuteron formation rate

$$\frac{dN_D}{d\mathbf{p}_D} = W_D \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p} \qquad \frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

$$W_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_n d^3\mathbf{r}_p\, D(\mathbf{r}_n) D(\mathbf{r}_p) \left|\psi_D(\mathbf{r}_n, \mathbf{r}_p)\right|^2 = \frac{3}{4}(2\pi)^3 \int d^3r_{np}\, D_r(\mathbf{r}_{np}) \left|\phi_D(\mathbf{r}_{np})\right|^2$$

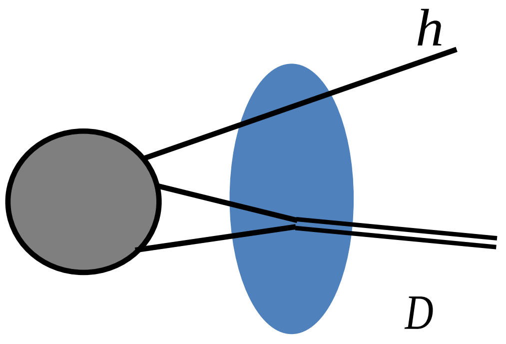
spin factor

$$\psi_D(\mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{PR}} \phi_D(\mathbf{r}_{np})$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton cont

Separation of CM and relative motion



$$\begin{cases} \mathbf{R} \equiv \dfrac{m_p \mathbf{r}_p + m_n \mathbf{r}_n + m_h \mathbf{r}_h}{m_p + m_n + m_h} \\ \mathbf{r}_{np} \equiv \mathbf{r}_p - \mathbf{r}_n \\ \mathbf{r} \equiv \mathbf{r}_h - \dfrac{m_p \mathbf{r}_p + m_n \mathbf{r}_n}{m_p + m_n} \end{cases}$$

$$\psi(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{PR}} \phi_{\mathbf{q}}(\mathbf{r}) \varphi_D(\mathbf{r}_{np})$$

$$R(\mathbf{q}) = \frac{1}{W_D} \int d^3R\, d^3r_{np}\, d^3r\, D(\mathbf{r}_h) D(\mathbf{r}_n)\, D(\mathbf{r}_p) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2 \left|\varphi_D(\mathbf{r}_{np})\right|^2$$

For Gaussian source

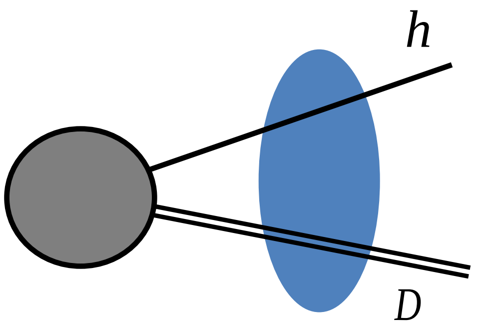
$$\boxed{R(\mathbf{q}) = \int d^3r\, D_{3r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2}$$

$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

Thermal vs. coalescence model

Thermal model

$$R(\mathbf{q}) = \int d^3r\, D_r(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$

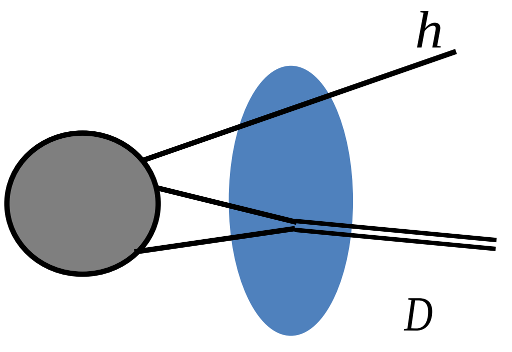


$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2}\right)$$

$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

Coalescence model

$$R(\mathbf{q}) = \int d^3r\, D_{3r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



h-D correlation function

The wave function in scattering asymptotic state

$$\phi_{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{qr}} + f(\mathbf{q})\frac{e^{iqr}}{r}$$

The *s*-wave amplidtue

$$f(\mathbf{q}) = -\frac{a}{1-iqa}$$

a – scattering length

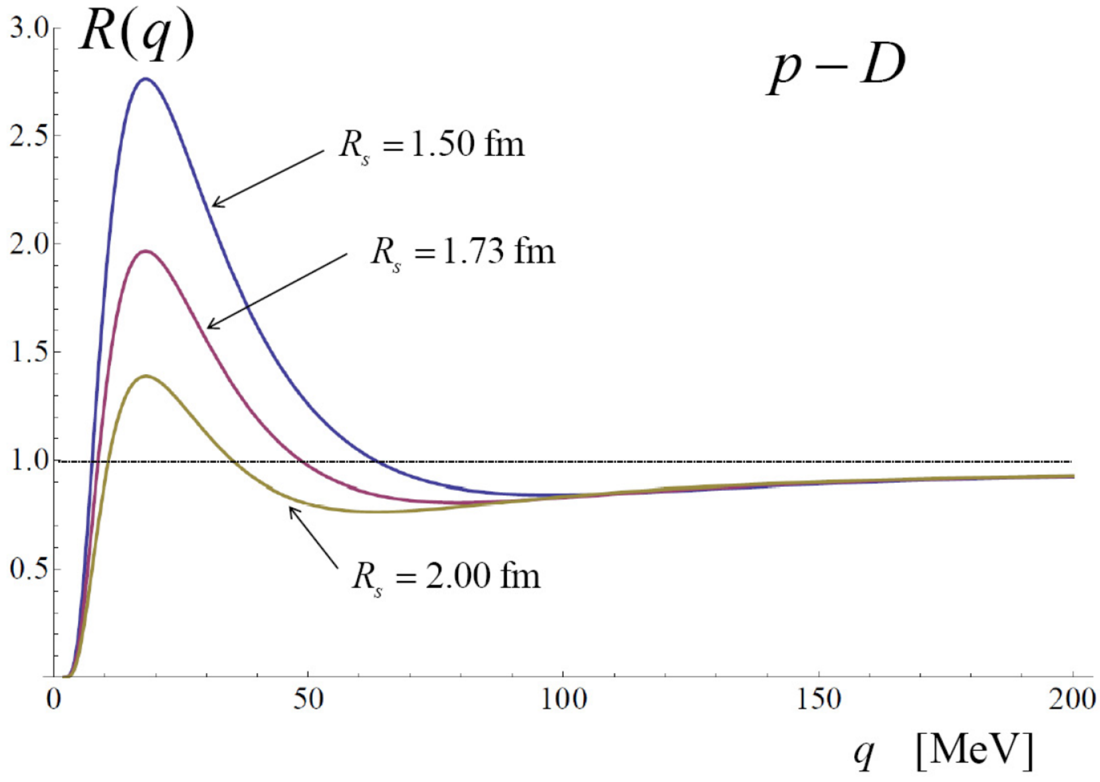
Coulomb interaction via Gamow factor

$$G(q) = \pm\frac{2\pi}{a_B q}\frac{1}{\exp\left(\pm\frac{2\pi}{a_B q}\right)-1}$$

$a_B = \frac{1}{\mu\alpha}$ – Bohr radius

R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

p-D correlation functions



$$R(q) = \frac{1}{3} R_{1/2}(q) + \frac{2}{3} R_{3/2}(q)$$

$$a_{1/2} = 4.0 \text{ fm}$$

$$a_{3/2} = 11.0 \text{ fm}$$

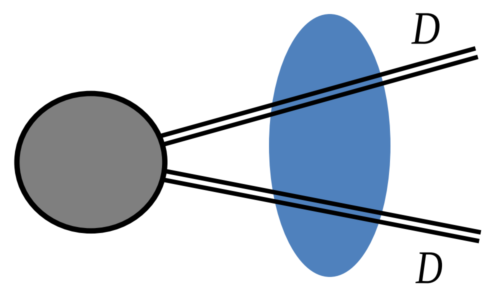
$$2.00 = \sqrt{\frac{4}{3}} 1.73 = \frac{4}{3} 1.50$$

R_s from *p-D* correlation function vs. R_s from *p-p* correlation function

Deuteron-deuteron correlation function

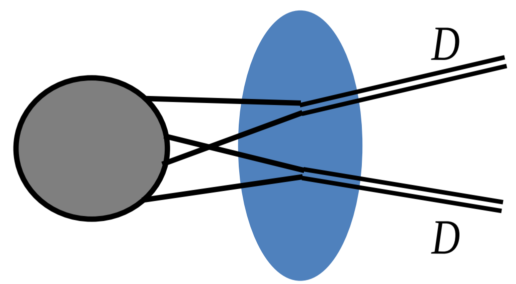
Direct production

$$R(\mathbf{q}) = \int d^3r\, D_r(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2}\right)$$

$$D_{4r}(\mathbf{r}) = \left(\frac{1}{2\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R^2}\right)$$

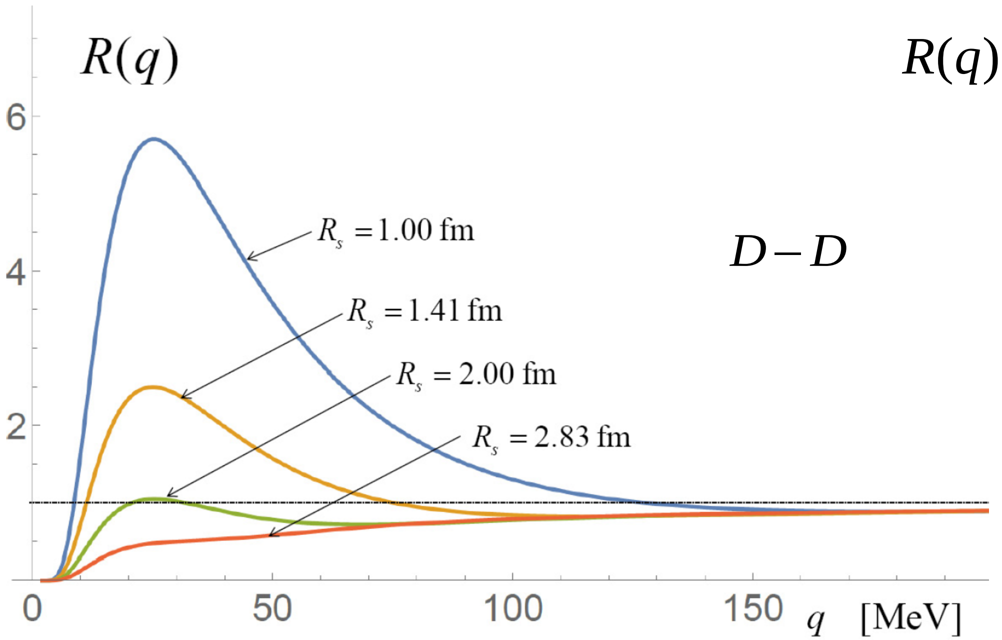


Final state interaction

$$R(\mathbf{q}) = \int d^3r\, D_{4r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$

St. Mrówczyński & P. Słoń, Physical Review C **104**, 024909 (2021)

Deuteron-deuteron correlation function



$$R(q) = \frac{1}{9} R_0(q) + \frac{3}{9} R_1(q) + + \frac{5}{9} R_2(q)$$

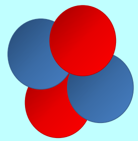
$$a_0 = (10.2 + 0.2i) \text{ fm}$$
$$a_2 = 7.5 \text{ fm}$$

$$2.83 = \sqrt{2} \cdot 2.00 = \left(\sqrt{2}\right)^2 \cdot 1.41 = \left(\sqrt{2}\right)^3 \cdot 1.00$$

R_s from *D-D* correlation function vs. R_s from *p-p* & *p-D* correlation function

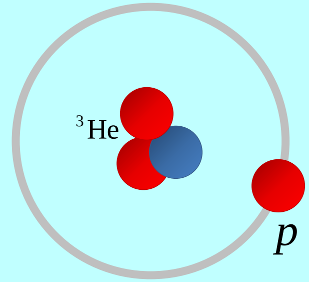
The second idea: ^{4}He vs. ^{4}Li

^{4}He



$$r_{\text{RMS}} = 1.68 \text{ fm}$$
$$\varepsilon_B = 28.3 \text{ MeV}$$
$$m = 3727.4 \text{ MeV}$$
$$s = 0$$

^{4}Li



$$^4\text{Li} \rightarrow {}^3\text{He} + p$$
$$\Gamma = 6 \text{ MeV}$$
$$m = m_{^3\text{He}} + m_p + 4.1 \text{ MeV}$$
$$m = 3749.7 \text{ MeV}$$
$$s = 2$$

- Thermal model $$\frac{\text{Yield}(^4\text{Li})}{\text{Yield}(^4\text{He})} = \frac{2S_{\text{Li}} + 1}{2S_{\text{He}} + 1} = 5$$

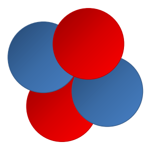
- Coalescence model $$\frac{\text{Yield}(^4\text{Li})}{\text{Yield}(^4\text{He})} = \frac{W_{\text{Li}}}{W_{\text{He}}}$$

S. Bazak & St. Mrówczyński, Mod. Phys. Lett. A **33**, 1850142 (2018)
S. Bazak & St. Mrówczyński, Eur. Phys. J. A **56**, 193 (2020)

Formation rates of ⁴He & ⁴Li

$$W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4 \, D(\mathbf{r}_1) D(\mathbf{r}_2) D(\mathbf{r}_3) D(\mathbf{r}_4) \left| \psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \right|^2$$

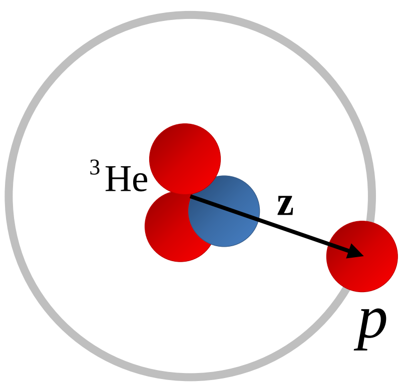
- ^{4}He



$$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

$$\left| \psi_{\mathrm{He}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \right|^2 \sim \exp\left[-\alpha (\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2) \right]$$

- ^{4}Li



J. C. Bergstrom, Nucl. Phys. A **327**, 458 (1979)

$$\mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3)$$

$$\left| \psi_{\mathrm{Li}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \right|^2 \sim \exp\left[-\beta (\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{23}^2) \right] \mathbf{z}^4 \exp\left(-\gamma \mathbf{z}^2 \right) \left| Y_{lm}(\Omega_{\mathbf{z}}) \right|^2$$

Formation rates of ^{4}He & ^{4}Li

$$W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4 \, D(\mathbf{r}_1)D(\mathbf{r}_2)D(\mathbf{r}_3)D(\mathbf{r}_4)\left|\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2$$

Source function

$$D(\mathbf{r}_i) = \frac{1}{(2\pi R_s^2)^{3/2}} \exp\left(-\frac{\mathbf{r}_i^2}{2R_s^2}\right) \qquad i = 1,2,3,4$$

If emission time included

$$R_s \to \sqrt{R_s^2 + v^2\tau^2}$$

Jacobi variables

$$\begin{cases} \mathbf{R} \equiv \frac{1}{4}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4) \\ \mathbf{x} \equiv \mathbf{r}_2 - \mathbf{r}_1 \\ \mathbf{y} \equiv \mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2) \\ \mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) \end{cases}$$

- $$\mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_3^2 + \mathbf{r}_4^2 = 4\mathbf{R}^2 + \frac{1}{2}\mathbf{x}^2 + \frac{2}{3}\mathbf{y}^2 + \frac{3}{4}\mathbf{z}^2$$
- $$\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2 = 2\mathbf{x}^2 + \frac{8}{3}\mathbf{y}^2 + 3\mathbf{z}^2$$

  $$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

Fully analytic calculations are possible!

Formation rates of ^{4}He & ^{4}Li

$$W_{\text{He}} = \frac{\pi^{9/2}}{2^{9/2}} \frac{1}{\left(R_s^2 + R_\alpha^2\right)^{9/2}}$$

$$W_{\text{Li}} = \frac{3\pi^{9/2}}{2^{11/2}} \begin{pmatrix} \frac{5}{2} \\ 1 \end{pmatrix} \frac{R_s^4}{\left(R_s^2 + \frac{1}{2}R_c^2\right)^3 \left(R_s^2 + \frac{4}{7}R_{\text{Li}}^2 - \frac{3}{7}R_c^2\right)^{7/2}}$$

$$\begin{pmatrix} l = 1 \\ l = 2 \end{pmatrix}$$

Since ^{4}Li is $J^P = 2^-$ then $l = 1$.

R_s – root mean square radius of the source

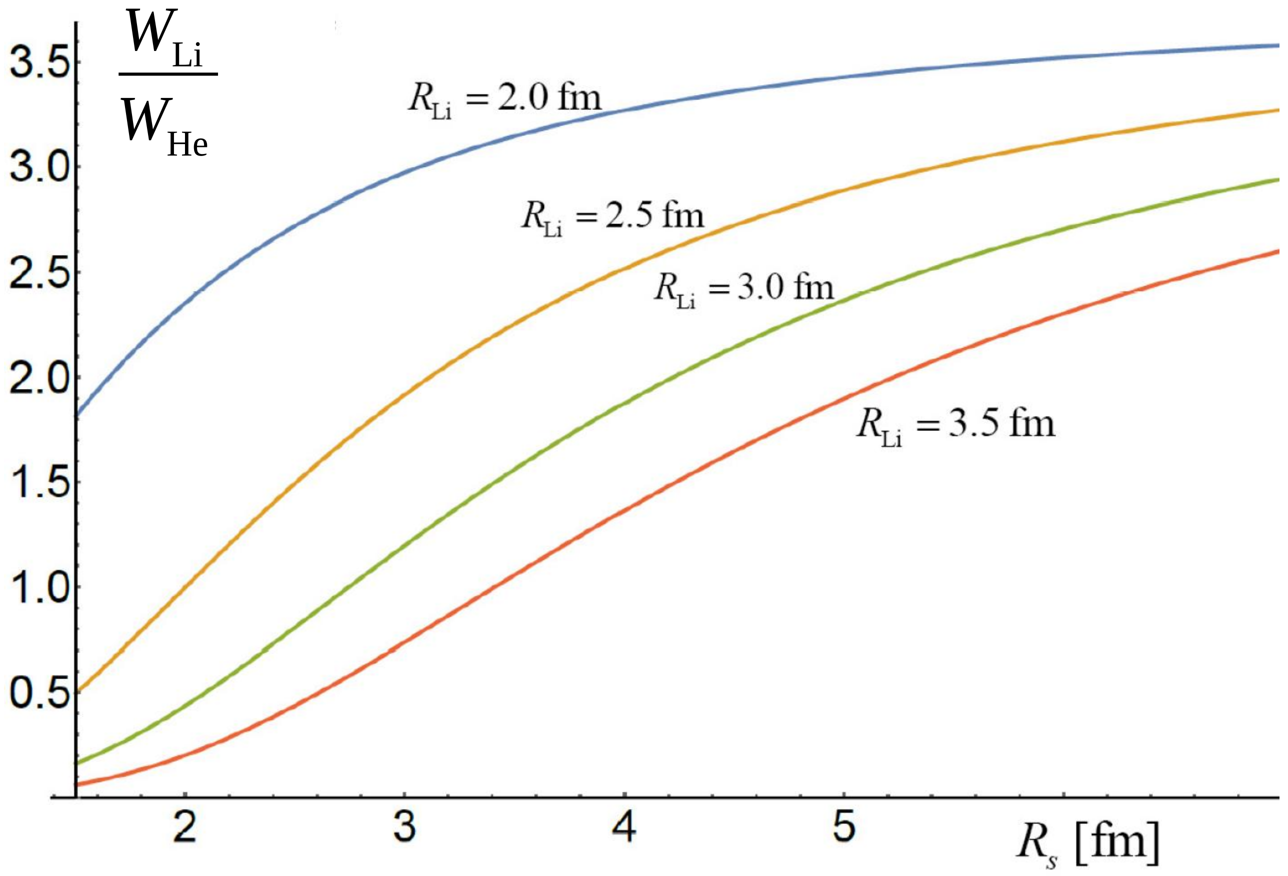
R_α – root mean square radius of ^{4}He

R_{Li} – root mean square radius of ^{4}Li

R_c – root mean square radius of ^{3}He cluster in ^{4}Li

Ratio of yields of ^{4}Li to ^{4}He

In the thermal model the ratio equals 5.

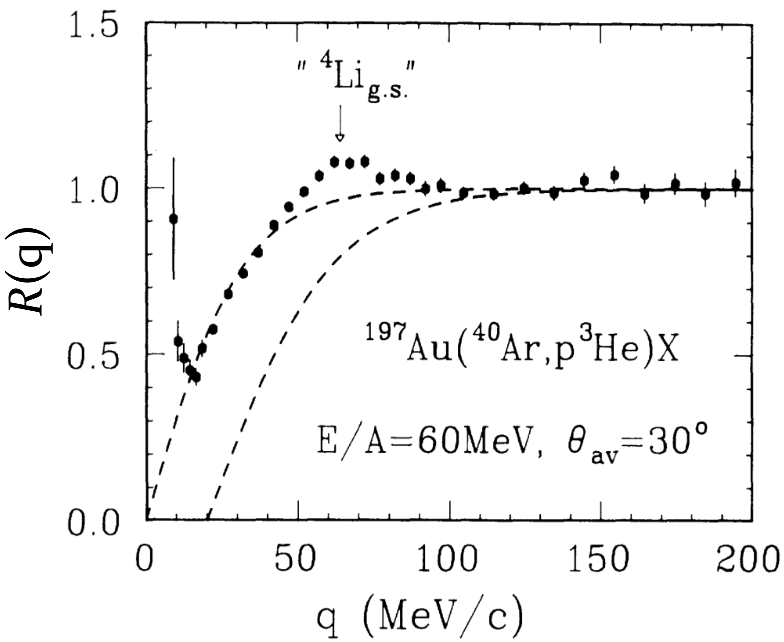


S. Bazak & St. Mrówczyński, Eur. Phys. J. A **56**, 193 (2020)

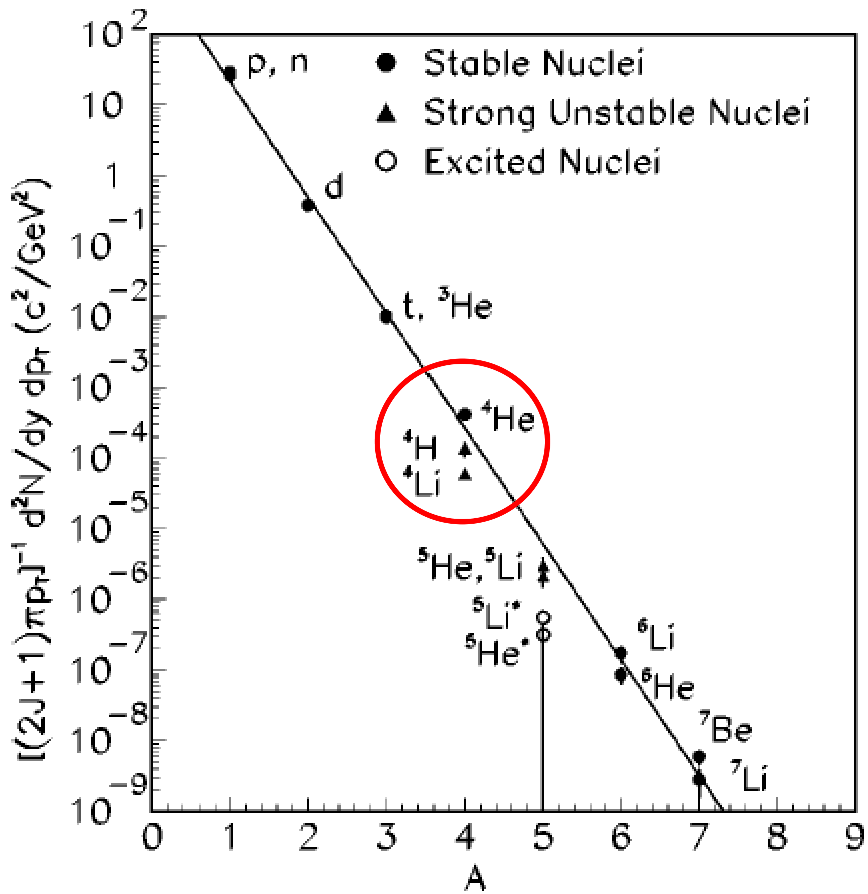
How to observe ^{4}Li?

Measurement of the correlation function of ^{3}He-*p* is needed

^{197}Au+^{197}Pt @ AGS



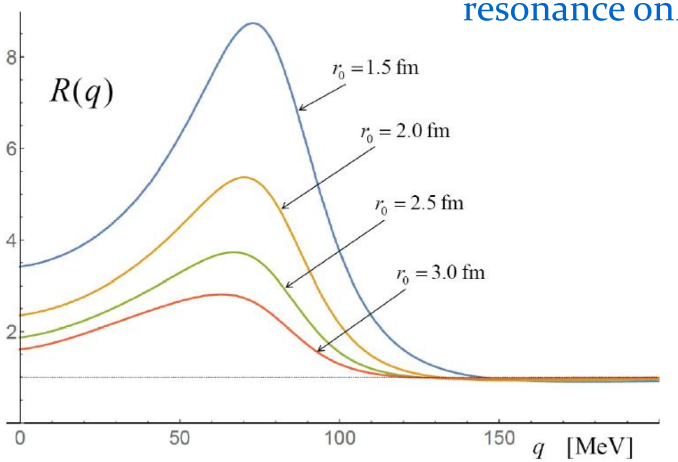
J. Pochodzala et al. Phys. Rev. C **35**, 1695 (1987)



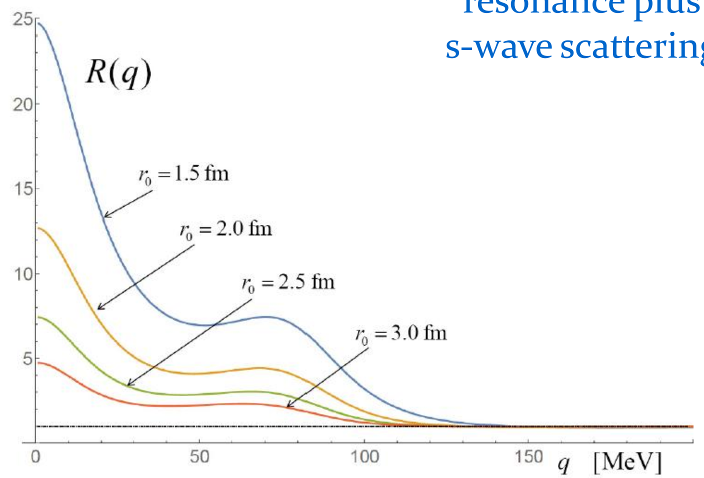
T. A. Armstrong et al. Phys. Rev. C **65**, 014906 (2001)

Correlation function p-^{3}He

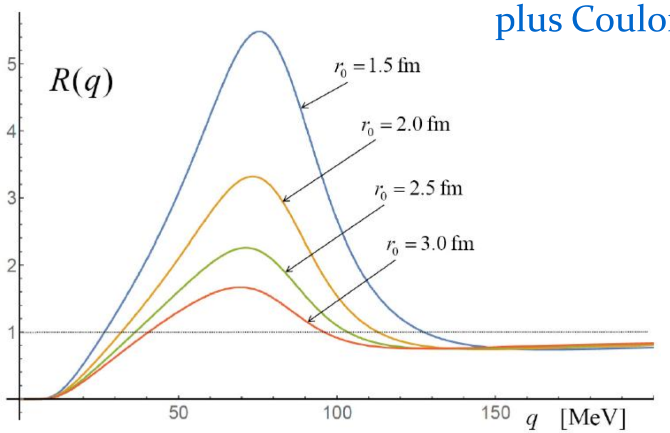
resonance only



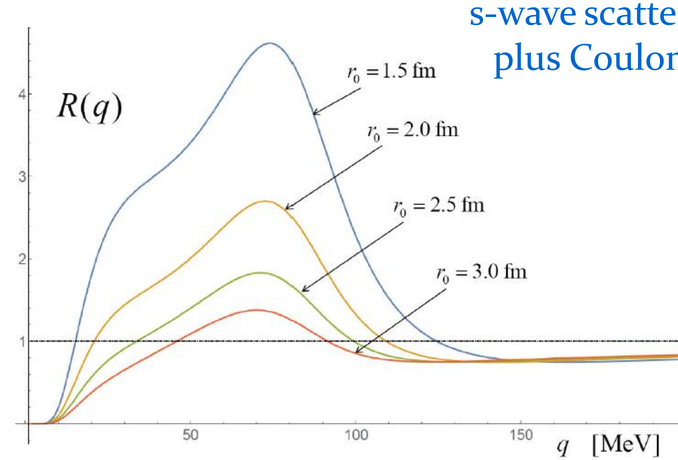
resonance plus
s-wave scattering



resonance
plus Coulomb



resonance plus
s-wave scattering
plus Coulomb



S. Bazak & St. Mrówczyński, Eur. Phys. J. A **56**, 193 (2020)

How to measure yield of ^{4}Li

$$\frac{dN_{\mathrm{Li}}}{d\mathbf{p}} = S_R \frac{dN_p}{d\mathbf{p}} \frac{dN_{^3\mathrm{He}}}{d\mathbf{p}}$$

p - momentum per nucleon

$$S_R \equiv \int d^3q \, R_R(\mathbf{q})$$

correlation function where only the ^{4}Li resonance contributes

Conclusions

p-D & D-D correlations

- Hadron-deuteron and deuteron-deuteron correlations carry information about source of deuterons.
- Measurement of *p-p, p-D* & *D-D* correlation functions can tell us whether deuterons are directly emitted from a fireball like protons or deuterons are formed due to final state interactions.
- *p-D* and *D-D* correlation functions show a sufficient sensitivity to a size of particle source to falsify the thermal or coalescence model.

^{4}He vs. ^{4}Li

- The thermal and coalescence models give different predictions on the ratio of yields of ^{4}Li to ^{4}He.
- In the thermal model the ratio of yields is independent of collision centrality.
- In the coalescence model the ratio is maximal for central collisions and rapidly decreases when one goes to peripheral collisions.
- Since ^{4}Li can be observed through the correlation function of ^{3}He-*p*, the correlation needs to be measured.

The career is not over!

There are still adventures ahead!