

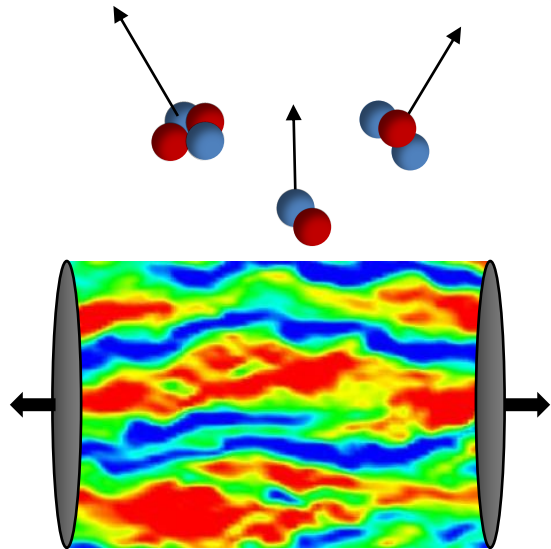
Production of light nuclei at colliders - coalescence vs. thermal model

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with **Sylwia Bazak & Patrycja Słoń**

Production of light nuclei at RHIC & LHC



baryonless matter

${}^2\text{H}$, ${}^2\bar{\text{H}}$, ${}^3\text{H}$, ${}^3\bar{\text{H}}$, ${}^3\text{He}$, ${}^3\bar{\text{He}}$, ${}^4\text{He}$, ${}^4\bar{\text{He}}$, ${}^3_{\Lambda}\text{H}$, ${}^3_{\Lambda}\bar{\text{H}}$

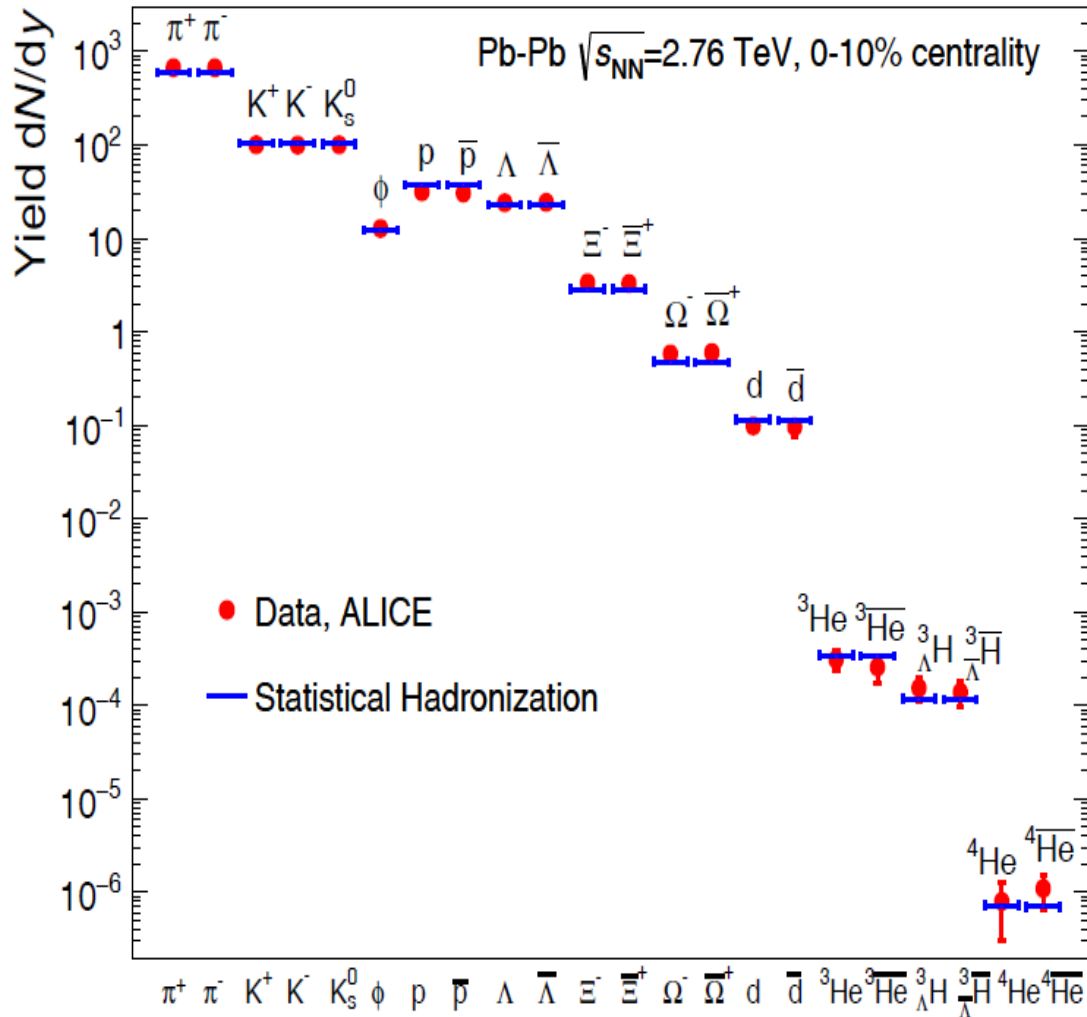
Genuine production!

Matter-antimatter symmetry!

Two approaches to production of light nuclei

- ▶ Coalescence model – final state interactions of nucleons
- ▶ Thermal model – direct production from thermalized hadron matter

Thermal model prediction



baryonless fireball

$$\text{Yield} \sim g e^{-\frac{m}{T}}$$

$$T = 156 \text{ MeV}$$

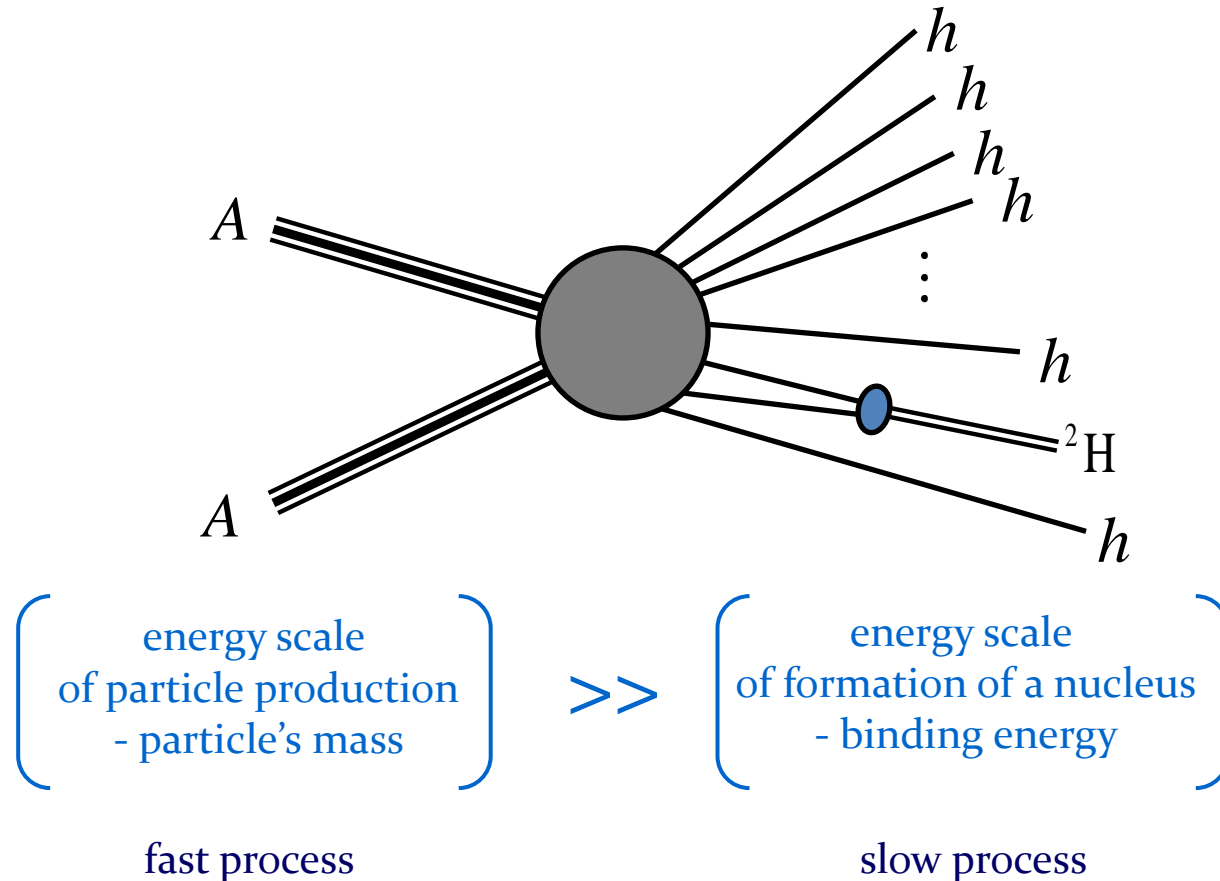
Can light nuclei exist in a fireball?

- ▶ Interparticle spacing in a hadron gas is about 1.5 fm at $T = 156$ MeV.
- ▶ Root mean square radius of a deuteron is 2.0 fm.
- ▶ Binding energy of a deuteron is $\varepsilon_B = 2.2$ MeV.
- ▶ A characteristic time of deuteron formation t is longer than 2 fm/c.
- ▶ A hadron gas at $T = 156$ MeV is essentially a classical system.

*Snowflakes in hell ?
or
Snowflakes from hell ?*



Final state interaction – conventional approach to production of light nuclei



S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963)

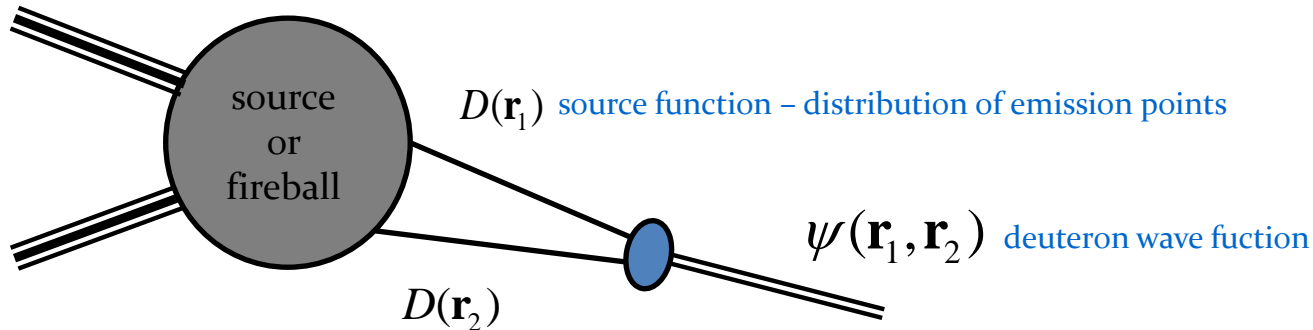
A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)

Factorization of production of nucleons and formation of a nucleus

Deuteron production cross section

$$\frac{d\sigma^D}{d^3\mathbf{P}_D} = W_D \frac{d\sigma^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p} \quad \frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

deuteron formation
production of np pair



$$W_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 D(\mathbf{r}_1) D(\mathbf{r}_2) |\psi(\mathbf{r}_1, \mathbf{r}_2)|^2$$

spin factor

Deuteron formation rate vs. n-p correlation

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}} \varphi(\mathbf{r}) \quad \mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$W_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r} D_r(\mathbf{r}) |\varphi(\mathbf{r})|^2$$

$$D_r(\mathbf{r}) \equiv \int d^3\mathbf{R} D\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) D\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right) \quad \text{distribution of relative distance of } n \text{ and } p$$

n-p – correlation function

$$C(\mathbf{q}) = \int d^3\mathbf{r} D_r(\mathbf{r}) |\varphi_{\mathbf{q}}(\mathbf{r})|^2$$

$\varphi(\mathbf{r})$ – wave function of a bound state

$\varphi_{\mathbf{q}}(\mathbf{r})$ – wave function of a scattering state

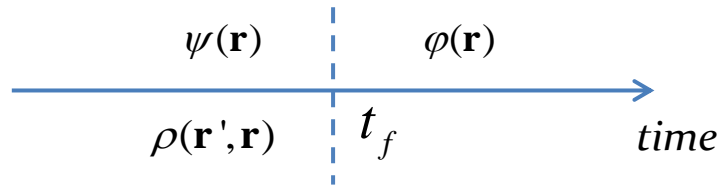
If emission time included

$$R_s \rightarrow \sqrt{R_s^2 + v^2 \tau^2}$$

St. Mrówczyński, Phys. Lett. B 277, 43 (1992)

Quantum-mechanical meaning of the formation rate formula

Sudden approximation



Transition matrix element

$$W = \left| \int d^3\mathbf{r} \psi^*(\mathbf{r}) \phi(\mathbf{r}) \right|^2 = \int d^3\mathbf{r} d^3\mathbf{r}' \phi^*(\mathbf{r}') \underbrace{\psi(\mathbf{r}') \psi^*(\mathbf{r})}_{\rho(\mathbf{r}', \mathbf{r})} \phi(\mathbf{r})$$

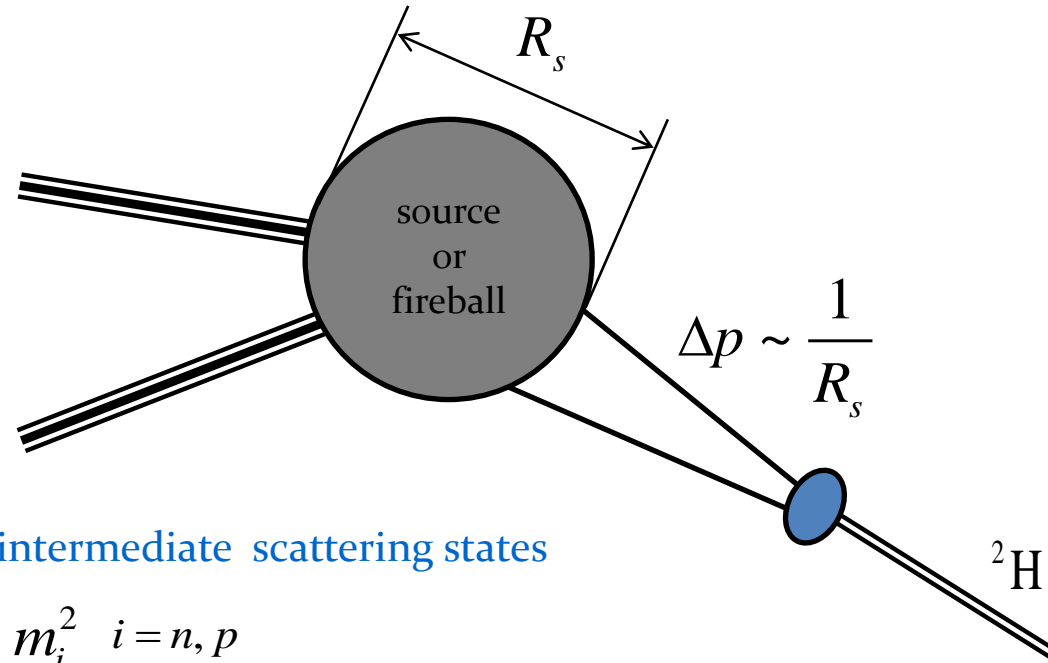
density matrix

$$W = \int d^3\mathbf{r} d^3\mathbf{r}' \phi^*(\mathbf{r}') \rho(\mathbf{r}', \mathbf{r}) \phi(\mathbf{r})$$

If density matrix is diagonal (random phase approximation)

$$\rho(\mathbf{r}', \mathbf{r}) = D(\mathbf{r}) \delta^{(3)}(\mathbf{r}' - \mathbf{r}) \quad \Rightarrow \quad \boxed{W = \int d^3\mathbf{r} D(\mathbf{r}) |\phi(\mathbf{r})|^2}$$

Energy-momentum conservation



Nucleons are intermediate scattering states

$$E_i^2 - \mathbf{p}_i^2 \neq m_i^2 \quad i = n, p$$

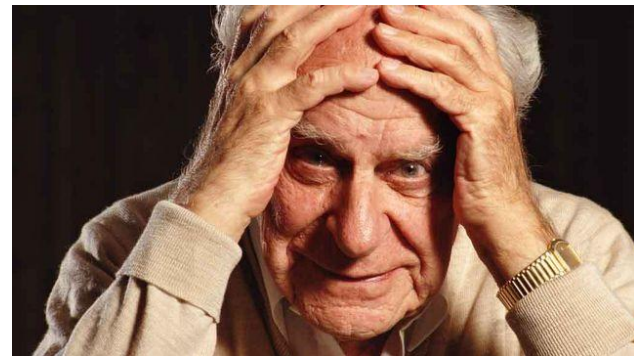
Energy-momentum conservation

$$\begin{cases} \mathbf{p}_p + \mathbf{p}_n = \mathbf{p}_D \\ E_p + E_n = E_D \end{cases}$$

St. Mrówczyński, J. Phys. G **11**, 1087 (1987)

Thermal vs. coalescence model

- ▶ The two models usually give quantitatively similar predictions.
- ▶ How to falsify one of the models experimentally?

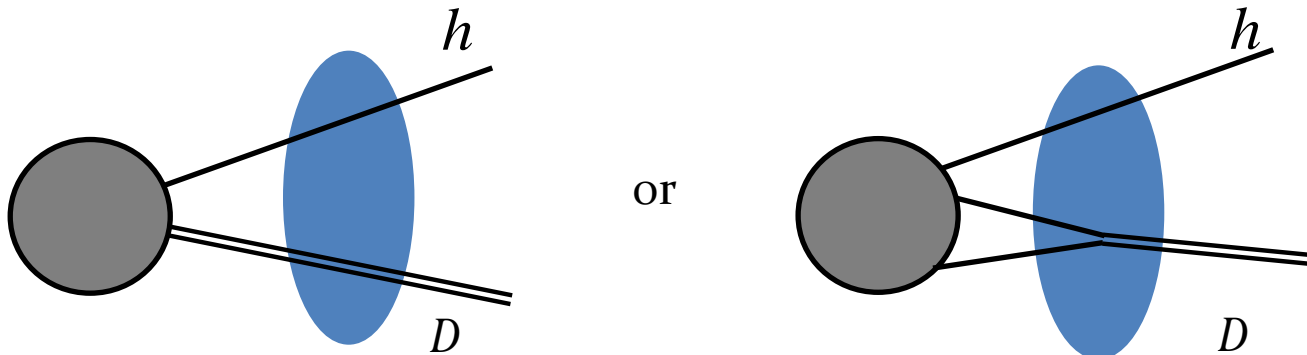


Karl Popper 1902-1994

St. Mrówczyński, Acta Phys. Pol. B **48**, 707 (2017); Eur. Phys. J. Special Topics **229**, 3559 (2020)

The first idea: h - D & D - D correlations

- ▶ Hadron-deuteron correlations carry information about a source of deuterons.
- ▶ A measurement of p - D & p - p correlation functions can falsify the thermal or coalescence model.

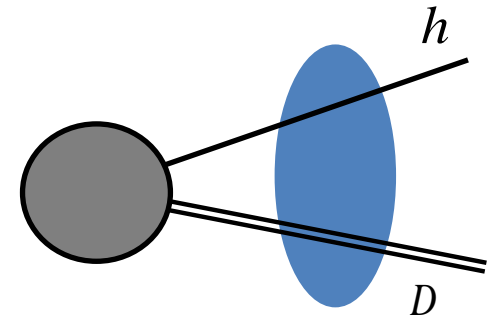


Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = R(\mathbf{p}_h, \mathbf{p}_D) \frac{dN_h}{d\mathbf{p}_h} \frac{dN_D}{d\mathbf{p}_D}$$



Theoretical formula

$$R(\mathbf{p}_h, \mathbf{p}_D) = \int d^3r_h d^3r_D D(\mathbf{r}_h) D(\mathbf{r}_D) |\psi(\mathbf{r}_h, \mathbf{r}_D)|^2$$

↑ ↑
distribution
of emission points

←
h-*D* wave function

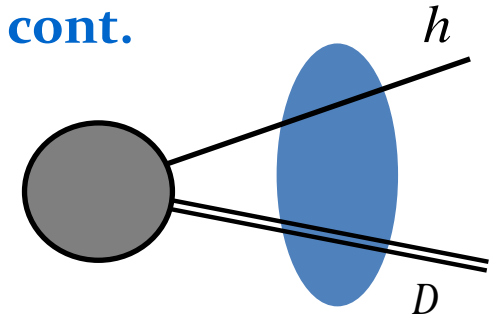
S.E. Koonin, Phys. Lett. B **70**, 43 (1977)

R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle cont.

Separation of CM and relative motion



$$\left\{ \begin{array}{l} \mathbf{R} \equiv \frac{m_D \mathbf{r}_D + m_h \mathbf{r}_h}{m_D + m_h} \\ \mathbf{r} \equiv \mathbf{r}_D - \mathbf{r}_h \end{array} \right. \quad \psi(\mathbf{r}_h, \mathbf{r}_D) = e^{i\mathbf{P}\mathbf{R}} \phi_{\mathbf{q}}(\mathbf{r})$$

$$R(\mathbf{q}) = \int d^3r D_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$$

„Relative” source function

$$D_r(\mathbf{r}) \equiv \int d^3R D\left(\mathbf{R} - \frac{m_D}{m_D + m_h} \mathbf{r}\right) D\left(\mathbf{R} + \frac{m_h}{m_D + m_h} \mathbf{r}\right) = \left(\frac{1}{4\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_s^2}\right)$$

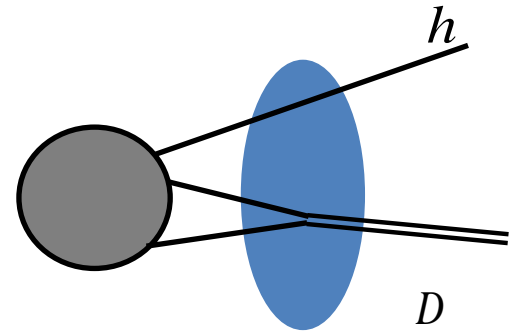
$$D(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = R(\mathbf{p}_h, \mathbf{p}_D) W_D \frac{dN_h}{d\mathbf{p}_h} \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$



Theoretical formula

$$R(\mathbf{p}_h, \mathbf{p}_D) W_D = \int d^3 r_h d^3 r_n d^3 r_p D(\mathbf{r}_h) D(\mathbf{r}_n) D(\mathbf{r}_p) |\psi_{hD}(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p)|^2$$

Deuteron formation rate

$$\frac{dN_D}{d\mathbf{p}_D} = W_D \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p} \quad \frac{1}{2} \mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

$$W_D = \frac{3}{4} (2\pi)^3 \int d^3 \mathbf{r}_n d^3 \mathbf{r}_p D(\mathbf{r}_n) D(\mathbf{r}_p) |\psi_D(\mathbf{r}_n, \mathbf{r}_p)|^2 = \frac{3}{4} (2\pi)^3 \int d^3 r_{np} D_r(\mathbf{r}_{np}) |\phi_D(\mathbf{r}_{np})|^2$$

spin factor

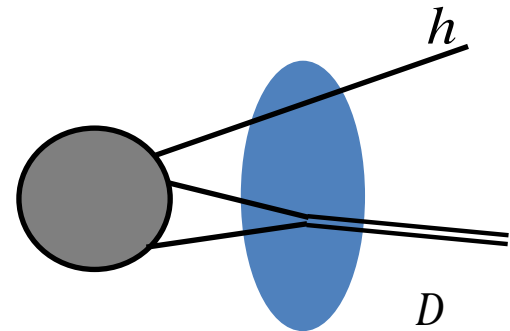
$$\psi_D(\mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_D(\mathbf{r}_{np})$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton cont

Separation of CM and relative motion

$$\left\{ \begin{array}{l} \mathbf{R} \equiv \frac{m_p \mathbf{r}_p + m_n \mathbf{r}_n + m_h \mathbf{r}_h}{m_p + m_n + m_h} \\ \mathbf{r}_{np} \equiv \mathbf{r}_p - \mathbf{r}_n \\ \mathbf{r} \equiv \mathbf{r}_h - \frac{m_p \mathbf{r}_p + m_n \mathbf{r}_n}{m_p + m_n} \end{array} \right.$$



$$\psi(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_{\mathbf{q}}(\mathbf{r}) \varphi_D(\mathbf{r}_{np})$$

$$R(\mathbf{q}) = \frac{1}{W_D} \int d^3 R d^3 r_{np} d^3 r D(\mathbf{r}_h) D(\mathbf{r}_n) D(\mathbf{r}_p) |\phi_{\mathbf{q}}(\mathbf{r})|^2 |\varphi_D(\mathbf{r}_{np})|^2$$

For Gaussian source

$$R(\mathbf{q}) = \int d^3 r D_{3r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2} \right)$$

For non-Gaussian source, W_D remains in the correlation function!

Thermal vs. coalescence model

Thermal model

$$R(\mathbf{q}) = \int d^3 r D_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

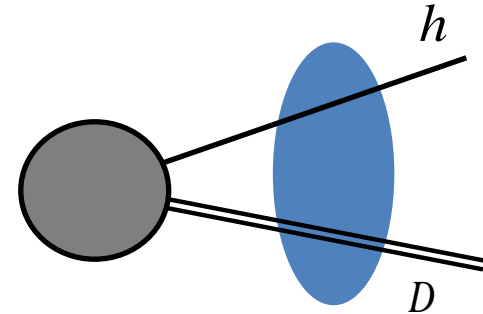


$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2} \right)$$

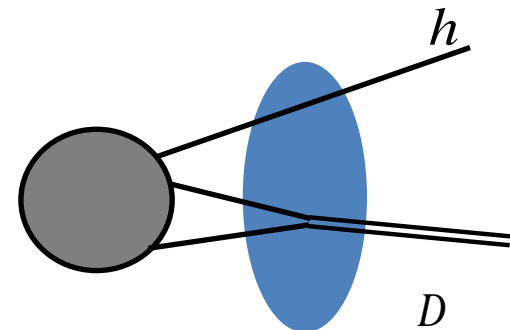
$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2} \right)$$

Coalescence model

$$R(\mathbf{q}) = \int d^3 r D_{3r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$



$$\sqrt{\frac{4}{3}} \approx 1.15$$



h-D correlation function

The wave function in scattering asymptotic state

$$\phi_{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{q}\mathbf{r}} + f(\mathbf{q}) \frac{e^{iqr}}{r}$$

The *s*-wave amplitude

$$f(\mathbf{q}) = -\frac{a}{1-iqa} \quad a - \text{scattering length}$$

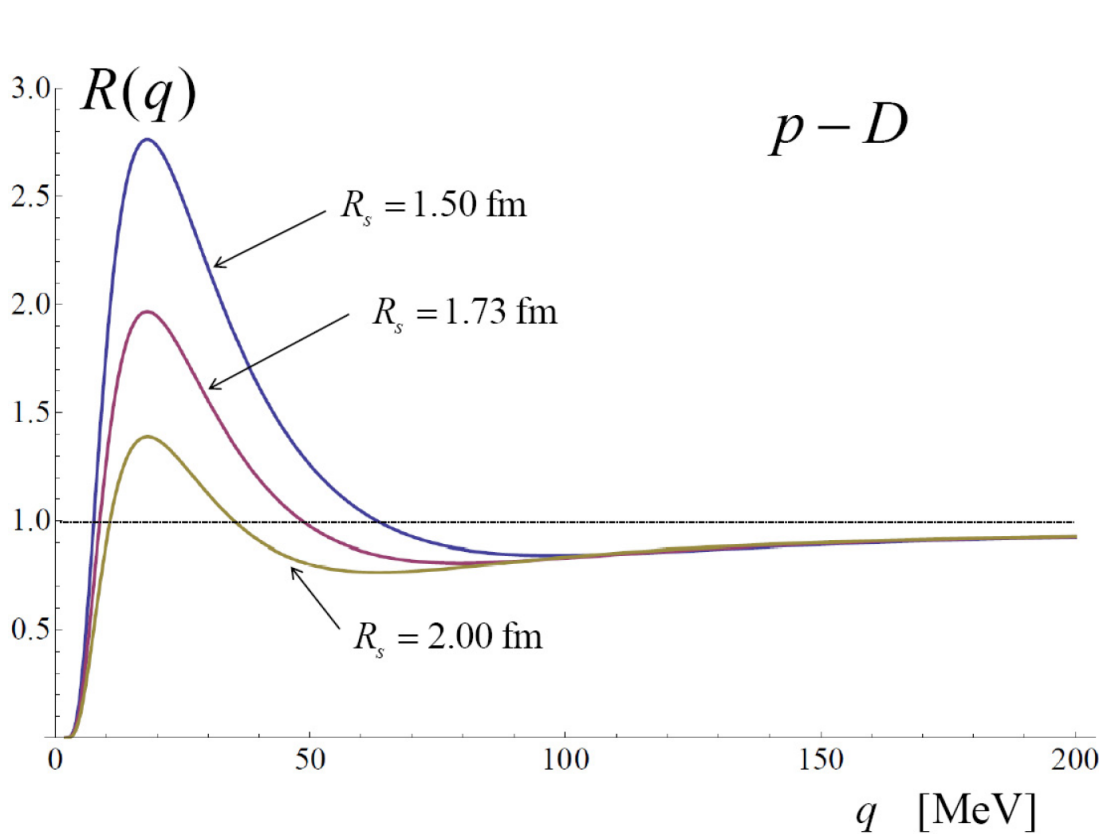
Coulomb interaction via Gamow factor

$$G(q) = \pm \frac{2\pi}{a_B q} \frac{1}{\exp\left(\pm \frac{2\pi}{a_B q}\right) - 1} \quad a_B = \frac{1}{\mu\alpha} - \text{Bohr radius}$$

Interference of strong and Coulomb interaction ignored!

R. Lednicky and V.L. Lyuboshitz, *Yad. Fiz.* **35**, 1316 (1982)

p-D correlation functions



$$R(q) = \frac{1}{3} R_{1/2}^{\text{spin } 1/2}(q) + \frac{2}{3} R_{3/2}^{\text{spin } 3/2}(q)$$

$$a_{1/2} = 4.0 \text{ fm}$$

$$a_{3/2} = 11.0 \text{ fm}$$

$$2.00 = \sqrt{\frac{4}{3}} 1.73 = \frac{4}{3} 1.50$$

R_s from *p-D* correlation function vs. *R_s* from *p-p* correlation function

Deuteron-deuteron correlation function

Direct production

$$R(\mathbf{q}) = \int d^3 r D_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$



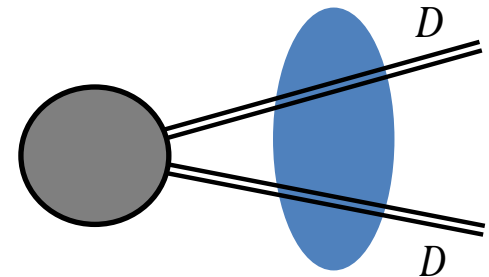
$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2} \right)$$

$$D_{4r}(\mathbf{r}) = \left(\frac{1}{2\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R^2} \right)$$

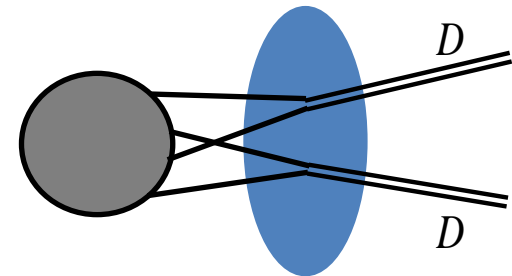
Final state interaction



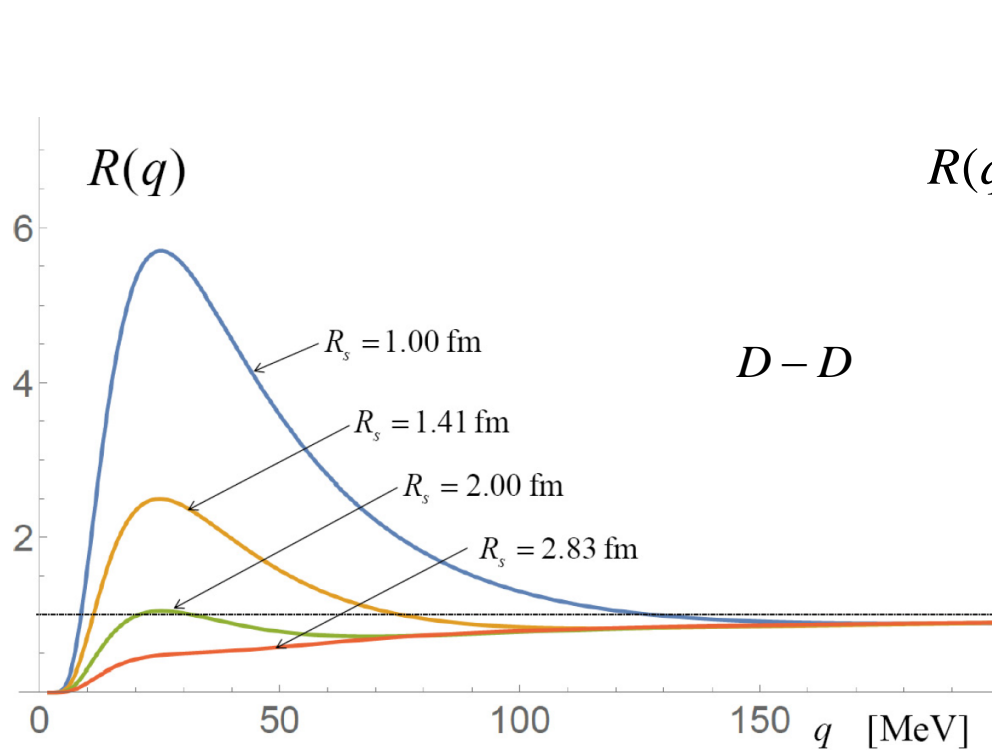
$$R(\mathbf{q}) = \int d^3 r D_{4r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$



$$\sqrt{2} \approx 1.41$$



Deuteron-deuteron correlation function



$$R(q) = \frac{1}{9} R_0(q) + \frac{3}{9} R_1(q) + \frac{5}{9} R_2(q)$$

spin 0
spin 1
spin 2

$$a_0 = (10.2 + 0.2i) \text{ fm}$$


$$a_2 = 7.5 \text{ fm}$$

$$2.83 = \sqrt{2} \cdot 2.00 = (\sqrt{2})^2 \cdot 1.41 = (\sqrt{2})^3 \cdot 1.00$$

R_s from $D-D$ correlation function vs. R_s from $p-p$ & $p-D$ correlation function

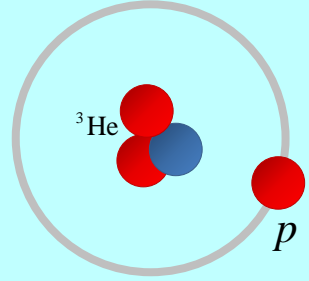
The second idea: ${}^4\text{He}$ vs. ${}^4\text{Li}$

${}^4\text{He}$



$r_{\text{RMS}} = 1.68 \text{ fm}$
 $\varepsilon_B = 28.3 \text{ MeV}$
 $m = 3727.4 \text{ MeV}$
 $s = 0$

${}^4\text{Li}$



${}^4\text{Li} \rightarrow {}^3\text{He} + p$
 $\Gamma = 6 \text{ MeV}$
 $m = m_{{}^3\text{He}} + m_p + 4.1 \text{ MeV}$
 $m = 3749.7 \text{ MeV}$
 $s = 2$

▶ Thermal model $\frac{\text{Yield}({}^4\text{Li})}{\text{Yield}({}^4\text{He})} = \frac{2S_{\text{Li}} + 1}{2S_{\text{He}} + 1} = 5$

▶ Coalescence model $\frac{\text{Yield}({}^4\text{Li})}{\text{Yield}({}^4\text{He})} = \frac{W_{\text{Li}}}{W_{\text{He}}}$

S. Bazak & St. Mrówczyński, Mod. Phys. Lett. A **33**, 1850142 (2018)

S. Bazak & St. Mrówczyński, Eur. Phys. J. A **56**, 193 (2020)

Formation rates of ${}^4\text{He}$ & ${}^4\text{Li}$

$$W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4 D(\mathbf{r}_1) D(\mathbf{r}_2) D(\mathbf{r}_3) D(\mathbf{r}_4) |\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2$$

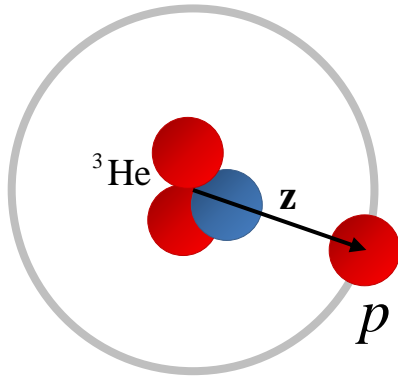
▶ ${}^4\text{He}$



$$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

$$|\psi_{\text{He}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2 \sim \exp\left[-\alpha(\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2)\right]$$

▶ ${}^4\text{Li}$



J. C. Bergstrom, Nucl. Phys. A **327**, 458 (1979)

$$\mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3)$$

$$|\psi_{\text{Li}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2 \sim \exp\left[-\beta(\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{23}^2)\right] \mathbf{z}^4 \exp(-\gamma \mathbf{z}^2) |Y_{lm}(\Omega_{\mathbf{z}})|^2$$

Formation rates of ${}^4\text{He}$ & ${}^4\text{Li}$

$$W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4 D(\mathbf{r}_1) D(\mathbf{r}_2) D(\mathbf{r}_3) D(\mathbf{r}_4) |\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2$$

Source function

$$D(\mathbf{r}_i) = \frac{1}{(2\pi R_s^2)^{3/2}} \exp\left(-\frac{\mathbf{r}_i^2}{2R_s^2}\right) \quad i = 1, 2, 3, 4$$

If emission time included

$$R_s \rightarrow \sqrt{R_s^2 + v^2 \tau^2}$$

Jacobi variables

$$\left\{ \begin{array}{l} \mathbf{R} \equiv \frac{1}{4}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4) \\ \mathbf{x} \equiv \mathbf{r}_2 - \mathbf{r}_1 \\ \mathbf{y} \equiv \mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2) \\ \mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) \end{array} \right.$$

$$\blacktriangleright \quad \mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_3^2 + \mathbf{r}_4^2 = 4\mathbf{R}^2 + \frac{1}{2}\mathbf{x}^2 + \frac{2}{3}\mathbf{y}^2 + \frac{3}{4}\mathbf{z}^2$$

$$\blacktriangleright \quad \mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2 = 2\mathbf{x}^2 + \frac{8}{3}\mathbf{y}^2 + 3\mathbf{z}^2$$

$$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

Fully analytic calculations
are possible!

Formation rates of ${}^4\text{He}$ & ${}^4\text{Li}$

$$\triangleright W_{\text{He}} = \frac{\pi^{9/2}}{2^{9/2}} \frac{1}{\left(R_s^2 + R_\alpha^2\right)^{9/2}}$$

$$\triangleright W_{\text{Li}} = \frac{3\pi^{9/2}}{2^{11/2}} \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \frac{R_s^4}{\left(R_s^2 + \frac{1}{2}R_c^2\right)^3 \left(R_s^2 + \frac{4}{7}R_{\text{Li}}^2 - \frac{3}{7}R_c^2\right)^{7/2}} \begin{pmatrix} l=1 \\ l=2 \end{pmatrix}$$

Since ${}^4\text{Li}$ is $J^P = 2^-$ then $l=1$.

R_s – root mean square radius of the source

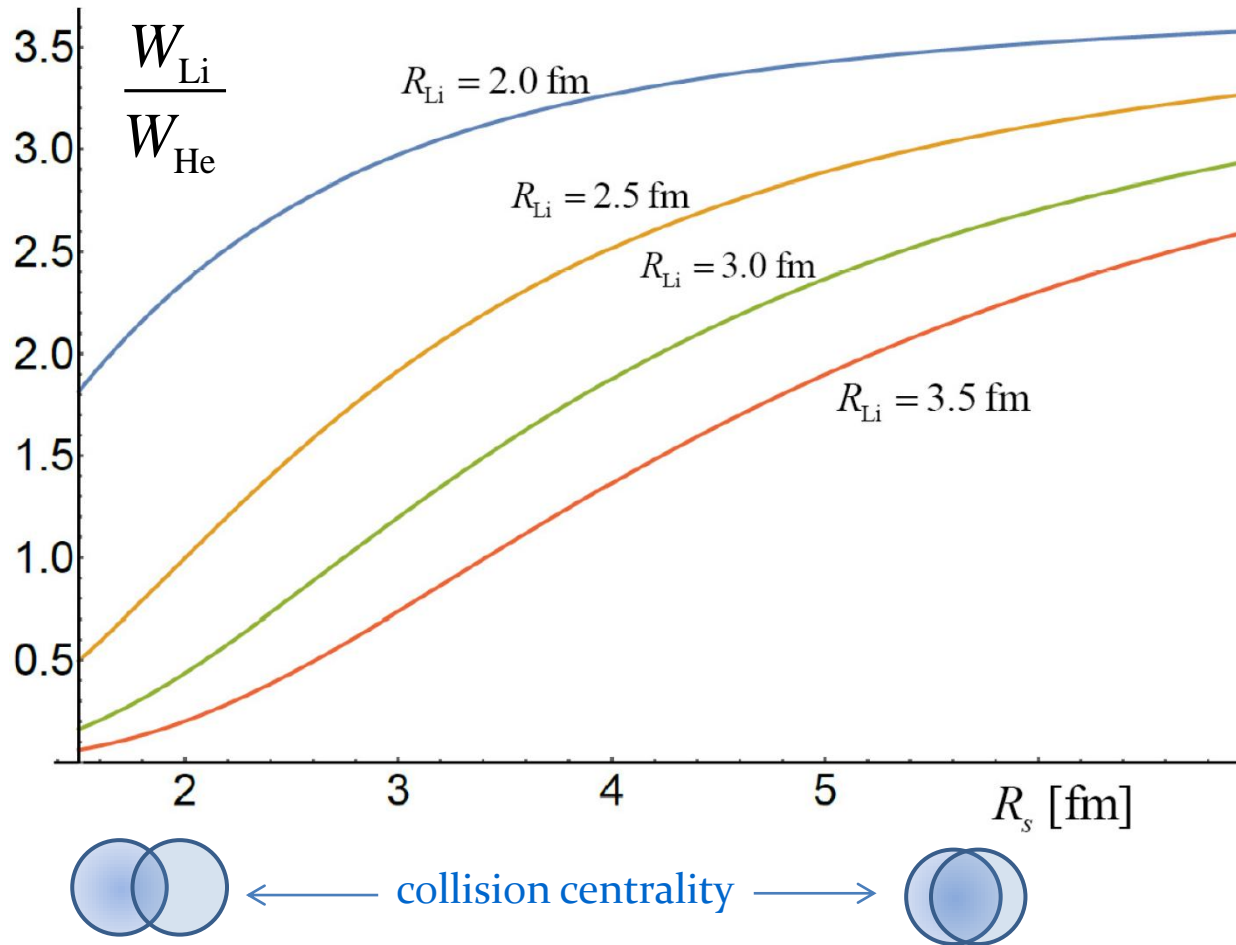
R_α – root mean square radius of ${}^4\text{He}$

R_{Li} – root mean square radius of ${}^4\text{Li}$

R_c – root mean square radius of ${}^3\text{He}$ cluster in ${}^4\text{Li}$

Ratio of yields of ${}^4\text{Li}$ to ${}^4\text{He}$

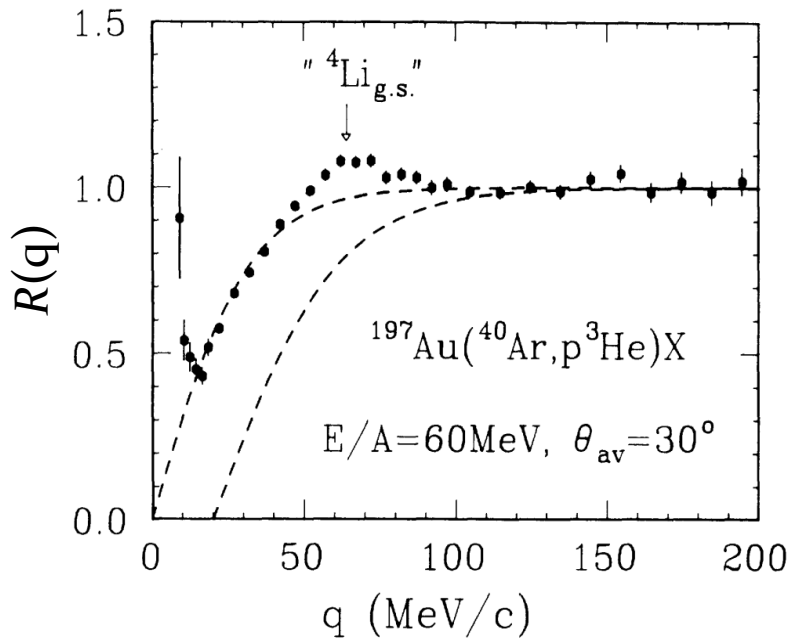
In the thermal model the ratio equals 5.



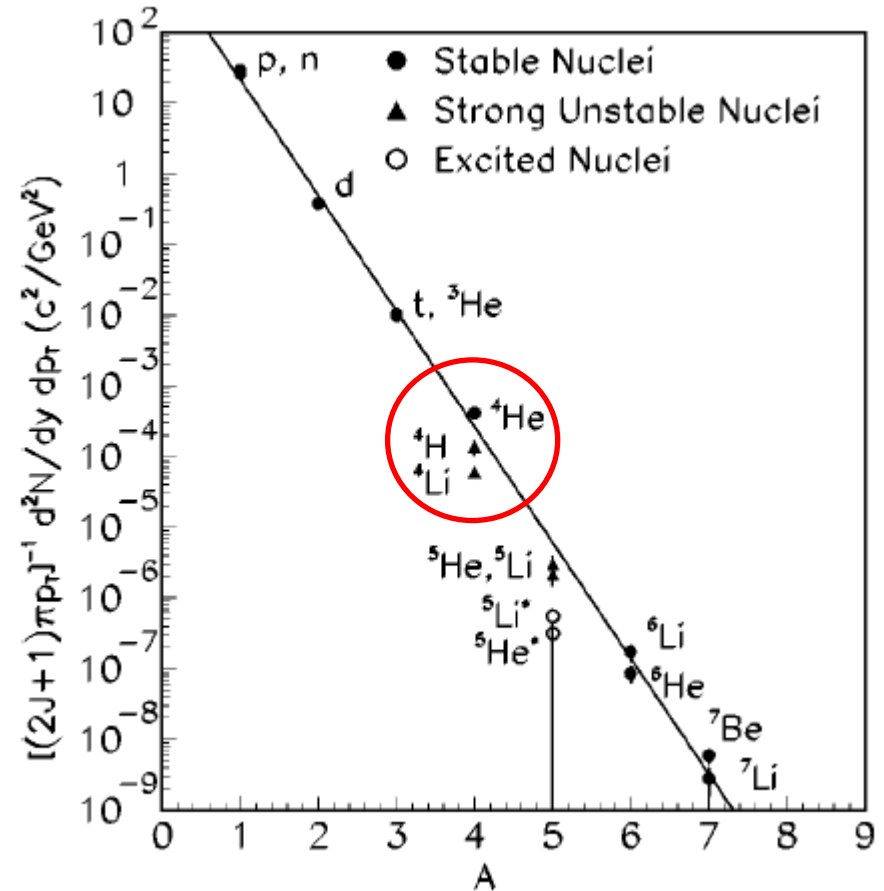
How to observe ${}^4\text{Li}$?

Measurement of the correlation function of ${}^3\text{He}$ - p is needed

${}^{197}\text{Au} + {}^{197}\text{Pt}$ @ AGS

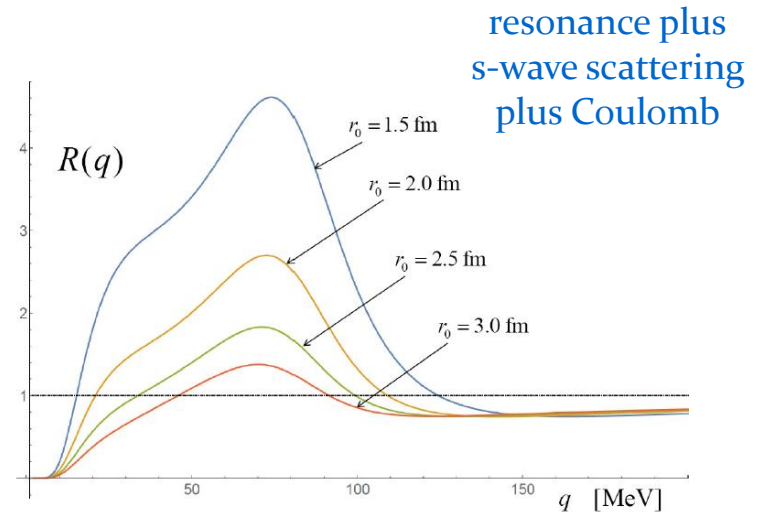
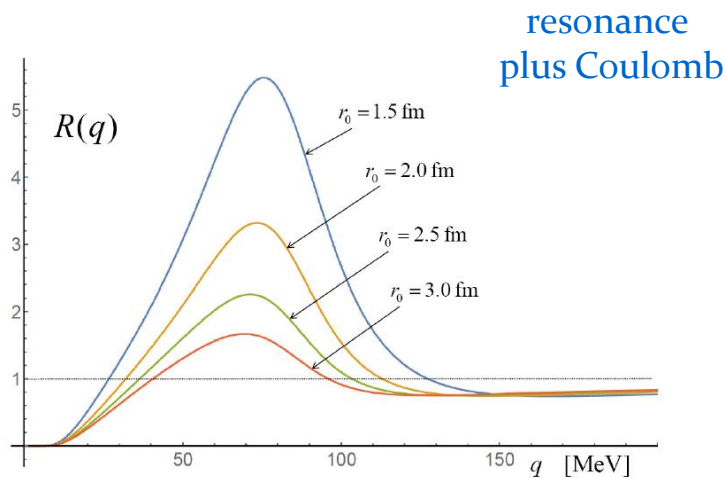
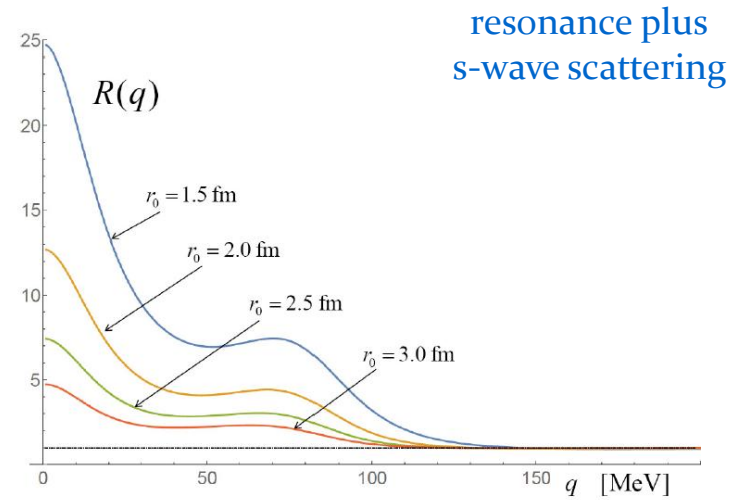
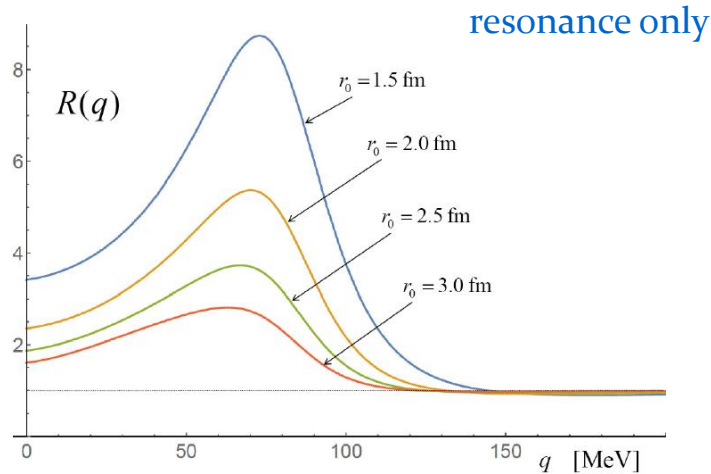


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Correlation function p - ^3He



How to measure yield of ${}^4\text{Li}$

$$\frac{dN_{\text{Li}}}{d\mathbf{p}} = S_R \frac{dN_p}{d\mathbf{p}} \frac{dN_{{}^3\text{He}}}{d\mathbf{p}}$$

\mathbf{p} - momentum per nucleon

$$S_R \equiv \int d^3q R_R(\mathbf{q})$$

↑
correlation function where only the ${}^4\text{Li}$ resonance contributes

Conclusions

p - D & D - D correlations



Hadron-deuteron and deuteron-deuteron correlations carry information about source of deuterons.



Measurement of p - p , p - D & D - D correlation functions can tell us whether deuterons are directly emitted from a fireball like protons or deuterons are formed due to final state interactions.



p - D and D - D correlation functions show a sufficient sensitivity to a size of particle source to falsify the thermal or coalescence model.

${}^4\text{He}$ vs. ${}^4\text{Li}$



The thermal and coalescence models give different predictions on the ratio of yields of ${}^4\text{Li}$ to ${}^4\text{He}$.



In the thermal model the ratio of yields is independent of collision centrality.



In the coalescence model the ratio is maximal for central collisions and rapidly decreases when one goes to peripheral collisions.



Since ${}^4\text{Li}$ can be observed through the correlation function of ${}^3\text{He}$ - p , the correlation needs to be measured.