A method to study “equilibration” in nucleus–nucleus collisions

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Abstract. A method to study “equilibration level” of a system created in high energy nucleus–nucleus collisions is proposed. The method is based on the fact that the correlation between momentum distribution and particle multiplicity observed in nucleon–nucleon interactions can be also measured in nucleus–nucleus collisions. It is argued that the magnitude of the residual correlation can be used as a measure of the system “equilibration level”. The method is effective even when changes of the “equilibration level” do not modify the inclusive distribution. The quantitative estimate of the effect to be measured is given. Finally, questions arising in practical application of the method are discussed.

1 Introduction

Recent experimental results on particle production in high energy nucleus–nucleus (A–A) collisions are somewhat surprising. The bulk characteristics like pion inclusive spectra or multiplicities are correctly described within the models treating A–A collisions as incoherent superposition of nucleon–nucleon (N–N) interactions [1]. On the other hand, there are observed in central collisions “exotic” phenomena such as the enhancement of the strangeness production, the suppression of J/ψ meson production or large radii of pion emission sources. They all seem to be incompatible with the assumption of incoherent superposition of N–N interactions [2]. Explanation of these effects requires a substantial role of the secondary interactions on the parton and/or hadron level, and consequently, a significant equilibration of the system created in the collisions, see e.g. [3].

In our previous paper [4] we have discussed fluctuations of a variable, further called a sum-variable, which is defined as a sum of single particle kinematical variables (transverse momentum, rapidity etc.) with the summation running over particles produced in a single collision in a specified phase space region. In this paper we develop a method of data analysis, which allows to estimate the “equilibration level” of the system generated in A–A collisions by studying the second moment of the sum-variable distribution.

The basic idea of the method is the following. The number of particles produced in high energy N–N collisions and particle momenta are correlated with each other [5]. In particular, the average transverse momentum, or more generally the transverse momentum distribution depends on the particle multiplicity [5, 6]. We demonstrate that under a specific choice of the sum-variable, Z, the second moment of Z-distribution is sensitive to the correlation between multiplicity and momenta of particles emitted from the single source providing that the independent particle sources constitute an A–A collision. Vanishing of this correlation signals “equilibration” of the system created in nucleus–nucleus collisions.

Let us explain the last statement. Particles produced in nucleon–nucleon collisions are assumed to originate from particle sources. Due to a specific property of these sources a correlation between particle multiplicities and momenta is observed. The number of particle sources is expected to be much higher in nucleus–nucleus collisions than in nucleon–nucleon ones, and the question arises whether this correlation survives. This is the case if the sources are created independently from each other at the initial stage of a nuclear collision and further on do not interact. However, one expects a significant role of secondary interactions driving the whole system towards the equilibrium. Then, the correlation between multiplicities and momenta of particles coming from the same source can be lost. We interpret the vanishing of the correlation as a signal of “equilibration”. Since the lack of correlation does not guarantee that the system reaches the thermodynamic equilibrium, we write down the word equilibration in the quotation marks.

The method requires large acceptance, high precision and large statistic measurements of particle production at various impact parameters. We believe that the analysis

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will be feasible with the data provided by the new experiments recently proposed at CERN and BNL and based on the TPC detectors [7].

The paper is organized as follows. In Sect. 2 the general description of the method is given. The Z-distribution is introduced and the values of its second moment in two limiting cases are discussed. In the first case the particle sources in A–A collisions are identical with those in N–N collisions, and then we show that the correlation between particle multiplicity and particle momenta, which is observed in N–N collisions, can be also measured in A–A collisions. In the second case we consider "equilibrated" sources as described above. The Z-distributions obtained in the above two cases are different from each other even when the particle inclusive distributions in N–N and A–A collisions are identical. The discussion of applicability of our method to the real data, which is presented in Sect. 3, starts with the estimate of the actual difference of the two limiting Z-distributions. The N–N data at √s = 20 GeV and √s = 200 GeV are considered here. These values of energy correspond to the energy regions of the heavy-ion experiments in CERN and the experiments planned at Relativistic Heavy Ion Collider in BNL. In the remaining part of Sect. 3 we discuss how to proceed with the data to determine the "equilibration level". We conclude our considerations in Sect. 4. In the Appendix we present two very simple pictures of high energy nucleus–nucleus collisions demonstrating at a qualitative level that the correlation between multiplicity and momenta of particles produced in nucleon–nucleon interactions is vanishing in nucleus–nucleus collisions when the whole system approaches thermodynamic equilibrium.

2 The method

Let variable x be any kinematical variable (p, p₁, ...) of a single particle. Let us introduce two new variables:

- a single particle variable
  \[ z = x - \langle x \rangle, \]  
  where \( \langle x \rangle \) is the mean value of (inclusive) x-distribution, and

- a corresponding sum-variable
  \[ Z = \sum_{i=1}^{N} z_i, \]  
  with summation running over all particles of a given sort which are produced in a specified phase space region in a single A–A collision.

As we shall discuss below, the quantity \( \Gamma \) defined as

\[ \Gamma = \frac{\langle Z^2 \rangle}{\langle N \rangle}, \]  

with \( N \) being the particle multiplicity in the event and \( \langle \ldots \rangle \) denoting the averaging over events, is sensitive to the correlation between multiplicity of particles and their momenta characteristic for particle sources constituting A–A collisions.

We propose to study \( \Gamma \) as a function of \( \langle N \rangle \) splitting the sample of A–A events into classes which differ from each other by the mean number of particle sources. Since this number is a model dependent quantity, which is not directly measured, we suggest to select events using the number of spectator nucleons from projectile and/or target nucleus. The details of this selection are not relevant, however it is important to stress that the selection criteria should not directly involve particle multiplicity.

In the following subsections two possible limits of \( \Gamma \) are considered.

2.1 The N–N limit

Let us assume that A–A collisions can be treated as incoherent superposition of N–N interactions. Under this assumption one gets obvious relation between \( \langle Z \rangle \) and the mean value of the sum-variable, \( Z \), for N–N sources, \( \langle Y \rangle \), namely:

\[ \langle Z \rangle = \langle k \rangle \langle Y \rangle, \]  

where \( \langle k \rangle \) is the number of N–N sources in an A–A collision.

The fluctuations of \( Z \) will, in principle, depend on the fluctuations of \( k \) and \( Y \). The variance of \( Z \), defined as \( V(Z) = \langle (Z - \langle Z \rangle)^2 \rangle \), can be easily derived:

\[ V(Z) = \langle k \rangle V(Y) + \langle Y \rangle^2 V(k), \]  

where \( V(Y) \) is the variance of \( Y \) and \( V(k) \) is the variance of \( k \).

In general, the variance \( V(k) \) can not be calculated in a model independent way and without simulation of the event selection criteria. However, due to a specific choice of the variable \( Z \), such that \( \langle Z \rangle = 0 \), and consequently \( \langle Y \rangle = 0 \) (cf. (4)), (5) can be rewritten as

\[ \langle Z^2 \rangle = \langle k \rangle \langle Y^2 \rangle. \]  

One sees that the dispersion of \( Z \)-distribution is now independent of the fluctuations of \( k \), i.e. of the distribution of number of sources coinciding with the number of N–N interactions in the assumed scenario of A–A collisions.

In the discussed approach the average number of sources in A–A collisions in a given event sample selected by the number of sources is

\[ \langle k \rangle = \frac{\langle N \rangle}{\langle n \rangle}, \]  

where \( \langle n \rangle \) is the mean number of particles produced in the N–N interactions and \( \langle N \rangle \) is the mean number of particles produced in the A–A collisions in the sample. Combining (6) and (7) one finds

\[ \frac{\langle Z^2 \rangle}{\langle N \rangle} = \frac{\langle Y^2 \rangle}{\langle n \rangle}, \]  

which can be also written as

\[ \frac{\langle Z^2 \rangle_{AA}}{\langle N \rangle_{AA}} = \frac{\langle Z^2 \rangle_{NN}}{\langle N \rangle_{NN}}. \]  

Thus, the value of \( \Gamma \equiv \frac{\langle Z^2 \rangle}{\langle N \rangle} \) for the A–A collisions is determined solely by the properties of the N–N sources. The value of \( \Gamma \) given by (9) is called the N–N limit.
2.2 The “Equilibrium” limit

The scenario of nucleus–nucleus collisions, which is called “equilibrium”, assumes that the correlations existing at the initial stage of the collision are lost and particles are emitted independently from each other*. In such a case (8) can be rewritten as

\[
\frac{\langle z^2 \rangle}{\langle N \rangle} = \langle z^2 \rangle,
\]

where \(\langle z^2 \rangle\) is the second moment of the inclusive (single particle) \(z\)-distribution for a given sample of A–A collisions. We assume here that single particle distribution is independent of \(N\) in the A–A collision sample. Thus, the value of \(F\) in this limit is determined uniquely by the second moment of the inclusive distribution for A–A collisions.

2.3 Are the two limits different?

The question arises whether the two limits represented by (8) and (10) differ from each other. The answer is positive. The point is that the distributions of kinematical variables of particles produced in nucleon–nucleon interactions depend on the particle multiplicity as already mentioned. The existence of this dependence introduces a specific type of correlation. The \(n\) particles produced in an interaction can be independent from each other, but adding one particle more to a system influences all \(n\) particles. Then, the distribution (probability density) of the variable \(Y\), which is the \(Z\) variable for particles originating from the single source can be written as

\[
\mathcal{P}(Y) = \sum_n \int \cdots \int P_n(z_1, \ldots, z_n) \times \delta(Y - (z_1 + \ldots + z_n)).
\]

where \(P_n\) is the multiplicity distribution and \(\rho_{(n)}(z)\) is a single particle distribution of a variable \(z\) in events of multiplicity \(n\); \(P_n\) and \(\rho_{(n)}(z)\) are normalized as \(\sum_n P_n = 1\) and \(\int dz \rho_{(n)}(z) = 1\). This guarantees the proper normalization of \(\mathcal{P}(Y)\) i.e. \(\int dY \mathcal{P}(Y) = 1\).

Substituting (12) into (11) one easily calculates the first two moments of \(\mathcal{P}(Y)\) as

\[
\langle Y \rangle = \sum_n P_n \langle z \rangle_{(n)},
\]

\[
\langle Y^2 \rangle = \sum_n P_n \left[ \langle z^2 \rangle_{(n)} + (n-1) \langle z \rangle_{(n)}^2 \right],
\]

where \(\langle z^2 \rangle_{(n)} = \int dz z^2 \rho_{(n)}(z)\). The first term in the right-hand-side of (14) equals \(\langle n \rangle \langle z^2 \rangle_{NN}\), where \(\langle z^2 \rangle_{NN}\) is the second moment of the inclusive distribution for nucleon–nucleon interactions. Although we have chosen \(z\) in such a way that \(\langle Y \rangle = 0\), the second term in rhs (14) does not vanish as long as \(\langle z \rangle_{(n)}\) depends on \(n\). Consequently, \(\langle Y^2 \rangle / \langle n \rangle\) differs from the second moment of the inclusive distribution. Therefore, the two limits of \(F = \langle Z^2 \rangle / \langle N \rangle\) are in general different from each other. The two limits are identical only in the particular case when \(\langle Y^2 \rangle / \langle n \rangle = \langle z^2 \rangle\).

One should note that the proposed method distinguishes the N–N and “equilibrium” limits even in such a case when the inclusive distributions of \(z\) are identical in N–N and A–A collisions. It is very important because the experimental inclusive distributions of pions are rather similar in both cases [8].

Equation (14) also suggests the choice of the variable \(z\) and acceptance region, which should be used in the analysis. The choice should be done in order to maximize the relative contribution of the second term in the right-hand-side of (14).

3 Applicability of the method

In this section we discuss how to apply our method to the data analysis. We start with the quantitative estimate of the difference of \(\langle Z^2 \rangle / \langle N \rangle\) in the two limits discussed in the previous section. We use the data concerning the mean transverse momentum dependence on multiplicity in \(p\bar{p}\) collisions at \(\sqrt{s} = 20\) GeV [9] and \(p\bar{p}\) at \(\sqrt{s} = 200\) GeV [10], which are presented in Figs. 1 and 2, respectively. These two energies correspond to energy regions of existing and recently proposed heavy ion experiments in CERN and BNL.

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*The short range correlations, such as those due to the quantum statistics are, of course, present in the final states of nuclear collisions. It is expected however that these correlations do not influence significantly the \(Z\)-distribution if the large phase space region is considered.
Fig. 2. The mean transverse momentum of charged particles produced in $p\bar{p}$ collisions at $\sqrt{s} = 200$ GeV versus charged particle multiplicity. The data are limited to the pseudorapidity interval $\pm 2.5$. The dashed lines correspond to several values of the parameter $\Delta T/\Delta n$, which controls the dependence of the transverse momentum distribution on particle multiplicity within the parameterization (17).

3.1 Quantitative estimate of the effect

In this subsection we estimate the magnitude of the effect to be measured. We calculate the difference between the two limits of $\Gamma$ assuming that single particle distributions are identical in both scenarios. As an example, the transverse momentum, $p_T$, as a variable $x$, cf. (1), (2) is chosen. Let us define variable $z = p_T - \langle p_T \rangle_{\text{NN}}$, where $\langle p_T \rangle_{\text{NN}}$ is the mean transverse momentum calculated from the inclusive distribution in N-N interactions. This variable satisfies the conditions required by the method: $\langle z \rangle = 0$ and $\langle z^2 \rangle_{\text{NN}} = \langle p_T \rangle_{\text{NN}}^2 - \langle p_T \rangle_{\text{NN}}^2 \neq 0$, where $\langle p_T \rangle_{\text{NN}}$ is the mean single particle transverse momentum in the N-N events of multiplicity $n$. The $p_T$-probability density for events with multiplicity $n$ can be parameterized as

$$
\rho_{\text{NN}}(p_T) = C_n p_T e^{-m_T/T_{\text{NN}}},
$$

where $C_n$ is a normalization factor, $m_T$ is the so-called transverse mass equal $\sqrt{p_T^2 + m^2}$, $m$ is pion mass and $T(n)$ is the multiplicity dependent slope parameter. In the case of $p-p$ collisions at $\sqrt{s} = 20$ GeV, which are further treated as the N-N interactions, the multiplicity distribution and mean multiplicity are calculated using parameterizations given in [11], whereas for the $p-\bar{p}$ collisions at $\sqrt{s} = 200$ GeV, also treated as the N-N interactions, the data from [12] are taken.

Let us define the quantity

$$
\Delta D = \frac{\langle Y^2 \rangle}{\langle N \rangle} - \sqrt{\langle z^2 \rangle},
$$

which measures the difference between the N-N and "equilibrium" limits of $\sqrt{\langle z^2 \rangle}/\langle N \rangle$ assuming the same inclusive distributions of $z$ in both scenarios. In order to study the dependence of $\Delta D$ on the strength of the $\langle p_T \rangle_{\text{NN}}$ versus $n$ correlation we use a simple parameterization of the slope parameter $T(n)$

$$
T(n) = T_0 + (n - 1) \frac{\Delta T}{\Delta n},
$$

where $\Delta T/\Delta n$ is the parameter which allows to change the correlation strength. The calculated $\langle p_T \rangle_{\text{NN}}$ versus $n$ dependence is shown in Figs. 1 and 2 for several values of the parameter $\Delta T/\Delta n$. Accidentally the same value of $T_0 = 173$ MeV can be used to describe two data samples. The parameterization of $T(n)$ given by (17) does not fit the $p-\bar{p}$ data quite well (see Fig. 2), but it is not important as we intend to estimate only the magnitude of $\Delta D$. In order to calculate $\langle Y^2 \rangle$ we assume that in the N-N collisions particles are emitted independently from each other according to the $\rho_{\text{NN}}(p_T)$ distribution.

The final results are shown in Figs. 3 and 4 where the dependence of $\Delta D$ on $\Delta T/\Delta n$ is plotted. For the values of
\( \Delta T/\Delta n \) which approximately characterize the experimental data the values of \( \Delta D \) are about 3.5 MeV for both energies \( \sqrt{s} = 20 \text{ GeV} \) and \( \sqrt{s} = 200 \text{ GeV} \). We estimated that in the case of the sample of \( 10^7 \text{ A} \text{ A} \) collisions the two limit values can be distinguished on the 6–7 standard deviation level.

3.2 The data analysis

Let us briefly discuss how to perform the data analysis. The first important point is a careful choice of the \( z \) variable and an acceptance region. It can be done on the basis of existing N–N data and Monte Carlo models of hadron–hadron interactions. For a given collision energy and combination of projectile and target nuclei several sample of events have to be taken. They should differ by impact parameter distributions. It is important to collect a high statistics sample with exclusively large impact parameters for which the N–N scenario should be applicable. This sample can provide the N–N limit of \( \langle Z^2 \rangle/\langle N \rangle \) in the case of absence of the corresponding N–N data.

Let us index each sample of A–A collisions by its mean multiplicity, \( \langle N \rangle \). We propose to study the dependence on \( \langle N \rangle \) of a new normalized variable defined as

\[
\alpha = \frac{\sqrt{\langle Z^2 \rangle/\langle N \rangle} - \sqrt{\langle z^2 \rangle/\langle N \rangle}}{\sqrt{\langle N \rangle} - \sqrt{\langle z^2 \rangle/\langle N \rangle}},
\]

where \( \langle z^2 \rangle/\langle n \rangle = \langle Z^2 \rangle_{\text{NN}}/\langle N \rangle_{\text{NN}} \) corresponds to N–N collisions.

The value of \( \alpha \) gives a quantitative measure of the “equilibration level” of the system understood as a level of disintegration of the correlation present in the N–N sources of particles. The variable \( \alpha \) takes into account possible changes of inclusive distribution in A–A collisions from sample to sample of events. The comparison between results obtained in different experiments and/or in different acceptance regions done in terms of \( \alpha \) variable reduces bias due to experimental systematical error and allows to compare results for different particles and acceptances.

4 Summary and final remarks

We propose a new method to study high energy hadron–nucleus and nucleus–nucleus collisions. The method is based on the analysis of the quantity \( \Gamma \equiv \langle Z^2 \rangle/\langle N \rangle \), where \( Z = \sum_{i=1}^{N} (x_i - \langle x \rangle) \) with \( x_i \) being the single particle kinematical variable such as rapidity or transverse momentum, as a function of the mean multiplicity, \( \langle N \rangle \). It is shown that \( \Gamma \) is sensitive to the correlation between momentum distribution and multiplicity characteristic for particle sources which constitute A–A collisions. It is argued that the vanishing of this correlation in central A–A collisions may be interpreted as higher (than in N–N interactions) “equilibration” of the system caused by secondary interactions.

It is shown on the basis of existing elementary (hadron–hadron) data that the magnitude of the effect is high enough to be observed by recently proposed heavy ion experiments in CERN and BNL. Finally, we suggest a procedure of data collecting and analysing which minimizes ambiguities with the interpretation of the results.

We would like to close the paper with two comments.

- There are many calorimetric measurements of transverse energy distribution in proton–nucleus and nucleus–nucleus high energy collisions [13]. The transverse energy is defined as a sum of transverse energies of particles in the given acceptance \( (E_T = \sum_i (E_i \sin \theta)) \), where \( \theta \) is a angle between projectile direction and momentum vector of a particle. Thus transverse energy may be used as a sum-variable. The large fluctuations of \( E_T \) were observed [14]. However the interpretation of this result depends on assumption concerning variation of the number of sources in the analyzed sample of events. The influence of this variation is removed in our method due to the specific choice of the variable. Unfortunately the method cannot be applied to the calorimetric transverse energy data because it does not provide information on particle multiplicity and single particle \( E_T \) distribution within the calorimeter acceptance. Very recently the \( E_T \) fluctuations have been interpreted as a manifestation of color fluctuations in the wave function of a nucleon [15]. If this interpretation is correct one may expect that data analysis performed using our method shall yield the value of \( \alpha \) (see (18)) greater than unity for central p–A and A–A collisions.

- There are repeated attempts to find a simple relation between \( e^+e^- \) and p–p collision dynamics, for critical review see [16]. One of the discussed scenarios treats p–p process as an incoherent superposition of several “more elementary” processes similar to those in \( e^+e^- \) [11]. Therefore our method may serve as a test for such models. The \( e^+e^- \) limit of N–N collisions should be treated as the N–N limit of A–A collisions and the collision inelasticity instead of the spectator number should be used as an event selection parameter.

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Appendix

We consider here two very simple pictures of high energy nucleus–nucleus collisions, which show that under plausible assumptions the correlation between multiplicity and momenta of particles produced in nucleon–nucleon interactions is lost in nucleus–nucleus collisions when the system created in these collisions evolves towards thermodynamic equilibrium.

- Picture A. Let us imagine that several particle sources are created in a nucleus–nucleus collisions. The sources are assumed to be isolated from each other and the source excitation energy is supposed not to vary from source to
source and from event to event. However, the number of particles emitted by a single source can fluctuate. Then, due to energy conservation, the average particle momentum (in the source rest frame) decreases when the particle multiplicity increases. Such a correlation has been observed in the full phase space hadron–hadron and $e^+e^-$ data [5]; an example is presented in Fig. 1. The system of several isolated sources is highly nonhomogeneous and consequently far from (global) equilibrium. Let us now assume that the number of sources per unit volume is high enough. Consequently the sources interact with each other and finally form a homogeneous equilibrium system. We consider particles originated from a subsystem of a size of the previous isolated source. The number of particles can still fluctuate but it does not influence their momentum distribution due to the thermal contact with the surrounding matter, which works as a thermostat. Consequently, the correlation between multiplicity and momenta of particles originated form the considered subsystem gets lost.

- **Picture B.** As in the Picture A, several isolated sources of particles are created in nucleus–nucleus collision. The source excitation energy however is assumed to fluctuate substantially. Therefore the higher excitation energy the higher number of particles and their momenta. The increase of the average transverse momentum with particle multiplicity has been observed in central rapidity region of hadron–hadron and $e^+e^-$ collisions [6] at very high energies; for example see Fig. 2. The set of several isolated sources represents again a non equilibrium system. As previously we further consider an equilibrium system created out of the initial sources. The correlation between multiplicity and momenta of particles emitted form the subsystem again vanishes because the excitation energy (temperature) is uniform over the whole system.

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