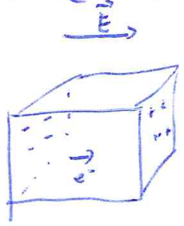
Confinement: an intuitive picture

Let us still remain in the sole realm of e.m.



$\vec{J} = \sigma \vec{E}$

σ is the 'conductivity'. Usually, it is some number, high for metals, but small for non-conducting material (wood, plastic).

However, there are superconducting materials in which

$\sigma \mapsto \infty$

(i.e., the resistance $R \sim 1/\sigma \mapsto 0$ ... you can have a endless flux of current in such a material).

Now, in order not to have $\vec{J} = \infty$ (∞ current), we realise that in a superconductor we have

$\vec{E} = 0$.

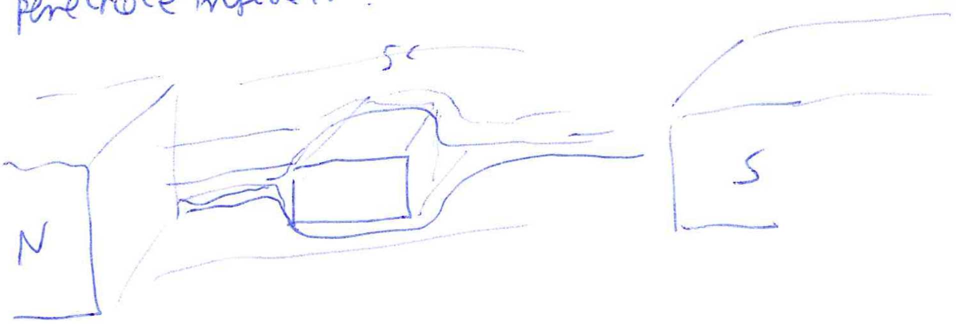
Moreover, being $\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} \rightarrow \vec{B} = \text{const} \rightarrow \vec{B}$ is fixed.

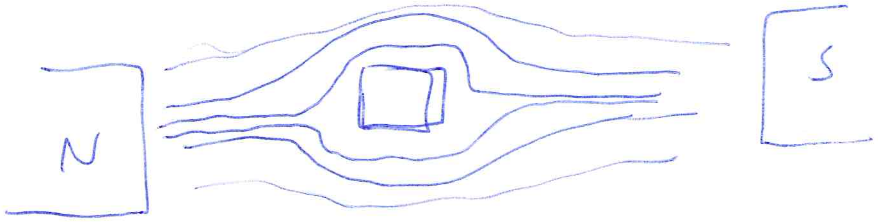
If no field is present at the beginning, we realise that in a superconductor

$\vec{B} = \vec{0}$.

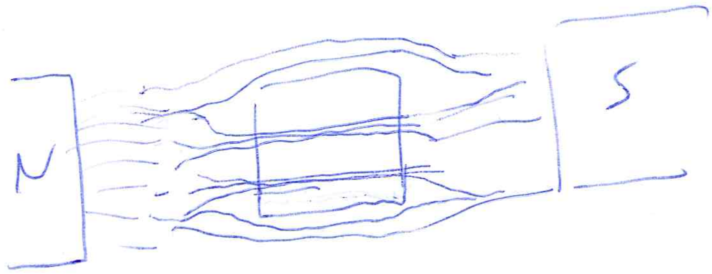
Meissner effect :

If you put a magnetic field around a superconducting material, $\vec{B}$ will not penetrate inside it.





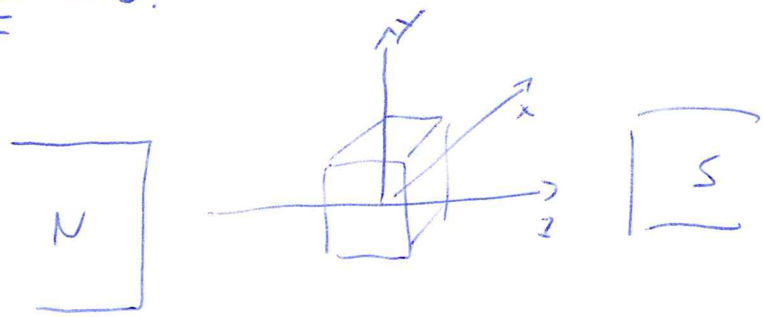
However, if we increase the intensity of the magnetic field very much, we may obtain that some magnetic lines go inside.



Here, there are two "thin" fluxes of magnetic field which penetrate into the material. A compromise is found between the Meissner effect and the penetration of $\vec{B}$. The compromise takes the form of "n" thin fluxes.

Note, the flux of the magnetic field around the superconductor is

not zero.



$$\phi = \int_{\text{surface}} \vec{B}\,\vec{n}\,dS$$

is the flux of the magnetic field through the superconductor.

Meissner: $\phi = 0$

Beyond Meissner $\boxed{\phi = \frac{n\pi}{e}}$ $\Rightarrow$ quantized

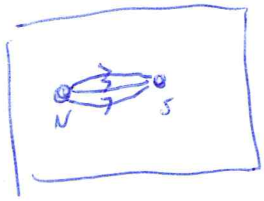
There is therefore a "minimal" flux, which is equal to $\frac{\pi}{e}$.

It is exactly as in the AB case for the AB-invisible coil, apart from a factor "2".

$e \mapsto 2e$

This has to do with the fact that the basic charge is "2e" for superconductivity.

But now: imagine that we take two magnetic monopoles and put inside a superconducting material:



→ very thin flux tube from N to S.

We get a "flux tube" and a linear rising potential between the magnetic monopoles.

That is, in a electric superconductor one has confinement of magnetic charges.

Recall: electric superconductor is obtained through a condensation of Cooper e^-e^- pairs.

Conjecture: Quark confinement in QCD is due to the fact that the QCD vacuum is a superconductor for (chromo)magnetic fields. Then, single color charges get confined (linear rising potential).

Superconductor → introduction of an "Higgs" like field.

The Higgs field is in this case given by a cooper pair

$$\underbrace{H \sim \psi^t C \gamma^5 \psi}$$

it is like a good diquark.

$S = L = 0.$

$Q = -2$

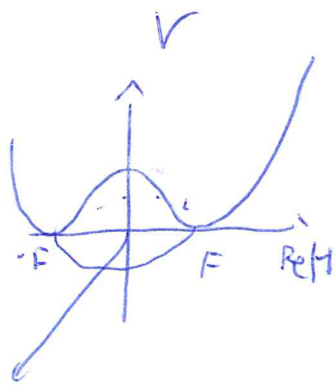
H, H^* complex scalar field.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 \longmapsto$$

$$\mathcal{L} = \frac{1}{2} D_\mu H^* D^\mu H - V(|H|) - \frac{1}{4} F_{\mu\nu}^2$$

$$D_\mu H = \partial_\mu H - i 2 e A_\mu H$$

If $V(|H|)$ takes the usual Mexican-hat form



$$V(H) = \frac{\lambda}{4}\left(|H|^2 - F^2\right)^2$$

We then get:

$H = F + h$

$(D_\mu H)^* D^\mu H = (\partial_\mu - i(2e) H A_\mu)^* (\partial^\mu - i(2e) A^\mu H)$

Isolate the term

$4e^2 H^2 A_\mu^2$

Upon the condensation of H, $H = F$, we get:

$4e^2 F^2 = \frac{\mu^2}{2}$

$$\boxed{\mu = 2\sqrt{2}\, e F}$$

Mass of the photon.

Achtung: No Goldstone particle, that d.o.f. is eaten up by the longitudinal mode of the photon.

No. of dof:

Re H, Im H A_μ

$2 + 2 = 4$

$1 + 3 = 4$

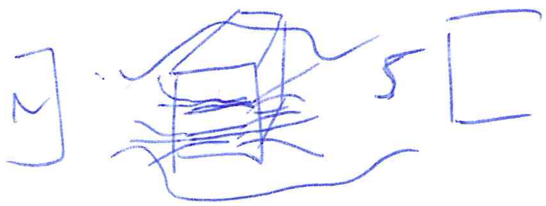
H A_μ with μ

The photon gets a mass → no propagation of $\vec{E}$ and $\vec{B}$ inside the superconductor.

This is the Meissner effect described above!

Apart from some border effects, you cannot have propagation, because it is exponentially suppressed as $e^{-\mu r}$.

However, how do we explain the flux thing?



This is also possible and corresponds indeed to a combination of H and A_μ.

Up to now: $H = F$ across constant... in this way, as we have seen, only the photon mass is generated, but no flux tube can be explained.

But, going further, we can get peculiar contribution.

Let us consider:

$$H = \frac{-y}{\sqrt{x^2+y^2}} F + i \frac{x}{\sqrt{x^2+y^2}} F$$

together with:

$$(A_x, A_y) = \frac{1}{2e(x^2+y^2)} \begin{pmatrix} y \\ -x \end{pmatrix}$$

We then get:

$$\left(\partial_i + 2ie A_i\right) H = 0 . \qquad i = x, y$$

Namely, $i = x$

$$\partial_x H = -y \frac{x}{(x^2+y^2)^{3/2}} F + i \frac{1\sqrt{} - x \cdot \frac{x}{\sqrt{}}}{(x^2+y^2)} F =$$

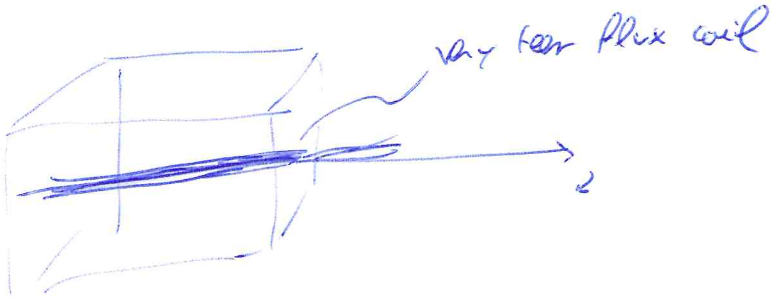
$$= -\frac{xyF}{r^{3/2}} + i \frac{y^2}{r^{3/2}} F$$

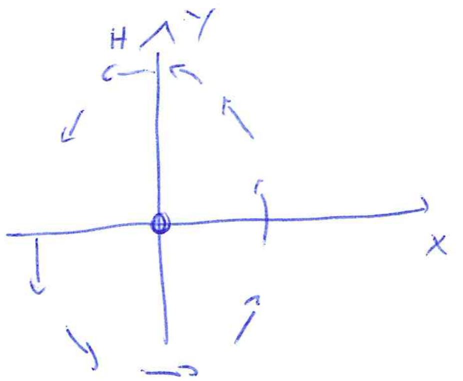
$$2ei A_x H = \cancel{2e} i \frac{1}{\cancel{2e} r^2} \cdot y \cdot \left(\frac{-y + ix}{r} \right) F$$

$$= \frac{i}{r^3} \left(-y^2 + ixy \right) F$$

and, indeed, you find that the constraint is fulfilled!!!

A_μ corresponds to





vortex of H ... H and A_μ cancel perfectly, in such a way that you get a flux tube.

$$\Phi = \frac{\pi}{e}$$

In general, $\Phi = n \frac{\pi}{e}$.

Then, going back to the confinement of electric charges, we should make sure that:

- we have magnetic monopoles
- they have an attractive interaction in such a way that they condense

Namely, if magnetic monopoles are present, we can use duality (symmetry between $\vec{E}$ and $\vec{B}$) and consider the very same discussion, but with fluxes of $\vec{E}$!

Problem: "usually" models with magnetic monopoles are such that they are heavy!!!!

This is why it is hard to make them condense.

Plenty of study in this directions:

- with supersymmetric models
- lattice
- Models of QCD

Topics

1) · KORTEWEG-DE-VRIES - Gleichung and ihre Lösungen

2) · (ϕ_1, ϕ_2) case in 1+1
(page 23-27 Raj.)

3) · CP^{N-1} model (generalisation of $O(3)$)

4) · Quantisation of static solution (cap. 5, Rajaraman)

5) Experimental status of monopole searches (which experiments...)

6) Natural examples of solitonic waves

7) AB → find the corresponding experiment and present it!

8) instanton in a periodic potential:
pages 303-318!

9) TSUNAMIS = WHY ARE THEY (NOT) A SOLITON?

10) Solitons in optics