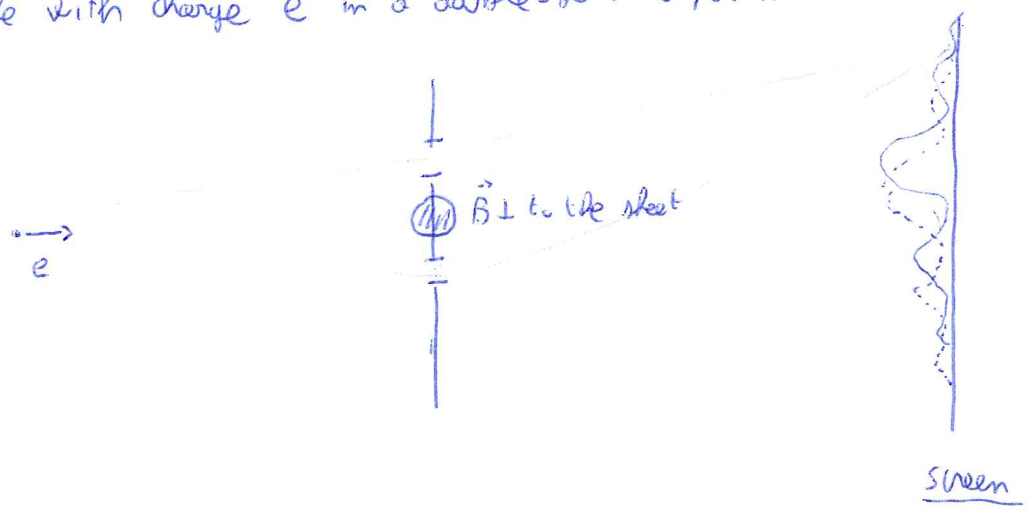
Recall

A-B effect

Particle with charge e in a double-slit experiment



Two paths ... No path goes inside the region with the nonzero $\vec{B}$.

However: $\vec{B} \neq 0$ in C, $= 0$ outside, but $\vec{A}$ nonzero everywhere.

The phase change is given by

$$\Delta\phi = ie \oint d\vec{x} \cdot \vec{A}$$

Interference given by

$$\sim \left| 1 + e^{i\Delta\phi(\vec{A}=\vec{0})} \cdot e^{ie\oint d\vec{x}\cdot\vec{A}} \right|^2$$

$$\oint d\vec{x}\,\vec{A} = \int \vec{B}\,\vec{n}\,dS = B_0 \pi R^2$$

$$\boxed{\Delta\phi = e B_0 \pi R^2}$$

what if $e B_0 \pi R^2 = 2n\pi$???

Then:

$$e^{ie\oint d\vec{x}\cdot\vec{A}} = e^{i2n\pi} = 1 \text{ !!!}$$

If we then consider the coil very thin, ie $R \mapsto 0$, we should also increase B_0 such that:

$$\boxed{(B_0 R^2) = \frac{1}{e} \cdot (2n)}$$

Then, we cannot see the coil. It becomes invisible!!!!!

In fact, also the AB effect is not capable to tells us the presence of this coil.

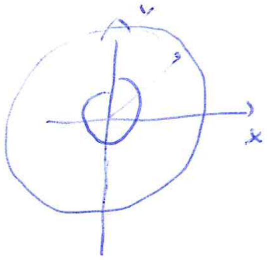
(Note, the flux of the magnetic field reads:

$$B_0 \pi R^2 = \frac{1}{e} 2\pi n$$
)

This is indeed still nonzero...

Now, the field $\vec{A}$ outside the circle C is given by:

$$\vec{A} = c\left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right)$$

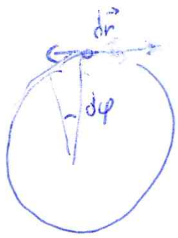


where

$$\boxed{c = \frac{B_0 R^2}{2}}$$

We can also calculate directly the integral $\oint \vec{A} d\vec{r}$.

$$\oint \vec{A}\cdot d\vec{r} =$$



$$|d\vec{r}| = r\, d\varphi$$

$$\vec{A}\cdot d\vec{r} = c\sqrt{\frac{y^2}{(x^2+y^2)^2} + \frac{x^2}{(x^2+y^2)^2}} \cdot r\, d\varphi = \frac{c}{r} r\, d\varphi$$ (indep. on r)

Ergo:

$$\oint \vec{A}\cdot d\vec{r} = c\cdot 2\pi$$

$$\Delta\phi = e\oint \vec{A}\cdot d\vec{r} = e\, c\, \cancel{2\pi} = \cancel{2\pi} n \to c = \frac{n}{e}$$

Ergo, if $\vec{A}$ is such that

$$\boxed{\vec{A} = \frac{n}{e}\left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right) \text{ is just a pure gauge}}$$

Even if it is singular on the z axis, there is no way we can see it..

Recall: magnetic monopole

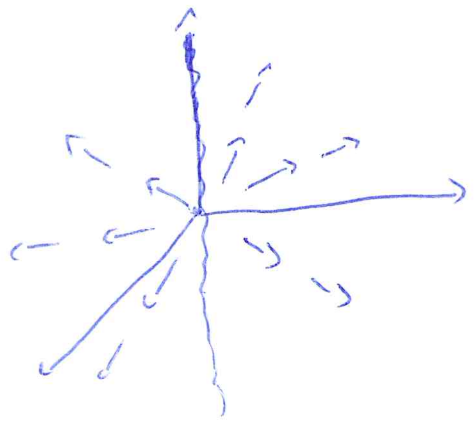
$$\vec{A} = \frac{g}{4\pi}\left(\frac{-yz}{r(x^2+y^2)}, \frac{xz}{r(x^2+y^2)}, 0\right)$$

Implies

$$\vec{B} = \frac{g}{4\pi}\frac{\vec{r}}{r^3}$$

$\to g$: magnetic charge

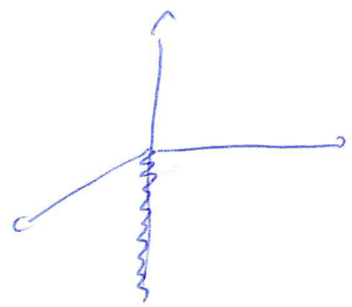
but is singular on the whole z axis.



However, we can go further and construct

$$\vec{A}_N = \frac{g}{4\pi}\left(\frac{z}{r} - 1\right)\left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right)$$

which still gives

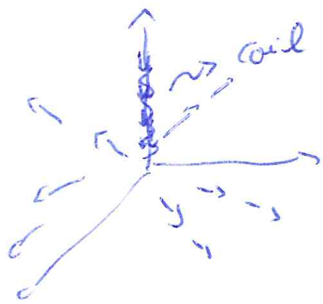


$$\vec{B} = \frac{g}{4\pi}\frac{\vec{r}}{r^3}$$

but now it is not defined on the negative z-axis.

Similarly, you could have constructed

$$\vec{A}_S = \frac{g}{4\pi}\left(\frac{z}{r}+1\right)\left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right)$$



which is valid in the south emisphere.

$\vec{B}$ is always given by $\frac{g}{4\pi}\frac{\vec{r}}{r^3}$, i.e. a magnetic monopole.

Now, according to which emisphere you are, you can chose to either use $\vec{A}_N$ or $\vec{A}_S$...

The key point is: if $\vec{A}_S - \vec{A}_N$ is a pure gauge, then there is no way to even see the coil.

$$\vec{A}_S - \vec{A}_N = \frac{g}{2\pi}\left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right)$$

But this is indeed a gauge if

$$\boxed{\frac{g}{2\pi} = \frac{n}{e}}$$

Ergo we come to the conclusion that

$$\boxed{ge = 2n\pi}$$

which implies:

- quantization of both the electric and the magnetic charges.
- inverse proportionality... for $n = 1$

$$eg = 2\pi$$

$$e \text{ small} \longleftrightarrow g \text{ large}$$

(perturbation theory is possible only for e-charges, but not for g... still, I would conceive the opposite situation in which g is small and e is large).

In models in which the calculation of the mass of the monopoles is doable, you shall find that they are very massive.

In conclusion, Dirac has shown that it is possible to construct a magnetic monopole in the usual ED; however, to this end it is necessary to look at his quantum version and consider the non-observability of a coil by using the AB effect, or better the fact that neither with the AB effect you can see that coil...

The quantization is a very interesting outcome.