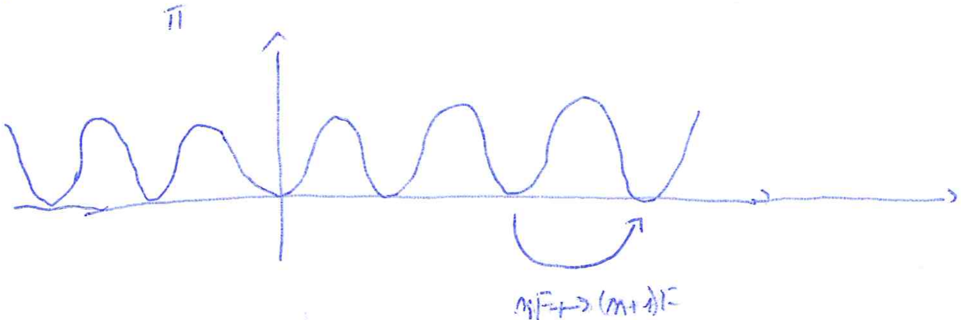
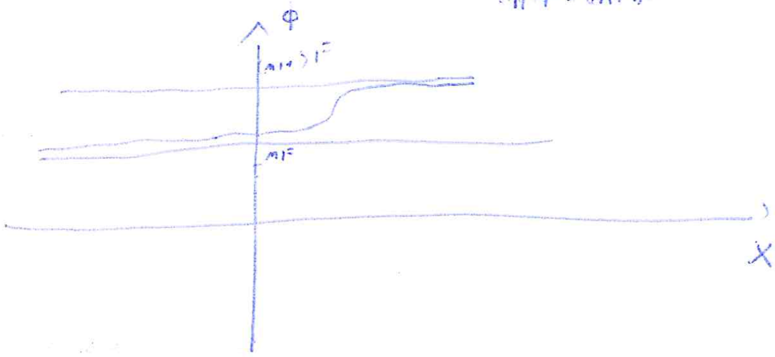
Sine-Gordon : recall and scattering.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) \qquad \phi(t,x) : \mathbb{R}^2 \mapsto \mathbb{R}$$

$$V(\phi) = A\left(1 - \cos\left(\frac{2\pi}{F}\phi\right)\right)$$

$$\phi(x) = \frac{2F}{\pi} \arctan\left(e^{m(x-x_0)}\right) + nF$$ is a soliton of this theory.





Composite soliton solutions... but before recall that:

$$m^2 = A\left(\frac{2\pi}{F}\right)^2$$

We define λ as:

$$\boxed{A = \frac{m^4}{\lambda}} \Rightarrow \boxed{\frac{2\pi}{F} = \frac{\sqrt{\lambda}}{m}}$$

$$\bar{\phi} = 4 \arctan\left[e^{\bar{x}}\right] = \frac{4\sqrt{\lambda}}{m} \arctan\left[e^{mx}\right] = \frac{\sqrt{\lambda}}{m}\phi$$

$\bar{x} = mx$

$\bar{t} = mt$

$$\mathcal{L} = \frac{m^4}{\lambda}\left[\frac{1}{2}(\bar{\partial}_\mu \bar{\phi})^2 - (1-\cos(\bar{\phi}))\right]$$

Ergo:

$$\phi = \frac{4m}{\sqrt{\lambda}} \arctan\left[e^{mx}\right] = \frac{\cancel{4}^{2} F}{\cancel{2}\pi} \arctan\left[e^{mx}\right] \quad \checkmark$$

Ergo:

$$\phi_{SA} = \frac{2F}{\pi}\left[\arctan\left(\frac{\sinh\left(\frac{v\bar{t}}{\sqrt{1-v^2}}\right)}{v\cosh\left(\frac{\bar{x}}{\sqrt{1-v^2}}\right)}\right)\right]$$

$\bar{t} = mt$

$\bar{x} = mx$

For $t \mapsto -\infty$

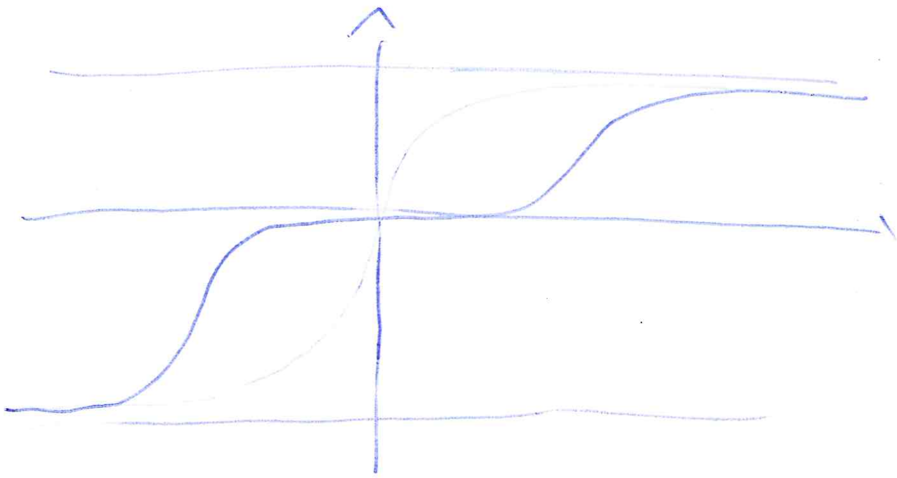
$$\phi_{SA} \cong \phi_{sol}\left(\frac{\bar{x} + v(\bar{t}+\Delta/2)}{\sqrt{1-v^2}}\right) + \phi_{antisol}\left(\frac{\bar{x} - v(\bar{t}+\Delta/2)}{\sqrt{1-v^2}}\right)$$

For $t \mapsto +\infty$

$$\phi_{SA} \cong \phi_{sol}\left(\frac{\bar{x} + v(\bar{t}-\Delta/2)}{\sqrt{1-v^2}}\right) + \phi_{antisol}\left(\frac{\bar{x} - v(\bar{t}-\Delta/2)}{\sqrt{1-v^2}}\right)$$

There is also a "time delay" Δ, which is the only effect of the collision between the soliton and the antisoliton.

$$\phi_{SS} = \frac{2F}{\pi} \arctan\left(\frac{v \sinh\left(\frac{x}{\sqrt{1-v^2}}\right)}{\cosh\left(\frac{vt}{\sqrt{1-v^2}}\right)} \right) \qquad Q = 2 \text{ solution.}$$



$$\phi_{BREATHER} = \frac{2F}{\pi} \arctan\left[\frac{\sin\left(\tilde{v}t/\sqrt{1+\tilde{v}^2}\right)}{\tilde{v}\cosh\left(\frac{x}{\sqrt{1+\tilde{v}^2}}\right)} \right] \qquad Q = 0.$$

formally from ϕ_{SA} by setting $v = i\tilde{v}$. Periodic with period

$$\Upsilon = \frac{2\pi\sqrt{1+v^2}}{v}$$

→ you still get

(If you do the same trick, $v = i\tilde{v}$, starting from the solution ϕ_{SS}, you get a complex-valued solution...)

⇒ still valid potential, but more complicated solutions should be presented...