Outlook and what is all about

$\psi(t,x)$ w. f.

$\psi(0,x)$ is a "in most cases" a complicated function.

Or in many cases: $\psi(t,x) = f(t)\,\varphi(x)$

↳ then is function of space.

$$\int_{-\infty}^{\infty} |\psi(0,x)|^2\, dx = 1$$ (and this is so for each x).

An observable is mathematically an operator. Let us call it $\hat{A}$. (Energy, momentum, position)

$\hat{A} : f(x) \mapsto g(x)$ (such as $\hat{P}_x = -i\hbar \frac{\partial}{\partial x}$).

$\hat{A}$ is "Hermitian"

$S = \{\varphi_i(x)\}$ ONC set of eigenfunctions of $\hat{A}$.

[let us suppose in $(-\infty,\infty)$ but it could be (a,b)].

$$\int_{-\infty}^{\infty} \varphi_i^*(x)\varphi_j(x)\,dx = \delta_{ij}$$

$$\sum_{i=1}^{\infty} \varphi_i^*(x')\varphi_i(x) = \delta(x-x')$$

$\hat{A}\,\varphi_i(x) = \lambda_i\,\varphi_i(x)$

↳ real!!!!! (This is so because $\hat{A}$ is Hermitian).

At $t=0$ we make a measurement of $\hat{A}$. Our wave function is $\psi(0,x)$.

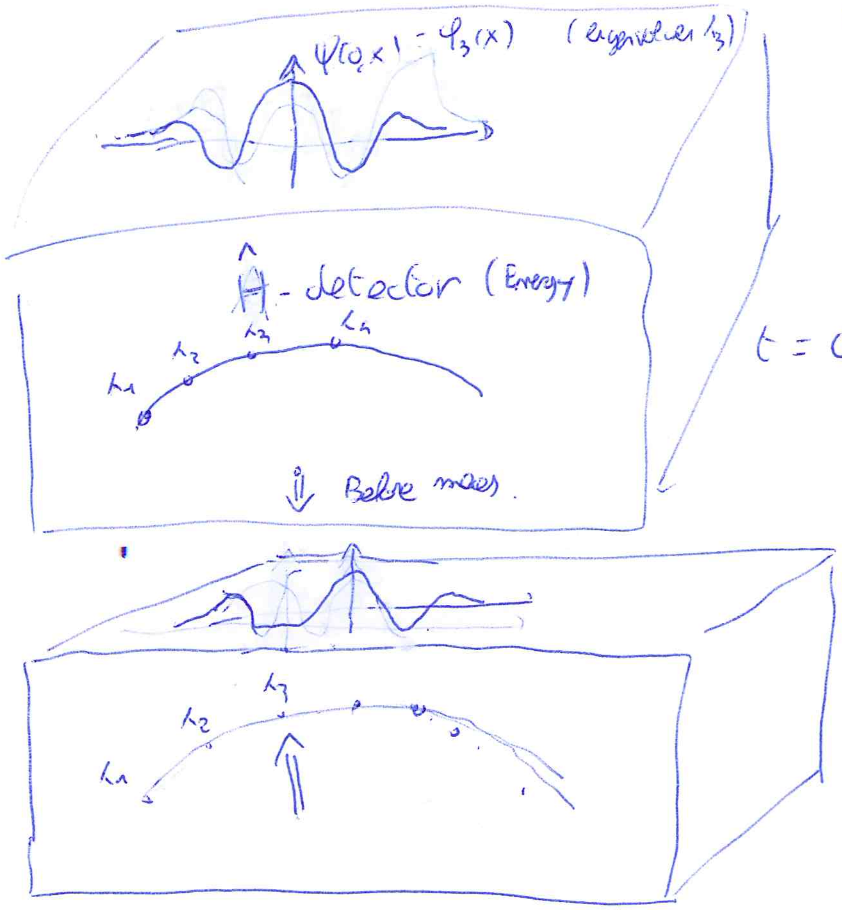
What we will get?

if

$\Psi(0,x) = \varphi_i(x)$

The result of the measurement is λ_i!!!

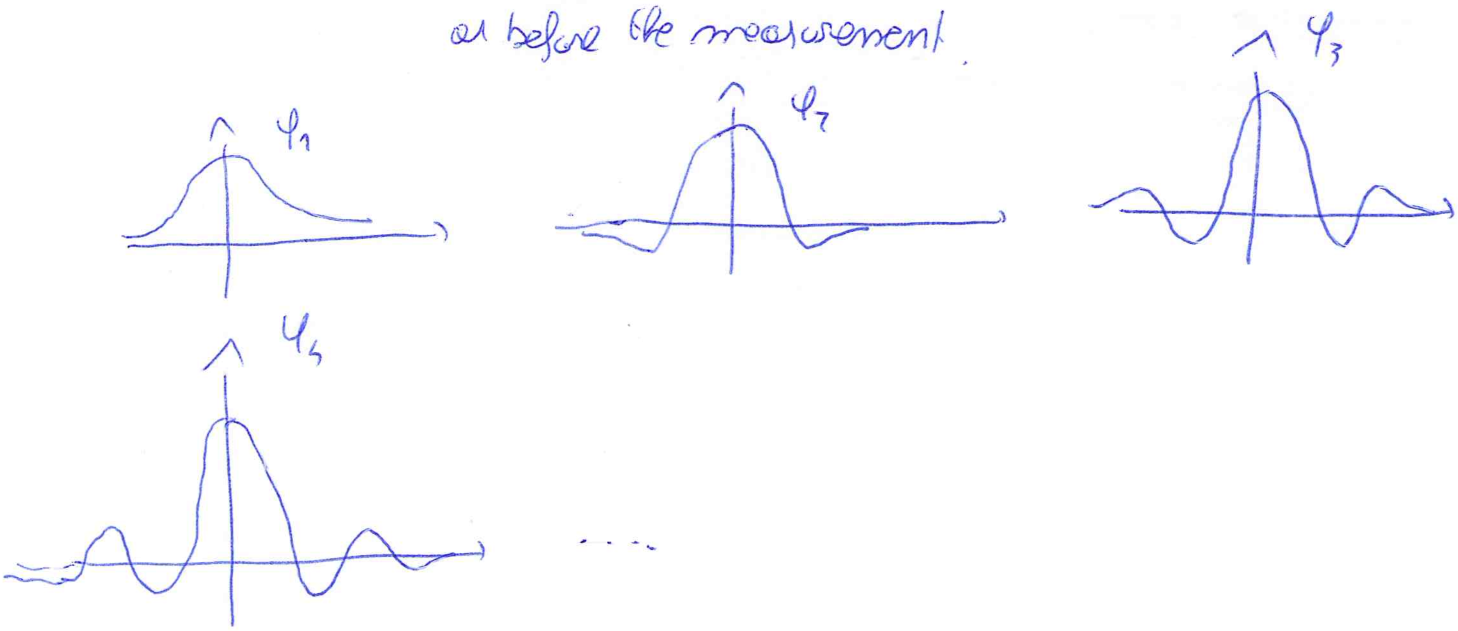
(For instance, energy E_3!!!)
$\hat{A} = \hat{H}$.



$t = 0^-$

$t = 0^+$

ψ has the same form!!!!
as before the measurement.



....

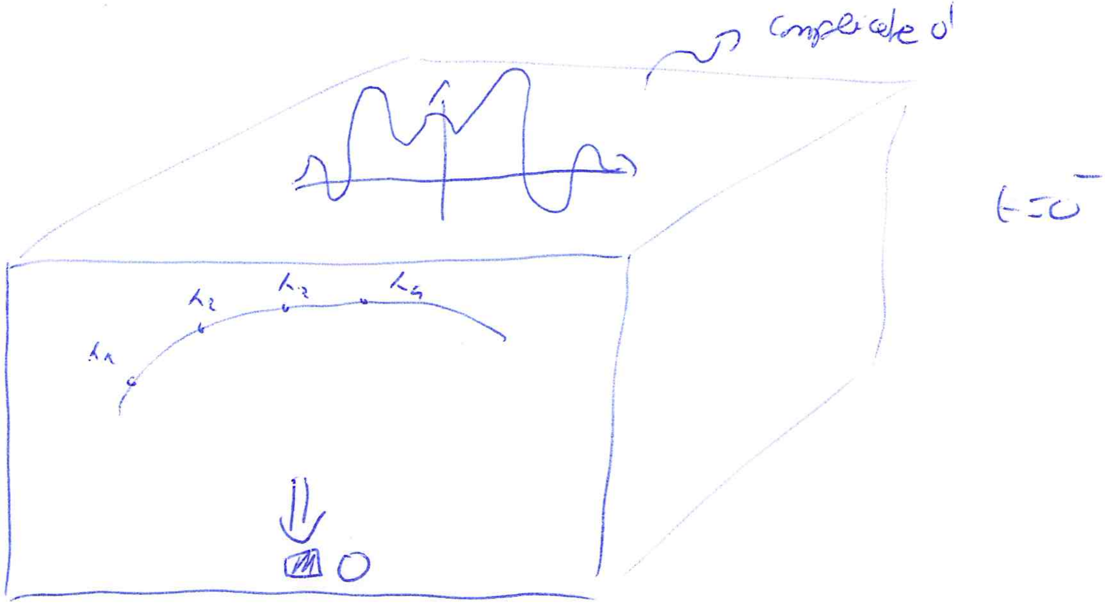
General case:

$$\psi(0,x) = \sum_{i=1}^{\infty} c_i \varphi_i(x)$$

$$(\psi(0,x), \psi(0,x)) = \int_{-\infty}^{\infty} |\psi(0,x)|^2 dx = 1$$

$$= \sum_{i=1}^{\infty} |c_i|^2 = 1$$

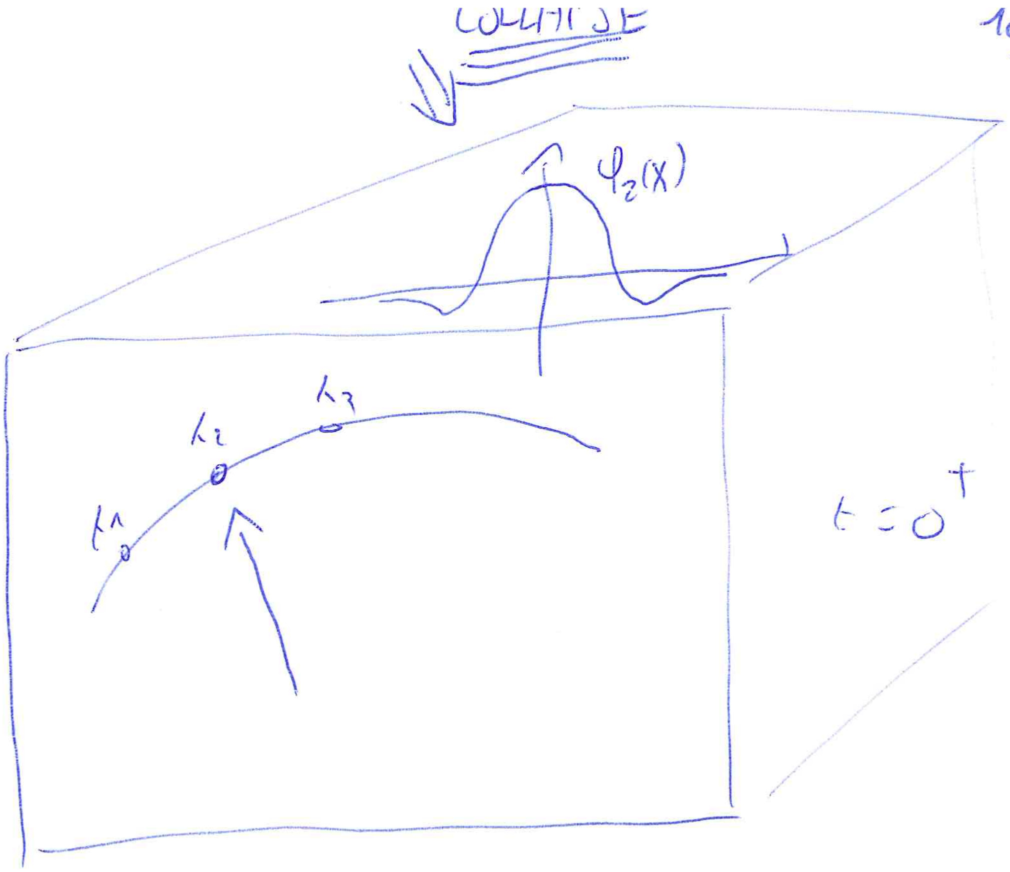
(PARSEVAL !!!!)



Now, as $\psi(0,x)$ is a superposition of $\neq$ eigenfunctions... as result will be λ_i with probability $|c_i|^2$.

For instance, with probability $|c_2|^2$ we find as a result λ_2!

That means:



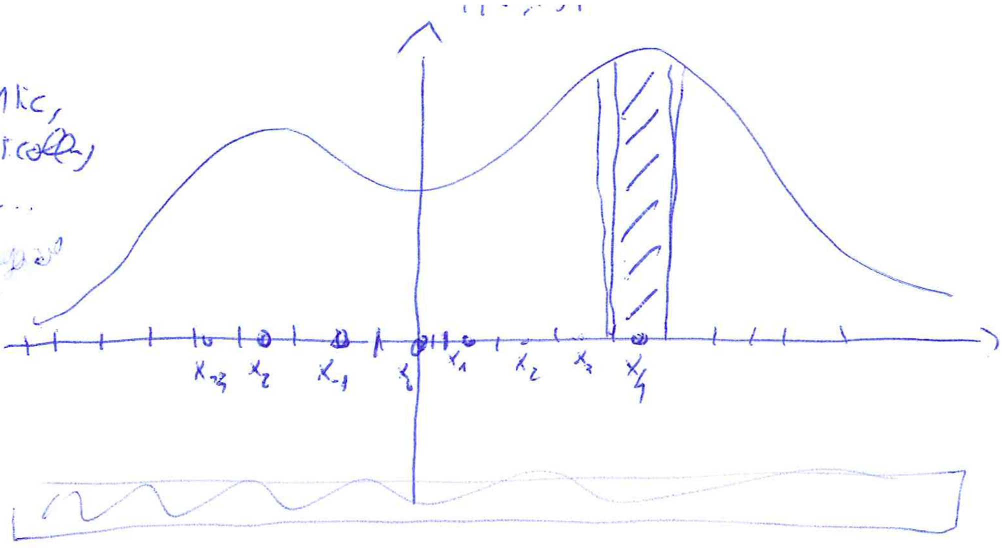
But this is only one of the "∞-ly many" possible outcomes!!!!

The wave function for $t=0^-$ was $\sum_{i=1}^{\infty} c_i \varphi_i(x) \longmapsto$ for $t=0^+$ is $\varphi_2(x)$ (In $|c_2|^2$ % of cases).

(Measurement of position: same story... but here it is a kind of pathological, cause we need the Dirac δ...) explain we are about it.

Position

Only hermitic, not mathematically correct yet... continuous eigenvalue!!!)



Position Detector

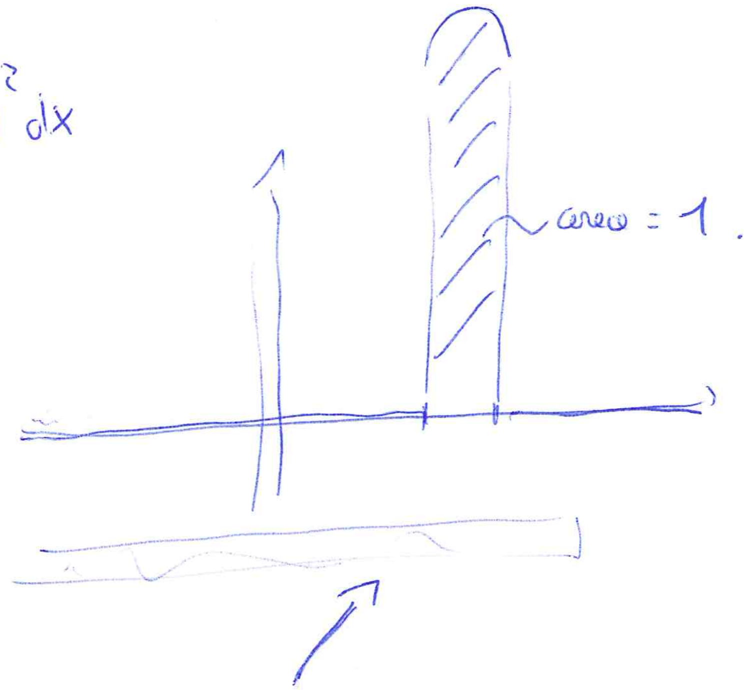
$\Downarrow$

Measurement position... let us suppose to have a limited precision...

which is the probability that our position detector will find x_4?

$$P_4 = \int_{x_4 - \delta x/2}^{x_4 + \delta x/2} |\psi(0,x)|^2 \, dx$$

if we find x_4



New PWf

$$\psi(0^+, x) = \begin{cases} 0 & x < x_4 - \delta x/2 \\ N \cdot \psi(0^-, x) & x_4 - \delta x/2 \le x \le x_4 + \delta x/2 \\ 0 & x > x_4 + \delta x/2 \end{cases}$$

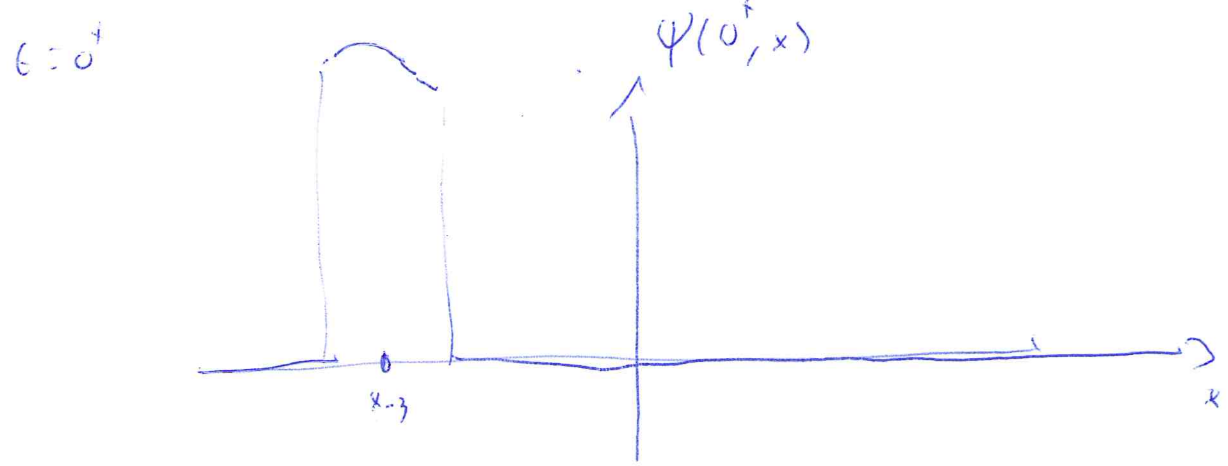
$$|N|^2 = \frac{1}{P_4}.$$

obviously:

$$\sum_{n=-\infty}^{\infty} P_n = \sum_{n=-\infty}^{\infty} \int_{x_n - \delta x/2}^{x_n + \delta x/2} |\Psi(0,x)|^2 dx = 1 \;!!!! \quad \underline{\text{important}} !!!$$

Obviously, if we repeat the experiment with the very same initial condition we may get a different result.

For instance, we may get as a result x_{-3}



Then, the wave function will evolve on its own!!!!!!