Sol. 2 AQM:

4) $\langle f_1 | \hat{p} | f_2 \rangle = \int_{-\infty}^{\infty} dx\, f_1^* \left(-i \frac{\partial f_2}{\partial x}\right) = (f_1, \hat{p}_x f_2)$

$\langle f_1 | \hat{p}_x^+ | f_2 \rangle = \langle f_2 | \hat{p}_x | f_1 \rangle^* = \left[\int_{-\infty}^{\infty} f_2^* \left(-i \frac{\partial}{\partial x} f_1\right) \right]^* =$

$= \int_{-\infty}^{\infty} f_2 \, i \frac{\partial f_1^*}{\partial x} dx = \left[i f_2 f_1 \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} i \frac{\partial f_2}{\partial x} f_1^* dx = \langle f_1 | \hat{p}_x | f_2 \rangle$

$[x, \hat{p}_x] \varphi(x) = x(-i \partial_x \varphi) + i \partial_x (x \varphi) = -i x \partial_x \varphi + i \varphi + x \cancel{\partial_x} \varphi$

$= +i \varphi \quad \forall \varphi$

Ergo $[x, \hat{p}_x] = i$.

2) a) $H | E_1 \rangle = E_1 | E_1 \rangle$
$H | E_2 \rangle = E_2 | E_2 \rangle$
follows by construction.

b) $| \psi(t) \rangle = e^{-iHt} | \psi(0) \rangle = a\, e^{-iE_1 t} | E_1 \rangle + b\, e^{-iE_2 t} | E_2 \rangle$.

The prob. to measure $| E_1 \rangle$ at t is $| a e^{-iE_1 t} |^2 = |a|^2$.

3) $\hat{A} = | f_1 \rangle \langle f_2 |$

$\hat{A} (\alpha_1 | \varphi_1 \rangle + \alpha_2 | \varphi_2 \rangle) = \alpha_1 \langle f_2 | \varphi_1 \rangle | f_1 \rangle + \alpha_2 \langle f_2 | \varphi_2 \rangle | f_1 \rangle$

$= \alpha_1 \hat{A} | \varphi_1 \rangle + \alpha_2 \hat{A} | \varphi_2 \rangle \quad \checkmark$

$\langle \varphi_n | \hat{A}^+ | \varphi_m \rangle = \langle \varphi_m | \hat{A} | \varphi_n \rangle^* = \langle \varphi_m | f_1 \rangle^* \langle f_2 | \varphi_n \rangle^* = \langle \varphi_n | f_2 \rangle \langle f_1 | \varphi_m \rangle$

Ergo: $\hat{A}^+ = | f_2 \rangle \langle f_1 |$

4) $[\hat{A}, \hat{B}]^+ = (\hat{A}\hat{B})^+ - (\hat{B}\hat{A})^+ = \hat{B}^+ \hat{A}^+ - \hat{A}^+ \hat{B}^+ = \hat{B}\hat{A} - \hat{A}\hat{B} = -[\hat{A}, \hat{B}]$ qed.