

Development of a hadronic model: general considerations

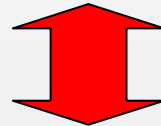
Objectives

- Development of a chirally symmetric model for mesons and baryons including (axial-)vector d.o.f.

‘Extended Linear Sigma Model (eLSM)’

- Study of the model for $T = \mu = 0$ (spectroscopy in vacuum)

(Masses, decay, scattering lengths,...)



Interrelation between
these two aspects!

- Second goal: properties at nonzero T and μ

(condensates and masses in thermal/matter medium,...)

Fields of the model

- Quark-antiquark mesons: **scalar**, pseudoscalar, vector and axial-vector quarkonia.
 - Additional mesons: The scalar and the pseudoscalar glueballs
 - Baryons: nucleon doublet and its partner
- (in the so-called mirror assignment)

How to construct the eLSM

- Confinement: only hadrons.
- Dilatation invariance and its anomalous breaking
- Chiral symmetry $SUR(N_f) \times SUL(N_f) \times UV(1)$ and its spontaneous as well as explicit breaking.
- Chiral anomaly

Development of a hadronic model: mesons

Confinement: from gluons to glueballs

$$m_{gluon} = 0 \rightarrow m_{gluon}^* \approx 500 - 800 \text{ MeV}$$

$$\langle G_{\mu\nu}^a G^{a,\mu\nu} \rangle \neq 0$$

Confinement implies glueballs.

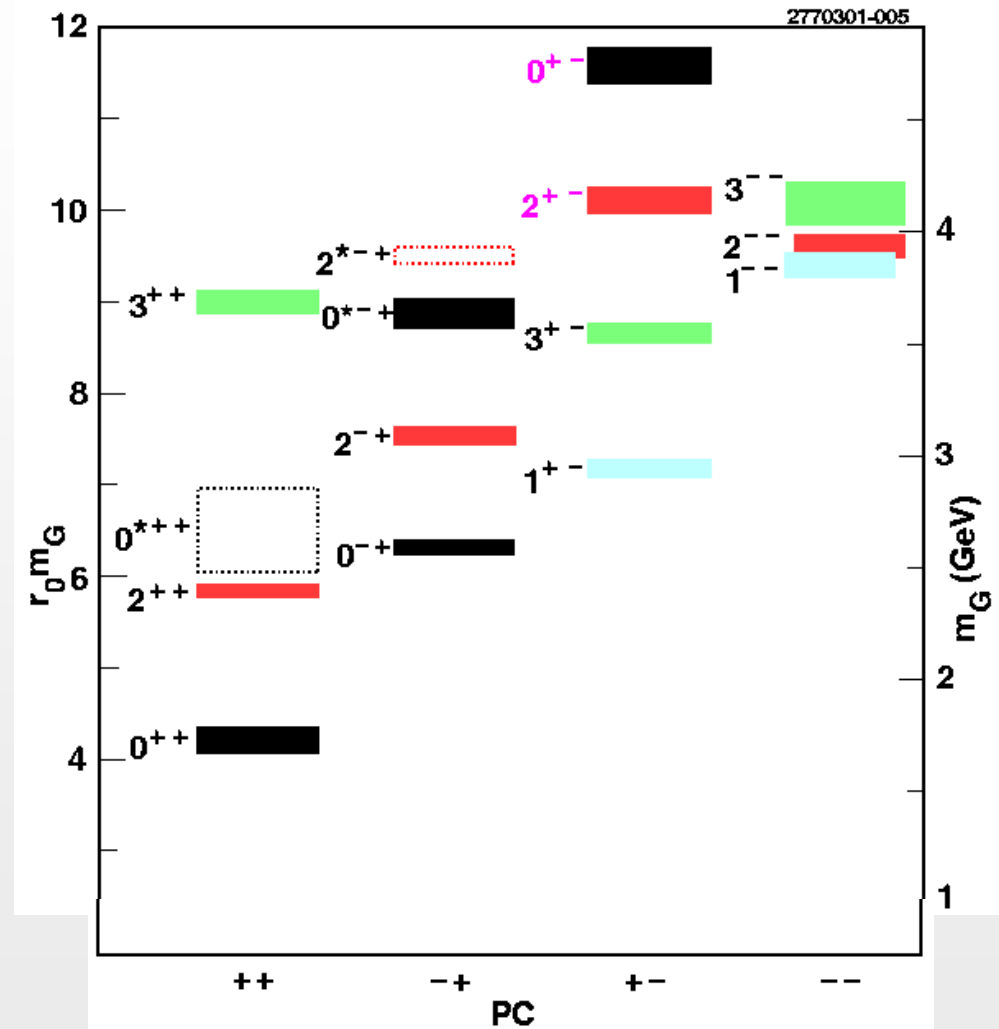
Where are they? We are still looking for them.

$$M_G = 1.6 - 1.8 \text{ GeV}$$

$$J^{PC} = 0^{++}$$

$$I = 0$$

*lightest predicted
glueball*



The lightest glueball is part of an effective Lagrangian which reproduces at a composite level the breaking of dilatation invariance.

Development of a dilaton potential.

Dilaton / Scalar glueball

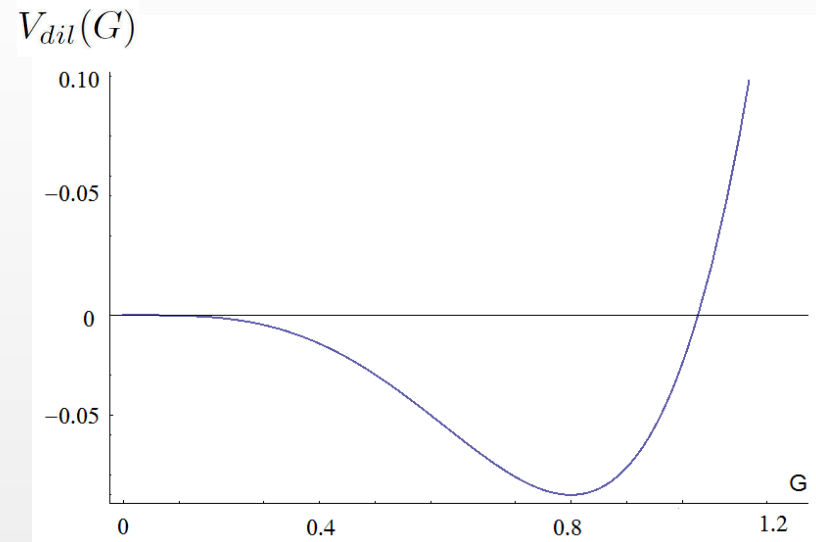
At the hadronic level, we describe these properties as:

$$G^4 \sim G_{\mu\nu}^a G^{a,\mu\nu}$$

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - V_{dil}(G)$$

$$V_{dil}(G) = \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left[G^4 \ln \left(\frac{G}{\Lambda_G} \right) - \frac{G^4}{4} \right]$$

Λ_G dimensionful param that breaks dilatation inv!



$$\partial_\mu J^\mu = T_\mu^\mu = -\frac{1}{4} \frac{m_G^2}{\Lambda_G^2} G^4$$

In QCD it is:

$$\partial_\mu J^\mu = T_\mu^\mu = \frac{\beta(g)}{4g} G_{\mu\nu}^a G^{a,\mu\nu} \neq 0$$

Dilaton / Scalar glueball (2)

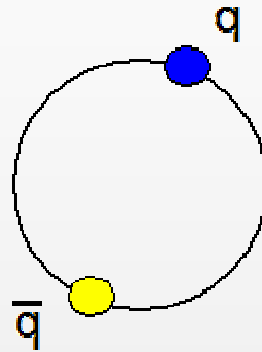
Where is the glueball G in the PDG ???

(Too many) candidates, most notably: $f_0(1500)$ and $f_0(1710)$

Quark-Antiquark mesons

Quark: u,d,s **R,G,B**

Quark-antiquark bound states: conventional mesons



$$|color\rangle = \sqrt{1/3} (\bar{R}R + \bar{B}B + \bar{G}G)$$

$$\vec{L}, \vec{S} \quad \longrightarrow \quad P = -(-1)^L \quad C = (-1)^{L+S}$$

$$\vec{L}, \vec{S} \quad \longrightarrow \quad \vec{J} = \vec{L} + \vec{S} \quad J^{PC}$$

(Pseudo)scalar sector

9 pseudoscalar fields: $L = S = 0 \rightarrow J^{PC} = 0^{-+}$

$$P = P_a \lambda^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_N}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_N}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \quad \Gamma = i\gamma^5$$

$$\pi^+ \equiv u\bar{d}$$

$$K^+ \equiv u\bar{s}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta_\eta & \sin \theta_\eta \\ -\sin \theta_\eta & \cos \theta_\eta \end{pmatrix} \begin{pmatrix} \eta_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S \equiv \bar{s}s \end{pmatrix}$$

$$-36^\circ < \theta_\eta < -45^\circ$$

...and 9 scalar fields: $L = S = 1 \rightarrow J^{PC} = 0^{++}$

$$S = S_a \lambda^a = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{\sigma_N}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{\sigma_N}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \quad \Gamma = 1$$

$$a_0^+ = a_0(1450) \equiv u\bar{d} \quad \text{and not } a_0(980)!!!$$

$$K_S^+ = K_0^{*+}(1430) \equiv u\bar{s} \quad \text{and not } k(700)!!!$$

$$\sigma_N \equiv \sqrt{1/2}(u\bar{u} + d\bar{d}) \approx f_0(1370)$$

$$\text{and not } f_0(500)!!!$$

$$\sigma_S \equiv u\bar{s} \approx f_0(1500) \text{ or } f_0(1710)$$

$$\text{and not } f_0(980)!!!$$

Chiral transformation of (pseudo)scalar mesons

$$q_i = q_{i,R} + q_{i,L} \rightarrow (U_R)_{ij} q_{j,R} + (U_L)_{ij} q_{j,L} \quad U_R, U_L \subset U(3)$$

$$\Phi = S + iP$$

$$\Phi_{ij} = \bar{q}_j q_i + i \bar{q}_j i \gamma^5 q_i = \sqrt{2} \bar{q}_{R,j} q_{L,i}$$

$$\Phi \rightarrow U_L \Phi U_R^+$$

Example of an invariant term

$$\Phi \rightarrow U_L \Phi U_R^+ \quad U_R, U_L \subset SU(3)$$

$$\lambda_2 \text{Tr}[\Phi^+ \Phi \Phi^+ \Phi] \rightarrow$$

$$\lambda_2 \text{Tr}[U_R \Phi^+ U_L^+ U_L \Phi U_R^+ U_R \Phi^+ U_L^+ U_L \Phi U_R^+] = \lambda_2 \text{Tr}[\Phi^+ \Phi \Phi^+ \Phi]$$

$$U_L^+ U_L = 1, U_R^+ U_R = 1$$

(Axial-)Vector sector

9 vector fields... $L = 0, S = 1 \rightarrow J^{PC} = 1^{--}$

$$V^\mu = V^\mu_a \lambda^a = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_N}{\sqrt{2}} & \rho^+ & K_*(892)^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_N}{\sqrt{2}} & K_*(892)^0 \\ K_*(892)^- & \bar{K}_*(892)^0 & \phi_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \quad \Gamma = \gamma^\mu$$

$$\rho^+ \equiv u\bar{d}$$

$$K_*^+(892) \equiv u\bar{s}$$

$$\omega \approx \omega_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$$

$$\phi \approx \phi_S \equiv \bar{s}s$$

...and 9 axial-vector fields... $L = S = 1 \rightarrow J^{PC} = 1^{++}$

$$A^\mu = A^\mu_a \lambda^a = \begin{pmatrix} \frac{a_1^0}{\sqrt{2}} + \frac{f_{1,N}}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & -\frac{a_1^0}{\sqrt{2}} + \frac{\omega_N}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1,S} \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \quad \Gamma = \gamma^\mu \gamma^5$$

$$a_1^+ = a_1^+(1260) \equiv u\bar{d}$$

$$K_1^+ = K_1^+(1270) \equiv u\bar{s}$$

$$f_1(1285) \approx f_{1,N} \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$$

$$f_1(1510) \approx f_{1,S} \equiv \bar{s}s$$

...where A and V are coupled in the following way:

$$L^\mu = V^\mu + A^\mu$$

$$R^\mu = V^\mu - A^\mu$$

which under chiral transformations transform as

$$R^\mu \rightarrow U_R R^\mu U_R^+$$

$$L^\mu \rightarrow U_L L^\mu U_L^+$$

Examples of further invariant objects:

$$G^2 Tr \left[L_{\mu}^+ L_{\mu} + R_{\mu}^+ R_{\mu} \right]$$

$$Tr \left[\Phi R_{\mu} \Phi^+ L_{\mu} \right]$$

...

Meson sector: how many fields do we have?

$$4N_f^2 + 2 \text{ fields}$$

For $N_f = 3$ there are 38 mesons

36 quark-antiquark fields + 2 glueballs

Criteria (repetita juvant!)

We construct the Lagrangian of the so-called Extended Linear Sigma Model (ELSM) according to:

dilatation symmetry

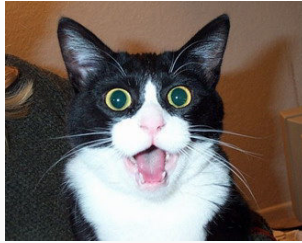
and

chiral invariance.

The breaking of the dilatation symmetry is only included in the „gluonic part“...(scalar glueball and axial anomaly)

Moreover, invariance under **C** and **P** is also taken into account.

Model of QCD – eLSM with scalar Glueball



$$\begin{aligned}\mathcal{L} = & \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right) + \text{Tr} [(D^\mu \Phi)^\dagger (D_\mu \Phi)] \\ & - m_0^2 \left(\frac{G}{G_0} \right)^2 \text{Tr} [\Phi^\dagger \Phi] - \lambda_1 (\text{Tr} [\Phi^\dagger \Phi])^2 - \lambda_2 \text{Tr} [(\Phi^\dagger \Phi)^2] \\ & + \left(\frac{G}{G_0} \right)^2 \text{Tr} \left[\left(\frac{m_1^2}{2} + \Delta \right) ((L^\mu)^2 + (R^\mu)^2) \right] \\ & - \frac{1}{4} \text{Tr} [(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \text{Tr} [H (\Phi^\dagger + \Phi)] \\ & + c_1 [\det(\Phi) - \det(\Phi^\dagger)]^2 + \frac{h_1}{2} \text{Tr} [\Phi^\dagger \Phi] \text{Tr} [L_\mu L^\mu + R_\mu R^\mu] \\ & + h_2 \text{Tr} [\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \text{Tr} [\Phi R_\mu \Phi^\dagger L^\mu]\end{aligned}$$

$$\begin{aligned}\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 \\ K_0^{*-} + iK^- & \bar{K}_0^{*0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix} \\ L^\mu, R^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N \pm \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \pm a_1^0}{\sqrt{2}} & \rho^+ \pm a_1^+ & K^{*+} \pm K_1^+ \\ \rho^- \pm a_1^- & \frac{\omega_N \mp \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \mp a_1^0}{\sqrt{2}} & K^{*0} \pm K_1^0 \\ K^{*-} \pm K_1^- & \bar{K}^{*0} \pm i\bar{K}_1^0 & \omega_S \pm f_{1S} \end{pmatrix}\end{aligned}$$

S. Janowski, D. Parganlija, F. Giacosa, D. H. Rischke, **Phys. Rev. D84, 054007 (2011)**

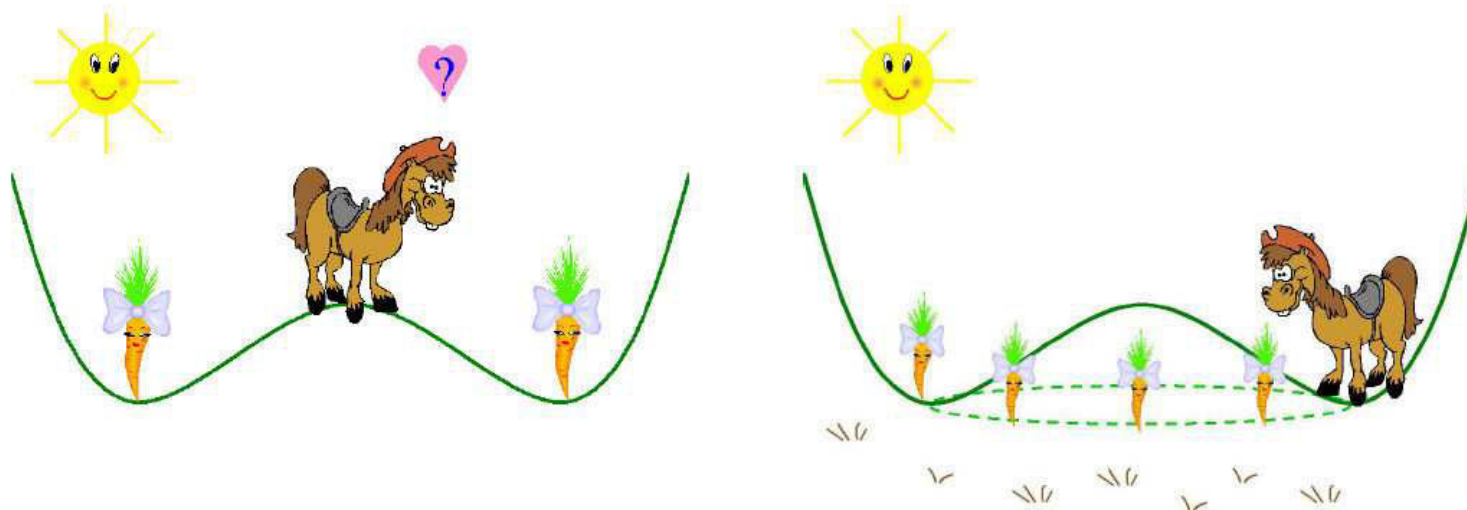
D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa, D. H. Rischke, **Phys.Rev. D87 (2013) 014011** arXiv:1208.0585

Francesco Giacosa

The donkey of Buridan

Jean Buridan (in Latin, *Johannes Buridanus*) (ca. 1300 – after 1358)

Spontaneous Symmetry Breaking



Although Nicolás likes the symmetric food configuration, he must break the symmetry deciding which carrot is more appealing. In three dimensions, there is a continuous valley where Nicolás can move from one carrot to the next without effort.

Technical remarks

Perform Spontaneous Symmetry Breaking (SSB):

$$\sigma_N \rightarrow \sigma_N + \phi_N, \quad \sigma_S \rightarrow \sigma_S + \phi_S$$

Explicit symmetry breaking terms:

$$H = \text{diag}\{h_1, h_2, h_3\} \quad \text{with } h_i \propto m_i \quad m_\pi^2 \propto (m_u + m_d) \langle \bar{q}q \rangle$$

$$\delta = \text{diag}\{\delta_1, \delta_2, \delta_3\} \quad \text{with } \delta_i \propto m_i^2$$

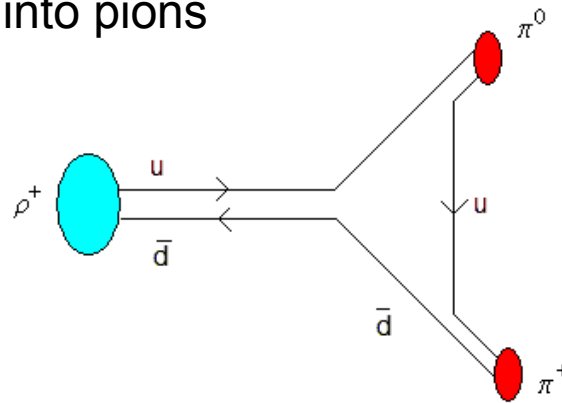
Parameter **c**: axial anomaly and eta-prime mass

But: **only a finite number of terms is allowed!**

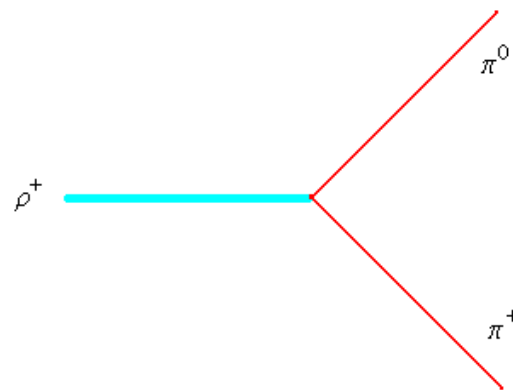
We can calculate: masses, decays, and scattering lengths.

Example: ρ -meson decay into pions

Microscopic



eLSM



Results of the fit (11 parameters, 21 exp. quantities)

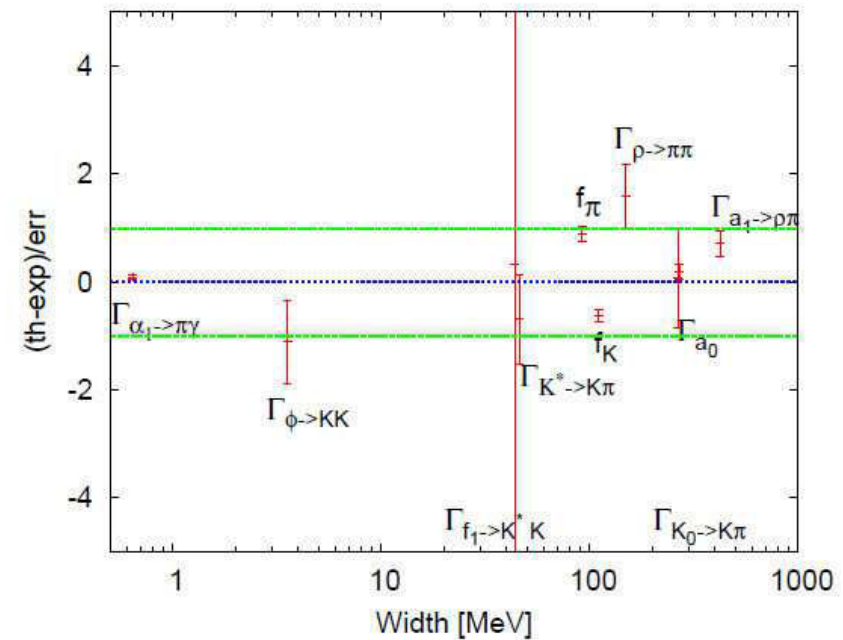
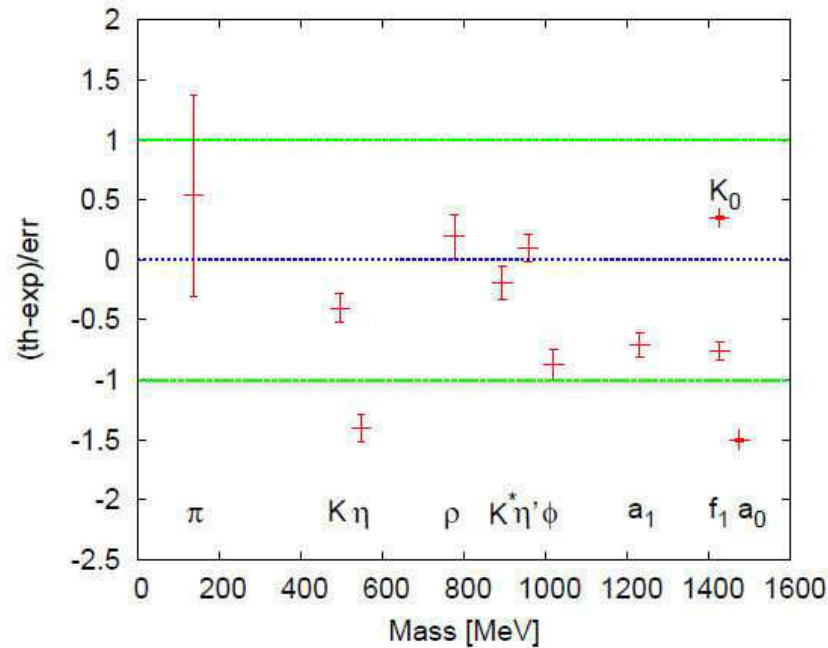
Error from PDG or 5%.
Scalar-isoscalar sector
not included.

$$\chi_{red}^2 = 1.2$$

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
m_{K^*}	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^*}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho \rightarrow \pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^* \rightarrow K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi \rightarrow \bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1 \rightarrow \rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1 \rightarrow \pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420) \rightarrow K^*K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^* \rightarrow K\pi}$	285 ± 12	270 ± 80

arXiv:1208.0585

Results of the fit: pictorial representation



arXiv:1208.0585

Overall phenomenology is good.

Scalar mesons $a_0(1450)$ and $K_0(1430)$ above 1 GeV and are quark-antiquark states.

Importance of the (axial-)vector mesons

Consequences/1: $a_0(1450)$

Theory

$$\frac{\Gamma_{a_0 \rightarrow \eta' \pi}}{\Gamma_{a_0 \rightarrow \eta \pi}} = 0.19 \pm 0.02, \quad \frac{\Gamma_{a_0 \rightarrow K K}}{\Gamma_{a_0 \rightarrow \eta \pi}} = 1.12 \pm 0.07$$

Exp (PDG)

$$\frac{\Gamma_{a_0(1450) \rightarrow \eta' \pi}}{\Gamma_{a_0(1450) \rightarrow \eta \pi}} = 0.35 \pm 0.16, \quad \frac{\Gamma_{a_0(1450) \rightarrow K K}}{\Gamma_{a_0(1450) \rightarrow \eta \pi}} = 0.88 \pm 0.23 .$$

Consequences/2: pseudoscalar mixing angle

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta_\eta & \sin \theta_\eta \\ -\sin \theta_\eta & \cos \theta_\eta \end{pmatrix} \begin{pmatrix} \eta_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}_S$$

Theory $\theta_\eta = -44^\circ$

Exp (KLOE) $\theta_\eta \cong -41^\circ$

Consequences/3: ρ -mass

$$m_{\rho}^2 = m_1^2 + \frac{1}{2}(h_1 + h_2 + h_3)\phi_N^2 + \frac{h_1}{2}\phi_S^2$$

$$m_1^2 \propto G_0^2 \quad m_1 = 0.643 \text{ GeV}$$

$$\sqrt{(h_2 + h_3)/2}\phi_N = 0.447 \text{ GeV}$$

Thus, the ρ -mass emerges from the interplay of both the chiral and gluon condensates!

Consequences/4: scalar-isoscalar quarkonia

$$\sigma \equiv \sigma_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \qquad \sigma_S = \bar{s}s$$

Theory $m_{\sigma_N} = 1360 \text{ MeV} \qquad m_{\sigma_S} = 1530 \text{ MeV}$

Exp (PDG)

$$m_{f_0(1370)} = 1350 \pm 150 \text{ MeV} \quad m_{f_0(1500)} = 1505 \pm 6 \text{ MeV} \quad m_{f_0(1710)} = 1720 \pm 6 \text{ MeV}$$

It then follows: the chiral partner of the pion is (predominantly) $f_0(1370)$ and not $f_0(500)$

The scalars below 1 GeV ($f_0(500)$, $k(700)$, $f_0(980)$, $a_0(980)$) are not quarkonia.
Possibility: tetraquarks.

(F. G. **Phys.Rev. D75 (2007) 054007**, hep-ph/0611388.)

An important ongoing work:

The calculation of the full mixing problem in the $I=J=0$ sector is ongoing:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = B \begin{pmatrix} \sigma_N \equiv \bar{n}n = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ G \equiv gg \\ \sigma_S \equiv \bar{s}s \end{pmatrix}$$

where B is a 3×3 orthogonal matrix

Our result within the $N_f=2$ case was:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} \sqrt{0.76} & \sqrt{0.24} & 0 \\ -\sqrt{0.24} & \sqrt{0.76} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_N \equiv \bar{n}n \\ G \equiv gg \\ \sigma_S \equiv \bar{s}s \end{pmatrix}$$

Details in S. Janowski, D. Parganlija, F.G., D. Rischke, **Phys.Rev. D84 (2011) 054007**, [arXiv:1103.3238](https://arxiv.org/abs/1103.3238)

A new entry: the pseudoscalar glueball

$$\mathcal{L}_{\tilde{G}\text{-mesons}}^{int} = ic_{\tilde{G}\Phi} \tilde{G} \left(\det\Phi - \det\Phi^\dagger \right)$$

Quantity	Value
$\Gamma_{\tilde{G} \rightarrow KK\eta} / \Gamma_{\tilde{G}}^{tot}$	0.049
$\Gamma_{\tilde{G} \rightarrow KK\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.019
$\Gamma_{\tilde{G} \rightarrow \eta\eta\eta} / \Gamma_{\tilde{G}}^{tot}$	0.016
$\Gamma_{\tilde{G} \rightarrow \eta\eta\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.0017
$\Gamma_{\tilde{G} \rightarrow \eta\eta'\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.00013
$\Gamma_{\tilde{G} \rightarrow KK\pi} / \Gamma_{\tilde{G}}^{tot}$	0.46
$\Gamma_{\tilde{G} \rightarrow \eta\pi\pi} / \Gamma_{\tilde{G}}^{tot}$	0.16
$\Gamma_{\tilde{G} \rightarrow \eta'\pi\pi} / \Gamma_{\tilde{G}}^{tot}$	0.094

Quantity	Value
$\Gamma_{\tilde{G} \rightarrow KK_S} / \Gamma_{\tilde{G}}^{tot}$	0.059
$\Gamma_{\tilde{G} \rightarrow a_0\pi} / \Gamma_{\tilde{G}}^{tot}$	0.083
$\Gamma_{\tilde{G} \rightarrow \eta\sigma_N} / \Gamma_{\tilde{G}}^{tot}$	0.028
$\Gamma_{\tilde{G} \rightarrow \eta\sigma_S} / \Gamma_{\tilde{G}}^{tot}$	0.012
$\Gamma_{\tilde{G} \rightarrow \eta'\sigma_N} / \Gamma_{\tilde{G}}^{tot}$	0.019

$$\Gamma_{\tilde{G} \rightarrow \pi\pi\pi} = 0$$

PANDA/FAIR will be able to scan the energy above 2.5 GeV

Details in:

W. Eshraim, S. Janowski, F.G., D. Rischke, **Phys.Rev. D87 (2013) 054036**. [arxiv: 1208.6474](#) .

W. Eschraim, S. Janowski, K. Neuschwander, A. Peters, F.G., **Acta Phys. Pol. B**, Proc. Suppl. 5/4, [arxiv: 1209.3976](#)

