a) $\psi = A(t) B(z)$

$$\Box \psi(t) = \left(\frac{1}{c^2}\partial_t^2 - \partial_z^2\right) A(t) B(z) = 0.$$

$$\frac{1}{c^2}\left(\partial_t^2 A(t)\right) \cdot B(z) = A(t)\, \partial_z^2 B(z)$$

$$\frac{1}{c^2}\frac{\partial_t^2 A(t)}{A(t)} = \frac{\partial_z^2 B(z)}{B(z)} = K = \text{const.}$$

(Da links = Funktion von t
rechts = " " " z

$\Downarrow$

Die einzige Möglichkeit ist, dass sie Konstant sind.

$$\frac{\partial_z^2 B(z)}{B(z)} = K$$

1) $K > 0$:

$$B(z) = b_1 e^{\sqrt{K} z} + b_2 e^{-\sqrt{K} z}$$

Genauso:

$$A(t) = b_1 e^{c\sqrt{K} t} + b_2 e^{-c\sqrt{K} t}$$

~~Man hat also folgende Möglichkeiten:~~
~~1) $K > 0$ → exponentielle Lösungen~~
~~2) $K = 0$ $B(z) =$~~

2) $K < 0$

$$B(z) = b_1 \cos(\sqrt{-K}\, z) + b_2 \sin(\sqrt{-K}\, z) \;\tilde{=}\; b_1 e^{i\sqrt{K_1}\, z} \;\tilde{+}\; b_2 e^{-i\sqrt{K}\ldots}$$

$$A(t) = b_1 \cancel{\sin} \ldots (c\sqrt{-K}\, t) + b_2 \sin(c\sqrt{-K}\, t)$$

3) $K = 0$

$$A(t) = a_0 + a_1 t$$
$$B(z) = b_0 + b_1 z$$

b) Wir wollen, dass die Felder endlich für $|z| \mapsto \infty$ bleiben

Daraus folgt: $K > 0$ ist unphysikalisch.

$K \leq 0 \rightarrow$ OK.

c)

$$\psi(t, z) = \int_{-\infty}^{\infty} dk\, C(k)\, e^{ik(ct - z)}$$

$$\Box \psi(t, z) = \int_{-\infty}^{\infty} dk\, C(k)\, \underbrace{\Box e^{ik(ct - z)}}_{= 0}$$

$$\Box e^{ik(ct - z)} = \left(\frac{1}{c^2}\partial_t^2 - \partial_z^2\right) e^{ik(ct - z)} =$$

$$= \left(\frac{(ik)^2 c^2}{c^2} - (ik)^2\right) e^{ik(ct - z)} = 0 \quad \forall k.$$

d) $C(k) = \delta(k-k_0)$

e) $C(k) = \frac{C_0}{2}\delta(k-k_0) + \frac{C_0}{2}\delta(k+k_0)$

f) $C(k) = C_0\theta(k_0^2-k^2)$

$$\psi = C_0\int_{-k_0}^{k_0} dk\, e^{ik(ct-z)} = \frac{C_0}{i(ct-z)}\left(e^{ik_0(ct-z)} - e^{-ik_0(ct-z)}\right)$$

$$= 2C_0\frac{\sin(k_0(ct-z))}{ct-z}$$