

Optimized expansion in quantum field theory of massive fermions with $(\bar{\psi}\psi)^2$ interaction

Anna Okopińska

Institute of Physics, Warsaw University, Białystok Branch, 15-424 Białystok, Lipowa 41, Poland

(Received 29 October 1987)

The optimized expansion for the generating functional for Green's functions in the path-integral representation is formulated in quantum field theory of fermions with a scalar Fermi interaction and compared with different expansion techniques. The effective action in n -dimensional space-time is calculated to the first order. Renormalization can be performed if the space-time dimension is less than four. In this case, the renormalized effective potential at a finite temperature is calculated to first order. The results suggest that the theory in two- and three-dimensional space-time is noninteracting or precarious.

I. INTRODUCTION

Quantum field theory (QFT) of a massless spinor field ψ with a scalar Fermi interaction $(\bar{\psi}\psi)^2$ has been proposed by Heisenberg¹ as a candidate for a unified theory of interactions. The lack of progress in developing the Heisenberg nonlinear theory is due to its perturbative nonrenormalizability in four-dimensional space-time. Even with the use of nonperturbative methods, Heisenberg and his co-workers¹ were not able to extract a physical content of the theory. Renewed interest in $(\bar{\psi}\psi)^2$ QFT was inspired by the work of Nambu and Jona-Lasinio,² who demonstrated that a fermion mass can be generated dynamically, in analogy with the energy gap in the microscopic theory of superconductivity. However, the value of the fermion mass, determined self-

consistently, is cutoff dependent and appears infinite, if the dimension of space-time is greater than two, i.e., when the $(\bar{\psi}\psi)^2$ QFT is perturbatively nonrenormalizable. The theory of N massless fermions in two dimensions has been solved by Gross and Neveu in the $N \rightarrow \infty$ limit and fermion mass generation has been shown.³ Recently, a dynamical mass generation for the Gross-Neveu model with any number of fields has been shown by Latorre and Soto⁴ in the Gaussian-effective-potential approximation.

In this work we will study the effective action in Fermi theory, extending the method of the optimized expansion (OE), developed in scalar QFT with $\lambda\phi^4$ interaction.⁵ We consider a massive fermion theory in n -dimensional Euclidean space with a classical action obtained by an ordinary Wick rotation:

$$S[\bar{\psi}, \psi] = \int d^n x \left[\sum_{i=1}^N \sum_{A,B=1}^d \bar{\psi}_A^i(x) (\not{\partial}_{AB} + m \delta_{AB}) \psi_B^i(x) + \frac{g}{2N} \left(\sum_{i=1}^N \sum_{A=1}^d \bar{\psi}_A^i(x) \psi_A^i(x) \right)^2 \right], \quad (1.1)$$

where ψ and $\bar{\psi}$ are independent Grassmann fields with a number of Dirac components $d = 2^{\lfloor n/2 \rfloor}$. The symbol $\not{\partial}$ denotes $\gamma^\mu \partial_\mu$ with Dirac matrices satisfying $\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu}$. In the following, for notational simplicity, the space arguments and the integrations over them will be suppressed. Also discrete indices and summation over them are shown only if necessary.

Quantization is done by representing the generating functional for Green's functions as a path integral over the fields:⁶

$$Z[\bar{\eta}, \eta] = \int D\bar{\psi} D\psi \exp(-S[\bar{\psi}, \psi] + \bar{\psi}\eta + \bar{\eta}\psi) \quad (1.2)$$

with Grassmann sources $\bar{\eta}$ and η introduced. The effective action is given by

$$\Gamma[\bar{\psi}, \psi] = \ln Z[\bar{\eta}, \eta] - \bar{\psi}\eta - \bar{\eta}\psi. \quad (1.3)$$

The background fields defined by means of left variational derivatives as

$$\bar{\psi} = -\frac{\delta \ln Z}{\delta \eta} \quad \text{and} \quad \psi = \frac{\delta \ln Z}{\delta \bar{\eta}} \quad (1.4)$$

are the vacuum expectation values of Grassmann fields in the presence of external sources. When the sources are turned off the background Grassmann fields should vanish, since Lorentz invariance is not expected to be broken. Therefore, the one-particle-irreducible (1PI) Green's functions are taken as derivatives of the effective action at $\bar{\psi}, \psi = 0$. The effective potential into one field defined as

$$V(\bar{\psi}, \psi) = -\frac{1}{N} \frac{\Gamma[\bar{\psi}, \psi]}{\int d^n x} \Big|_{\bar{\psi}, \psi = \text{const}} \quad (1.5)$$

generates 1PI vertices at vanishing external momenta and is useful to study vacuum structure and renormalized masses and couplings.

In the path-integral approach the statistical field theory of equilibrium can be treated in a similar way as vacuum Euclidean QFT. The same path integral (1.2) with appropriate boundary conditions can be used to represent a finite-temperature generating functional, which gives a partition function at the temperature T . In the imaginary-time formalism⁷ the integral is taken over the functions with periodic (antiperiodic) boundary conditions for boson (fermion) fields, with a period T^{-1} in the time coordinate. For $T=0$ the period becomes infinite and the partition function becomes the generating functional for Green's functions with vanishing external momenta in vacuum QFT. The finite-temperature effective potential is defined by Eq. (1.5) and can be expressed by momentum integrals with the aid of the Fourier transform. In vacuum QFT the integral (1.2) is taken over all functions in Euclidean space; therefore, the Fourier transform is given by an integral over

$$\int \frac{d_n p}{(2\pi)^n} . \quad (1.6)$$

In equilibrium thermodynamics of Fermi fields the integral in (1.2) is taken over antiperiodic functions with a period T^{-1} . Therefore, the momentum vector p has an imaginary-time component $p_0 = (2j+1)\pi T$, where $j=0, \pm 1, \dots$, and the Fourier transform becomes a combined sum over j and integral in $(v=n-1)$ -dimensional momentum space:

$$T \sum_j \int \frac{d_v p}{(2\pi)^v} . \quad (1.7)$$

Generally, the path integral for generating functional (1.2) cannot be evaluated analytically; therefore, systematic methods of approximation are developed. For Bose fields the steepest-descent method proves to be very useful to represent the path integral for generating functional as a series of calculable Gaussian functional integrals. It is remarkable that a path integral for Fermi fields bears some similarities with the Bose one. The Gaussian integrals are calculable and the results are very similar as in the bosonic case, in spite of so different a nature of the Grassmannian integration. Although there is no justification that the mean contribution to the integral comes from the stationary point of the exponential, the steepest-descent method "works" also in the fermionic case.⁶ As discussed in Sec. II, similarly as for boson fields, the steepest-descent method can be used in the QFT of N fields with a Fermi interaction to generate (a) the conventional loop expansion (LE), (b) the optimized expansion (QE), and (c) the mean-field (MF) expansion.⁸ In the above expansions the effective potential and all 1PI vertices given by the derivatives of the effective action at $\bar{\psi}=\psi=0$ can be found in terms of ordinary integrals in momentum space. The finite-temperature effective potential in any expansion generated by the steepest-descent method has the same structure as the effective potential and can be obtained after replacing the integrals (1.6) by

(1.7) in the given order approximation. Just as for boson theories, the LE gives a conventional perturbation theory; the MF expansion is equivalent to the large- N expansion (with N set equal to the number of fields). The effective potential in the first-order OE for Fermi theory coincides with the Gaussian effective potential obtained using the variational principle for the functional Schrödinger equation.⁴ The method is an extension of the Gaussian effective potential studied in $\lambda\phi^4$ QFT (Refs. 9 and 10).

Renormalizability of the theory requires the effective potential and vertices at arbitrary momenta to be finite. In the variational approach only the effective potential is calculated. To find the vertices the effective action has to be calculated. It can be done using a time-dependent variational principle, but the Schrödinger approach is not convenient to study renormalization. For a long time it has been claimed¹¹ that renormalization does not remove infinities from the Schrödinger equation, even in renormalizable QFT. Only recently, the correct additional renormalization was found by Symanzik in his rigorous construction of the Schrödinger representation.¹² As admitted by Symanzik, the proof of renormalizability is rather of conceptual value, but uninviting to compute. The Lagrangian formulation is usually preferred because it maintains explicit Lorentz covariance and the discussion of renormalizability is more clear. The use of the Lagrangian method is the main advantage of the OE under the variational method in Hamiltonian theory. The OE possesses other advantages also, as it gives a systematic approximation procedure and enables an easy extension to finite temperature.

The Fermi theory is perturbatively nonrenormalizable, if $n > 2$. However, it has been pointed out that renormalization can be performed to the leading order of the large- N expansion, if $n < 4$ (Refs. 3 and 13). Moreover, renormalizability in $n = 2 + \epsilon$ dimensions has been shown rigorously.¹⁴ In Sec. III we discuss the Fermi theory to the first-order OE. To this approximation, renormalization can be explicitly performed, if only $n < 4$. The result is compared with the leading order of the $1/N$ expansion. In Sec. IV the finite-temperature behavior of the Fermi theory is studied in the same approximations. As concluded in Sec. V the renormalized theory appears trivial or precarious, depending on the choice of a bare coupling constant. At a critical temperature the precarious theory undergoes a phase transition to the massless noninteracting phase.

II. FORMAL METHODS OF EVALUATION OF THE EFFECTIVE ACTION

The steepest-descent method for the path integral (1.2) consists in translating the integration variables $\bar{\psi}$ by $\bar{\psi}^0$, and ψ by ψ^0 , which minimize the exponent:

$$\frac{\delta S}{\delta \bar{\psi}^0} = \eta \quad \text{and} \quad \frac{\delta S}{\delta \psi^0} = -\bar{\eta} . \quad (2.1)$$

$Z[J]$ as a series in a formal parameter is obtained after expanding the exponential into a Taylor series and performing the Gaussian integration term by term. The effective action, to the given order in the formal param-

ter, can be obtained using an implicit definition given by Eqs. (1.3) and (1.4).

$$Z[\bar{\eta}, \eta] = \int D\bar{\psi} D\psi \exp \left[\frac{1}{\hbar} (-S[\bar{\psi}, \psi] + \bar{\psi}\eta + \bar{\eta}\psi) \right]. \quad (2.2)$$

A. Loop expansion

The conventional LE (Ref. 6) in Fermi theory can be generated applying the steepest-descent method to the generating functional written in the form

Upon translating the integration variables and rescaling them by $\hbar^{1/2}$, the Planck constant can be used as the formal expansion parameter and we obtain

$$\begin{aligned} Z[\bar{\eta}, \eta] &= \exp \left[\frac{1}{\hbar} (-S[\bar{\psi}^0, \psi^0] + \bar{\psi}^0\eta + \bar{\eta}\psi^0) \right] \int D\bar{\psi} D\psi \exp(-\bar{\psi}G^{-1}[\bar{\psi}^0, \psi^0]\psi + \bar{\psi}\eta + \bar{\eta}\psi) [1 + O(\hbar^{1/2})] \\ &= \exp \left[\frac{1}{\hbar} (-S[\bar{\psi}^0, \psi^0] + \bar{\psi}^0\eta + \bar{\eta}\psi^0) \right] \text{Det}(G^{-1}[\bar{\psi}^0, \psi^0]) [1 + O(\hbar)], \end{aligned} \quad (2.3)$$

where the inverse fermion propagator in the background fields $\bar{\psi}$ and ψ is

$$G_{ABij}^{-1}[\bar{\psi}, \psi, x, y] = \frac{\delta^2 S}{\delta\bar{\psi}_A^i(x)\delta\psi_B^j(y)} = (\not{\partial}_{AB}\delta^{ij} + M_{AB}^{ij}[\bar{\psi}, \psi])\delta(x, y) \quad (2.4)$$

with mass matrix given by

$$M_{AB}^{ij}[\bar{\psi}, \psi] = \left[m + \frac{g}{N} \bar{\psi}_C^k \psi_C^k \right] \delta_{AB} \delta^{ij} - \frac{g}{N} \bar{\psi}_A^i \psi_B^j. \quad (2.5)$$

The determinant in Eq. (2.3) is taken with respect to space arguments and to discrete indices. The effective action to first order in \hbar becomes

$$\Gamma[\bar{\psi}, \psi] = -\bar{\psi}(\not{\partial} + m)\psi - \frac{g}{2N} (\bar{\psi}\psi)^2 + \text{Tr Ln}(G^{-1}), \quad (2.6)$$

and the effective potential can be written as

$$V(\bar{\psi}, \psi) = \frac{1}{N} \left[m\bar{\psi}\psi + \frac{g}{2N} (\bar{\psi}\psi)^2 - \int \frac{d^n p}{(2\pi)^n} \ln \det(\not{p} + M) \right]. \quad (2.7)$$

B. Optimized expansion

In the calculation of the effective action we use a trick of Nambu and Jona-Lasinio,² writing a classical action as

$$S_\epsilon[\bar{\psi}, \psi] = S^{(0)}[\bar{\psi}, \psi] + \epsilon S^{(1)}[\bar{\psi}, \psi] = \bar{\psi}G^{-1}\psi + \epsilon \left[\bar{\psi}(m - \Omega)\psi + \frac{g}{2N} (\bar{\psi}\psi)^2 \right], \quad (2.8)$$

where the inverse fermion propagator is given by

$$G_{AB}^{-1}(x, y) = [\not{\partial}_{AB} + \Omega(x)\delta_{AB}]\delta(x, y). \quad (2.9)$$

The parameter ϵ has been introduced to identify the order of the perturbation and is set equal to one at the end. In the path integral (1.2) with the modified classical action (2.8) we translate integration variables to obtain a minimum of the exponent. After expanding in ϵ we obtain

$$\begin{aligned} Z[\bar{\eta}, \eta] &= \exp \left[-\bar{\psi}^0 G^{-1} \psi^0 - \epsilon \left[\bar{\psi}^0 (m - \Omega) \psi^0 + \frac{g}{2N} (\bar{\psi}^0 \psi^0)^2 \right] + \bar{\eta} \psi^0 + \bar{\psi}^0 \eta \right] \\ &\quad \times \int D\bar{\psi} D\psi \exp(-\bar{\psi}G^{-1}\psi) \left[1 - \epsilon \left[\bar{\psi} \frac{\delta^2 S^{(1)}}{\delta\bar{\psi}^0 \delta\psi^0} \psi + \frac{1}{4} \frac{\delta^4 S^{(1)}}{\delta\psi^0 \delta\bar{\psi}^0 \delta\psi^0 \delta\bar{\psi}^0} \bar{\psi}\psi\bar{\psi}\psi \right] + O(\epsilon^2) \right] \\ &= \exp \left[-\bar{\psi}_A^0 G_{AB}^{-1} \psi_B^0 - \epsilon \left[\bar{\psi}_A^0 (m - \Omega) \psi_A^0 + \frac{g}{2N} (\bar{\psi}_A^0 \psi_A^0)^2 \right] + \bar{\eta}_A \psi_A^0 + \bar{\psi}_A^0 \eta_A \right] \text{Det}^N G^{-1} \\ &\quad \times \left[1 - \epsilon \left[N \left[\Omega - m - \frac{g}{N} \bar{\psi}_A^0 \psi_A^0 \right] G_{BB} - g \bar{\psi}_A^0 G_{AB} \psi_A^0 - \frac{g}{2N} (N^2 G_{AA} G_{BB} - N G_{AB} G_{BA}) \right] + O(\epsilon^2) \right]. \end{aligned} \quad (2.10)$$

The effective action to first order in ϵ becomes

$$\begin{aligned} \Gamma(\bar{\psi}, \psi, \Omega) = & -\bar{\psi}_A^i (\partial_{AB} + m \delta_{AB}) \psi_B^i - \frac{g}{2N} (\bar{\psi}_A^i \psi_A^i)^2 + N \text{Tr} \text{Ln}(G^{-1}) - N(\Omega - m) G_{BB} + g \bar{\psi}_A^i G_{BB} \psi_A^i \\ & - g \bar{\psi}_A^i G_{AB} \psi_B^i - \frac{g}{2N} (N^2 G_{AA} G_{BB} - N G_{AB} G_{BA}) . \end{aligned} \quad (2.11)$$

The auxiliary field $\Omega(x)$ has been introduced in such a way that the generating functionals do not depend on them. However, in the truncated series the dependence appears. Using Stevenson's principle of minimal sensitivity,¹⁵ we make the k th-order approximant of the effective action, as insensitive as possible to the small variation of Ω , choosing Ω to satisfy

$$\frac{\delta \Gamma_k}{\delta \Omega} = 0 , \quad (2.12)$$

which can also be regarded as a requirement that the sources for the auxiliary field vanish. It would be possible to improve the lowest-order approximation within the same scheme, introducing more auxiliary fields in the matrix Ω_{AB}^{ij} . For the simplest choice $\Omega_{AB}^{ij} = \Omega \delta^{ij} \delta_{AB}$ the effective action is given by (2.11) with Ω satisfying

$$\begin{aligned} (\Omega - m) G_{AB} G_{BA} - \frac{g}{N} \bar{\psi}_A G_{AB} G_{BA} \psi_A + \frac{g}{N} \bar{\psi}_A G_{AC} G_{CB} \psi_B \\ + \frac{g}{N} (N G_{AA} G_{BC} G_{CB} - G_{AB} G_{BC} G_{CA}) = 0 . \end{aligned} \quad (2.13)$$

It is not possible to obtain a more explicit expression for the effective action. However, this implicit expression enables us to calculate all derivatives of the effective action with respect to background fields $\bar{\psi}$ and ψ . When taken at $\bar{\psi} = \psi = 0$, they give 1PI vertices expressed in terms of momentum integrals.

The effective potential depends only on $\alpha = \bar{\psi}\psi/N$ (which can be treated approximately as a real, positive number). To first order in ϵ it becomes

$$\begin{aligned} V(\alpha, \Omega) = & m\alpha + g\alpha^2 - dI_1(\Omega) + [d(\Omega - m) - rg\alpha] \Omega I_0(\Omega) \\ & + \frac{d}{2} rg [\Omega I_0(\Omega)]^2 , \end{aligned} \quad (2.14)$$

where $r = (Nd - 1)/N$. The gap equation (2.13) turns into

$$\Omega - m - \frac{g}{d} r\alpha + gr\Omega I_0(\Omega) = 0 , \quad (2.15)$$

which is the same as obtained requiring $\partial V/\partial \Omega = 0$.

Here and in the following we use the notation

$$I_1(\Omega) = \frac{1}{2} \int \frac{d^n p}{(2\pi)^n} \ln(p^2 + \Omega^2) = \int \frac{d^\nu p}{(2\pi)^\nu} \frac{\omega_p}{2} , \quad (2.16a)$$

$$I_0(\Omega) = \int \frac{d^n p}{(2\pi)^n} \frac{1}{p^2 + \Omega^2} = \int \frac{d^\nu p}{(2\pi)^\nu} \frac{1}{2\omega_p} , \quad (2.16b)$$

$$I_{-1}(\Omega) = 2 \int \frac{d^n p}{(2\pi)^n} \frac{1}{(p^2 + \Omega^2)^2} = \int \frac{d^\nu p}{(2\pi)^\nu} \frac{1}{2\omega_p^3} , \quad (2.16c)$$

$$I(\Omega) = \int \frac{d^n p}{(2\pi)^n} \frac{1}{(p + \Omega)^2} = I_0(\Omega) - \Omega^2 I_{-1}(\Omega) , \quad (2.16d)$$

where $\nu = n - 1$ and $\omega_p^2 = \Omega^2 + p_1^2 + p_2^2 + \dots + p_\nu^2$. The integrals I_N have been introduced in Ref. 10 with some useful relation between the integrals I_N . In Table I we quote the relations used in our work.

In two-dimensional space-time the effective potential given by Eq. (2.14) coincides (for $m=0$) with the Gaussian effective potential obtained in the Gross-Neveu model by Latorre and Soto.⁴ They extended the variational method for the functional Schrödinger equation in scalar QFT (Ref. 9) to Fermi fields, by taking delta trial vacuum functional. Our approach confirms their result and offers a possibility of systematically improving them.

For $\alpha=0$ our Eq. (2.15) coincides with the gap equation obtained by Nambu and Jona-Lasinio,² requiring that radiative corrections to the self-energy vanish. It can be shown that their approach is equivalent to calculating the two-particle vertex as a sum of perturbative Feynman diagrams without overlapping divergences. Other vertices are calculated perturbatively with the use of the improved propagator. In our approach the effective action is calculated as a sum of vacuum Feynman diagrams without overlapping divergences. The result, which is renormalization-group invariant, is used to derive self-energy and other 1PI vertices. Only a self-energy taken at $\bar{\psi} = \psi = 0$ happens to be the same in both approaches; higher 1PI vertices differ. As will be seen in Sec. III B the difference in the four-vertex is crucial for renormalizability—the $(\bar{\psi}\psi)^2$ QFT in three-dimensional space-time is renormalizable in the first-order OE, but

TABLE I. Relations between I_N integrals in n dimensions: $x = \Omega^2/m_R^2$.

$n = 2$ or 3	
$I_1(\Omega) - I_1(m_R) = \frac{1}{2}(\Omega^2 - m_R^2)I_0(m_R) - \frac{1}{8\pi} m_R^n L_2(x)$	
$\Omega I_0(\Omega) - m_R I_0(m_R) = (\Omega - m_R)I_0(m_R) - \frac{1}{4\pi} \Omega m_R^{n-2} L_1(x)$	
$I_{-1}(\Omega) = \begin{cases} 1/(2\pi\Omega^2), & n=1, \\ 1/(4\pi\Omega), & n=2. \end{cases}$	

not in the first order of the Nambu and Jona-Lasinio approach.

In QFT of Fermi fields at finite temperature the integrals (1.6) should be replaced by (1.7), then the integrals given in Eqs. (2.16a) and (2.16b) turn into their finite-temperature counterparts:

$$\begin{aligned} I_1^T(\Omega) &= \frac{1}{2} T \sum_J \int \frac{d_\nu p}{(2\pi)^\nu} \ln(p^2 + \Omega^2) \\ &= \int \frac{d_\nu p}{(2\pi)^\nu} \left[\frac{\omega_p}{2} + T \ln(1 + e^{-\omega_p/T}) \right] \\ &= I_1(\Omega) + J_1^T(\Omega), \end{aligned} \quad (2.17a)$$

$$\begin{aligned} I_0^T(\Omega) &= T \sum_J \int \frac{d_\nu p}{(2\pi)^\nu} \frac{1}{p^2 + \Omega^2} \\ &= \int \frac{d_\nu p}{(2\pi)^\nu} \frac{1}{\omega_p} \left[\frac{1}{2} - \frac{1}{e^{\omega_p/T} + 1} \right] \\ &= I_0(\Omega) + J_0^T(\Omega). \end{aligned} \quad (2.17b)$$

The temperature-dependent contributions $J_1^T(\Omega)$ and $J_0^T(\Omega)$ are finite in any space dimensions ν and vanish for $T=0$. The first-order optimized finite-temperature effective potential

$$\begin{aligned} V^T(\alpha, \Omega) &= m\alpha + g\alpha^2 - dI_1^T(\Omega) \\ &\quad + [d(\Omega - m) - rg\alpha] \Omega I_0^T(\Omega) \\ &\quad + \frac{d}{2} rg [\Omega I_0^T(\Omega)]^2, \end{aligned} \quad (2.18)$$

with Ω satisfying the gap equation

$$\Omega - m - \frac{g}{d} r\alpha + gr\Omega I_0^T(\Omega) = 0, \quad (2.19)$$

gives a finite-temperature Hartree approximation in QFT.

C. Mean-field theory

The mean-field (MF) expansion⁸ is obtained formally by setting N equal to the number of fields in the given order of the large- N expansion, even if N is not large. It can be justified by a path-integral derivation of the expansion, when the role of $1/N$ as the formal parameter becomes clear. Multiplying the generating functional (1.2) by a constant factor represented as

$$C = \int d\theta \exp \left\{ -\frac{N}{2g} \left[\theta - i \left[m + \frac{g}{N} \bar{\psi}\psi \right] \right]^2 \right\}, \quad (2.20)$$

and introducing a source $S(x)$ for an auxiliary field θ , the functional

$$Z[\bar{\eta}, \eta, S] = \int D\theta \exp \left[-\frac{N}{2g} (\theta^2 - 2im^2\theta) + N\frac{m^2}{2g} \right] \int D\bar{\psi} D\psi \exp[-\bar{\psi}(\not{\partial} - i\theta)\psi + \bar{\eta}\psi + \bar{\psi}\eta + S\theta] \quad (2.21)$$

is defined. Integration over N fields $\bar{\psi}$ and ψ can be performed, leading to

$$Z[\bar{\eta}, \eta, S] = \int D\theta \exp(-NF[\theta, \bar{\eta}, \eta, S]), \quad (2.22)$$

where

$$F[\theta, \bar{\eta}, \eta, S] = -\frac{m^2}{2g} + \frac{1}{2g}\theta^2 - \frac{1}{g}im\theta - \frac{1}{N}\bar{\eta}G\eta - \text{Tr Ln}G^{-1} - \frac{1}{N}S\theta \quad (2.23)$$

and

$$G_{AB}^{-1}(x, y) = [\not{\partial}_{AB} - i\theta(x)\delta_{AB}]\delta(x - y). \quad (2.24)$$

The steepest-descent method to the integral (2.22) gives a series in $1/N$:

$$Z[\bar{\eta}, \eta, S] = \exp(-NF[\vartheta_0, \bar{\eta}, \eta, S]) \int D\theta \exp \left[-\frac{1}{2} \frac{\delta^2 F}{\delta\vartheta_0^2} \theta^2 \right] [1 - O(1/N)] = \exp(NF[\vartheta_0, \bar{\eta}, \eta, S] + O(1/N)). \quad (2.25)$$

After replacing the sources by the background fields $\bar{\psi}, \psi$ given by Eq. (1.4) and $\Omega = -i\delta \ln Z / \delta S$, with the aid of a Legendre transform the effective action is obtained. To leading order we have

$$\begin{aligned} \Gamma(\bar{\psi}, \psi, \Omega) &= N \left[-\frac{1}{N} \bar{\psi}(\not{\partial} + \Omega)\psi + \frac{1}{2g}\Omega^2 - \frac{1}{g}\Omega m \right. \\ &\quad \left. + \text{Tr Ln}G^{-1} + \frac{m^2}{2g} \right], \end{aligned} \quad (2.26)$$

where

$$G_{AB}^{-1}(x, y) = [\not{\partial}_{AB} + \Omega(x)\delta_{AB}]\delta(x - y). \quad (2.27)$$

In the physical theory the sources are absent, and the derivative of the effective action over Ω should vanish, leading to the gap equation

$$\Omega - m - g\alpha + gd\Omega I_0(\Omega) = 0, \quad (2.28)$$

where $\alpha = \bar{\psi}\psi/N$ and a consistency of the method requires $\Omega > 0$. For constant fields the effective potential

can be expressed as

$$V(\alpha, \Omega) = \Omega\alpha - \frac{1}{2g}\Omega^2 + \frac{1}{g}\Omega m - dI_1(\Omega) - \frac{m^2}{2g} \quad (2.29)$$

with Ω given by Eq. (2.28).

It is worthwhile to note that to the first order of the OE the gap equation (2.15) for large N turns into (2.28) and the effective potential (2.14) reduces to (2.29) after using the gap equation. Similarly as for scalar fields,⁹ the Gaussian effective potential in the large- N limit is exact. Therefore, for infinite N the leading-order MF and first-order OE give an exact result and for finite N both methods provide different approximation schemes. The accuracy of the approximations can be estimated by calculating higher-order contributions. The similarities of the MF and OE are due to the fact that in both methods the auxiliary field Ω includes some effects of composite fields. In the leading-order MF calculation of the effective action the same class of perturbative Feynman diagrams without overlapping divergences is summed, as in the first-order OE, only the coupling constant g is replaced by $gNd/(Nd-1)$, because only diagrams with a coupling between different fermions are included in MF and Pauli principle does not operate.

The finite-temperature effective potential to the leading order in $1/N$ is given by

$$V^T(\alpha, \Omega) = \Omega\alpha - \frac{1}{2g}\Omega^2 + \frac{1}{g}\Omega m - dI_1^T(\Omega) - \frac{m^2}{2g}, \quad (2.31)$$

with Ω chosen to satisfy

$$\Omega - m - g\alpha + gd\Omega I_0^T(\Omega) = 0, \quad (2.32)$$

III. RENORMALIZATION OF $(\bar{\psi}\psi)^2$ GFT

The $(\bar{\psi}\psi)^2$ theory can be quantized in n -dimensional space-time. If the dimension is greater than one, the momentum integrals in proper vertices become divergent and should be regularized, e.g., with the help of UV cutoff Λ . The theory is renormalizable, if a finite and Λ -independent content is obtained after reparametrization in terms of renormalized fields $\bar{\psi}_R = Z^{1/2}\bar{\psi}$, $\psi_R = Z^{-1/2}\psi$, and physical quantities, instead of bare parameters. In this section we will compare the renormalization procedure in different space-time dimensions to the lowest order of the discussed expansions. The renormalized mass and coupling are defined, as a second and fourth derivative of the effective potential at vanishing background.

A. Renormalization in the loop expansion

The two-point proper vertex can be obtained as a Fourier transform of the second derivative of the effective action over renormalized fields. To the first order of the LE the renormalized self-energy becomes

$$\Gamma_{AB}^2(p) = Z\{p_{AB} + [m + grmI_0(m)]\delta_{AB}\}. \quad (3.1)$$

Renormalization is accomplished taking $Z=1$, the renormalized mass

$$m_R = m + grmI_0(m) \quad (3.2)$$

and the renormalized coupling

$$g_R = g \left[1 - \frac{1}{d}gr^2I(m) \right]. \quad (3.3)$$

Solving iteratively Eqs. (3.2) and (3.3) gives the bare mass

$$m = m_R - grm_R I_0(m_R) \quad (3.4)$$

and the bare coupling constant

$$g = g_R \left[1 - \frac{1}{d}g_R r^2 I(m_R) \right]. \quad (3.5)$$

Elimination of bare parameters with the aid of (3.4) and (3.5) removes all infinities from the effective potential given by Eq. (2.7), only if the divergence of the integral $I(m_R)$ in Eq. (3.5) is at most logarithmic. Therefore, if the space-time dimension is greater than two, the $(\bar{\psi}\psi)^2$ QFT appears nonrenormalizable already in the first order of the LE.

B. Renormalization in the optimized expansion

In the OE derivatives of the effective potential (2.14) at $\alpha=0$ will be used as renormalized parameters. Since Ω as a function of α has been obtained requiring $\partial V/\partial\Omega=0$, the first derivative of the effective potential is simply

$$\frac{dV}{d\alpha} = \frac{\partial V}{\partial\alpha} = m + g\alpha - gr\Omega I_0(\Omega) = \Omega + \frac{g\alpha}{Nd}, \quad (3.6)$$

where the last equality has been obtained with the aid of the gap equation (2.15). This can be used to define the renormalized mass by

$$m_R = \left. \frac{dV}{d\alpha} \right|_{\alpha=0} = m - grm_R I_0(m_R). \quad (3.7)$$

Taking into account that

$$\frac{d\Omega}{d\alpha} = \frac{gr}{d[1+grI(\Omega)]}, \quad (3.8)$$

as found from the gap equation, we obtain

$$\begin{aligned} \frac{d^2V}{d\alpha^2} &= g \left[1 - gr \frac{d}{d\Omega} [\Omega I(\Omega)] \frac{d\Omega}{d\alpha} \right] \\ &= g \frac{1 + (gr/Nd)I(\Omega)}{1 + grI(\Omega)} \end{aligned} \quad (3.9)$$

and the renormalized coupling constant becomes

$$g_R = \left. \frac{d^2V}{d\alpha^2} \right|_{\alpha=0} = g \frac{1 + (gr/Nd)I(m_R)}{1 + grI(m_R)}. \quad (3.10)$$

Taking the effective action to the first-order OE (2.12) with Ω given by (2.13) we find the renormalized two-point proper vertex at vanishing background fields equal to

$$\Gamma_{AB}^2(p) = Z\{p_{AB} + [m - grm_R I_0(m_R)]\delta_{AB}\}. \quad (3.11)$$

Therefore, taking mass-shell normalization conditions, we have $Z=1$ and the renormalized mass given by Eq. (3.7).

The effective potential to the first-order OE coincides with the Gaussian effective potential, which renormalization in the two-dimensional case has been studied by Latorre and Soto for the massless theory.⁴ We can extend their analysis to an arbitrary bare mass and arbitrary space-time dimension. Solving Eqs. (3.7) and (3.10) with respect to bare parameters gives

$$m = m_R + g r m_R I_0(m_R) \quad (3.12)$$

and

$$g = \frac{Nd}{2rI(m_R)} \left[-1 + g_R r I(m_R) \pm \left[1 + \frac{2r}{Nd} g_R I(m_R) (2 - Nd) + g_R^2 r^2 [I(m_R)]^2 \right]^{1/2} \right]. \quad (3.13)$$

If the dimension $n \geq 2$, the integral $I(m_R)$ defined in Eq. (2.16d) diverges in the limit of an infinite cutoff. In this limit the solutions (3.13) for g become finite

$$g = N dg_R, \quad (3.14)$$

or infinitesimal and negative:

$$V(\alpha, \Omega) = m_R \alpha + g \alpha^2 - 2[I_1(\Omega) - I_1(m_R)] + 2(\Omega - m_R) \Omega I_0(\Omega) - r g \alpha [\Omega I_0(\Omega) - m_R I_0(m_R)] + g r [\Omega I_0(\Omega) - m_R I_0(m_R)]^2 \quad (3.18)$$

and the gap equation turns into

$$\Omega - m_R - \frac{g}{2} r \alpha + g r [\Omega I_0(\Omega) - m_R I_0(m_R)] = 0. \quad (3.19)$$

With the relations of Table I, the effective potential becomes

$$V(\alpha, \Omega) = m_R \alpha + g \alpha^2 - (\Omega^2 - m_R^2) I_0(m_R) + \frac{1}{4\pi} m_R^n L_2(x) + 2(\Omega - m_R) \Omega I_0(\Omega) + r g \alpha \left[\frac{1}{4\pi} \Omega m_R^{n-2} L_1(x) - (\Omega - m_R) I_0(m_R) \right] + r g \left[\frac{1}{4\pi} \Omega m_R^{n-2} L_1(x) - (\Omega - m_R) I_0(m_R) \right]^2 \quad (3.20)$$

and the gap equation can be written as

$$\Omega - m_R - \frac{g}{2} r \alpha - g r \left[\frac{1}{4\pi} \Omega m_R^{n-2} L_1(x) - (\Omega - m_R) I_0(m_R) \right] = 0, \quad (3.21)$$

$$g = -\frac{1}{rI(m_R)} - \frac{1}{r dg_R [I(m_R)]^2} = -\frac{1}{rI_0(m_R)} - \frac{G}{r[I_0(m_R)]^2} \quad (3.15)$$

with G defined by

$$G = m_R^2 I_{-1}(m_R) + \frac{1}{dg_R}. \quad (3.16)$$

It is worthwhile to observe that requiring the bare mass to vanish, as in the original Gross-Neveu model, requires the bare coupling $g = -1/rI_0(m_R)$, according to Eq. (3.12). As can be seen from Eq. (3.15) it corresponds to $G=0$ and Eq. (3.16) gives the renormalized coupling

$$g_R = -\frac{1}{m_R d I_{-1}(m_R)} = \begin{cases} -\pi, & n=2, \\ -2\pi/m_R, & n=3, \\ 0, & n \geq 4. \end{cases} \quad (3.17)$$

Therefore, fixing a bare mass to zero, fixes the renormalized coupling, while the dynamically generated mass m_R remains arbitrary. That is unlikely with the scalar $\lambda\varphi^4$ GFT (Refs. 10 and 5), when the relation between bare and renormalized mass is given by $m = m_R + 12\lambda I_0(m_R)$ and although the bare coupling is vanishingly small $\lambda = -1/6I_{-1}(m_R)$, the bare mass has to be infinite in order to have finite renormalized parameters.

In two- and three-dimensional space-time the Dirac spinor has $d=2$ components; therefore, the effective potential is very similar. After eliminating the bare mass and adding an infinite-constant term the effective potential can be written as

where $x = \Omega^2/m_R^2$ and the functions L_i are different in two- and three-dimensional space-time, as given in Table II.

It remains to eliminate the bare coupling constant. If we take it to be finite $g = 2Ng_R$, the gap equations can be satisfied with finite Ω , only if

$$\alpha = 2(\Omega - m_R) I_0(m_R) \quad (3.22)$$

and after eliminating Ω the effective potential becomes

TABLE II. The $L_i(x)$ functions used in relations given in Table I.

$n = \text{even}$	$n = \text{odd}$
$L_1(x) = \ln(x)$	$L_1(x) = (\sqrt{x} - 1)$
$L_2(x) = x \ln(x) - (x - 1)$	$L_2(x) = \frac{1}{3}(\sqrt{x} - 1)^2(2\sqrt{x} + 1)$

finite:

$$V(\alpha) = m_R \alpha + \frac{1}{2} g_R \alpha^2 . \quad (3.23)$$

In this case the finite-temperature effective potential, after using the gap equation becomes

$$V^T(\alpha) = m_R \alpha + \frac{1}{2} g_R \alpha^2 - 2J_1^T(m_R) . \quad (3.24)$$

The first-order effective potential suggests that the renormalized theory differs only in replacing bare parameters by renormalized ones. However, the finite-temperature behavior indicates that the theory is noninteracting, as the temperature contribution does not depend on the background fields and is the same as for free particle with a mass m_R .

If the infinitesimal solution (3.15) for the bare coupling constant is taken, the precarious effective potential

$$V(\alpha) = \alpha \Omega + \frac{1}{4\pi} m_R^n L_2(x) - G(\Omega - m_R)^2 \quad (3.25)$$

is finite for all Ω . Requiring $dV/d\Omega = 0$, gives the gap equation

$$\alpha + \frac{1}{2\pi} \Omega m_R^{n-1} L_1(x) - 2G(\Omega - m_R) = 0 . \quad (3.26)$$

At finite temperature the renormalized effective potential to the first-order OE is given by

$$V^T(\alpha) = \alpha \Omega + \frac{1}{4\pi} m_R^n L_2(x) - G(\Omega - m_R)^2 - 2J_1(\Omega) , \quad (3.27)$$

where Ω satisfies

$$\alpha + \frac{1}{2\pi} \Omega m_R^{n-2} L_1(x) - 2G(\Omega - m_R) - 2\Omega J_0^T(\Omega) = 0 . \quad (3.28)$$

For $T=0$, the temperature-dependent contributions vanish and the result turns into a precarious effective potential (3.25).

In the $(\bar{\psi}\psi)^2$ theory in four- and greater-dimensional space-time it can be found by inspection that elimination of the bare parameters with the aid of renormalized ones does not remove infinities from the effective potential for either choice of bare coupling, since the integral $I_{-1}(\Omega)$ diverges.

C. Renormalization in the mean-field expansion

In the MF approximation the renormalized mass and coupling constant defined by the first and second derivatives of the effective potential are equal:

$$m_R = m - g dm_R I_0(m_R) \quad (3.29)$$

and

$$g_R = \frac{g}{1 + g dI(m_R)} . \quad (3.30)$$

From the leading-order expression for the effective action (2.26) we find the renormalized two-point proper vertex for a vanishing background to be equal:

$$\Gamma^2(p) = Z \{ p_{AB} + [m - g dm_R I_0(m_R)] \delta_{AB} \} . \quad (3.31)$$

Therefore, with $Z=1$ and m_R given by Eq. (3.29), the mass-shell renormalization is accomplished. Hence, the bare mass has to be chosen as

$$m = m_R + g dm_R I_0(m_R) . \quad (3.32)$$

Keeping the bare coupling constant g nonvanishing, the gap equation (2.28) can be satisfied with finite Ω , only if

$$\alpha = 2(\Omega - m_R) I_0(m_R) . \quad (3.33)$$

In this case the renormalized coupling constant approaches zero and after eliminating Ω the effective potential becomes trivial,

$$V(\alpha) = m_R \alpha , \quad (3.34)$$

and the finite-temperature effective potential

$$V^T(\alpha) = m_R \alpha - 2J_1^T(m_R) \quad (3.35)$$

is a free energy of the free particle with a mass m_R .

The other possibility is to choose the bare coupling constant as the infinitesimal solution of Eq. (3.30):

$$g = \frac{g_R}{1 - g_R dI(m_R)} , \quad (3.36)$$

which in the limit of the infinite cutoff becomes

$$g = -\frac{1}{dI_0(m_R)} - \frac{G}{d[I_0(m_R)]^2} \quad (3.37)$$

with G defined by (3.16). In two- and three-dimensional space-time, after eliminating the bare parameters with the aid of (3.32) and (3.37), the same precarious theory as in the OE is obtained; the finite-temperature effective potential is given by (3.27) with Ω satisfying (3.28). If the dimension of space-time is greater than or equal to four, the theory appears nonrenormalizable in the leading order MF.

It is worthwhile to mention that similar features have been found in $\lambda\phi^4$ theory in four dimensions, studied in similar approximations at zero¹⁰ and finite temperature,^{16,17} and they persist in five dimensions.

IV. FINITE-TEMPERATURE BEHAVIOR OF $(\bar{\psi}\psi)^2$ QFT IN TWO AND THREE DIMENSIONS

As discussed in the previous section, renormalization of the QFT with $(\bar{\psi}\psi)^2$ interaction can be performed in the leading order of the MF expansion, as well as in the first order of the OE, if the space-time dimension is less

than four. However, the only possibility to obtain non-trivial renormalized theory is to choose the bare coupling infinitesimal negative, which gives the same precarious theory in both approximations. In this section we will discuss numerical results in two- and three-dimensional space-time. For arbitrary bare mass, the renormalized theory can be parametrized by m_R and g_R . The massless Gross-Neveu model will be recovered for g_R given by (3.17). To get rid of one parameter we express all dimensional quantities in units of an arbitrary m_R .

It is necessary to stress that our analysis is limited to the lowest order of the considered expansions. In two di-

mensions, when the perturbative proof of renormalizability exist, it provides an approximation of the renormalized theory. However, in three dimensions the results can appear meaningless, if renormalizability of the theory would not be maintained in higher orders.

A. Precarious theory in two dimensions

In two-dimensional QFT the precarious finite-temperature effective potential (3.27), after taking the functions L_i from Table II (n even), becomes

$$V^T(\alpha) = \alpha\Omega + \frac{1}{2\pi}\Omega^2 \ln(\Omega) - \frac{1}{4\pi}(\Omega^2 - 1) - G(\Omega - 1)^2 - 2\frac{T}{\pi} \int_0^\infty dp \ln(1 + e^{-(\Omega^2 + p^2)^{1/2}/T}) \quad (4.1)$$

with Ω determined by the gap equation (3.28), which becomes

$$\alpha + \frac{1}{\pi}\Omega \ln(\Omega) - 2G(\Omega - 1) + \frac{2}{\pi} \int_0^\infty \frac{dp}{(\Omega^2 + p^2)^{1/2} \{ \exp[(\Omega^2 + p^2)^{1/2}/T] + 1 \}} = 0, \quad (4.2)$$

where

$$G = \frac{1}{2} \left[\frac{1}{\pi} + \frac{1}{g_R} \right]. \quad (4.3)$$

For $T=0$ the gap equation (4.2) has a solution only for

$$\alpha < \alpha_{\max} = \frac{1}{\pi} e^{2\pi G - 1} - 2G = \frac{1}{\pi} e^{\pi/g_R} - \frac{1}{\pi} - \frac{1}{g_R}. \quad (4.4)$$

Therefore the range in which the precarious effective potential is determined in the discussed approximation is finite and $\alpha_{\max} \rightarrow \infty$, when $|g_R| \rightarrow 0$. For fixed renormalized coupling constant α_{\max} decreases with increasing temperature. The critical temperature T_{cr} is that at which $\alpha_{\max} = 0$; i.e., the gap equation has a solution only for $\alpha = 0$. We will study the case $G = 0$ as an example. In Fig. 1 we show the effective potential at $T=0$ and

$T=0.3$. The zero-temperature result coincides with that obtained by Latorre and Soto.⁴ The critical temperature is $T_{\text{cr}} = (1/\pi)e^\gamma \simeq 0.567$ ($\gamma = 0.577 \dots$ is Euler's constant), which agrees with the value obtained in the papers on the thermal behavior of the Gross-Neveu model in the leading order of the $1/N$ expansion,¹⁸ although their discussion has been done in a different way—studying the finite-temperature effective potential for a composite field. The phase transition is of the second order because the second derivative of the effective potential, which plays a role of the order parameter, decreases smoothly with increasing temperature and approaches zero at T_{cr} . That is unlikely with $\lambda\phi^4$ theory, where the order parameter jumps from a finite value to zero at critical temperature, because the value of the finite-temperature effective potential at the end point $\Omega=0$ becomes lower than the value obtained using the gap equation.¹⁷

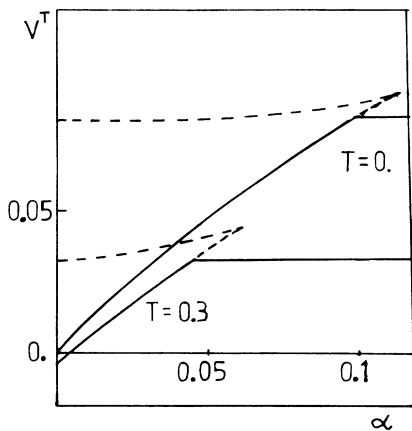


FIG. 1. The effective potential for $(\bar{\psi}\psi)^2$ QFT in two-dimensional space-time at $T=0$ and $T=0.3$. The critical temperature $T_{\text{cr}}=0.567$. All variables are in the units of renormalized mass m_R .

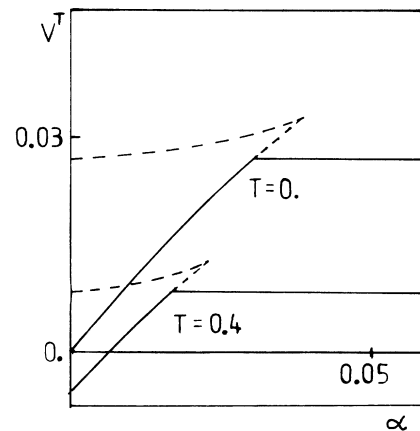


FIG. 2. The effective potential for $(\bar{\psi}\psi)^2$ QFT in three-dimensional space-time at $T=0$ and $T=0.4$. The critical temperature $T_{\text{cr}}=0.721$. All variables are in the units of renormalized mass m_R .

B. Precarious theory in three dimensions

The precarious finite-temperature effective potential in three-dimensional QFT, after taking the functions L_i from Table II (n odd), becomes

$$V^T(\alpha) = \alpha\Omega + \frac{1}{12\pi}(\Omega-1)^2(2\Omega+1) - G(\Omega-1)^2 - \frac{T}{\pi} \int_0^\infty p dp \ln(1 + e^{-(\Omega^2+p^2)^{1/2}/T})$$

with the gap equation given by

$$\alpha + \frac{1}{2\pi}\Omega(\Omega-1) - 2G(\Omega-1) + \frac{T}{\pi} \int_0^\infty \frac{dp}{(\Omega^2+p^2)^{1/2} \{ \exp[(\Omega^2+p^2)^{1/2}T] + 1 \}} = \alpha + \frac{1}{2\pi}(\Omega-1) - 2G(\Omega-1) + \frac{T}{2\pi} \ln(1 + e^{-\Omega/T}) = 0. \quad (4.6)$$

These equations are simpler than in the two-dimensional case. For zero temperature the gap equation is quadratic with two solutions:

$$\Omega^\pm = \frac{1}{2} \{ 1 + 4\pi G \pm [(1 - 4\pi G)^2 - 8\pi\alpha]^{1/2} \} \quad (4.7)$$

and the corresponding branches of the effective potential

$$V^\pm = \frac{\alpha}{2} \pm \left[\frac{\alpha}{3} - \frac{1}{24\pi} \right] [(1 - 4\pi G)^2 - 8\pi\alpha]^{1/2} + \frac{1}{24\pi} \quad (4.8)$$

are defined for

$$\alpha < \alpha_{\max} = \frac{1}{8\pi} (1 - 4\pi G)^2 = \frac{\pi}{2g_R^2}.$$

Therefore, α_{\max} increases infinitely when $|g_R| \rightarrow 0$. The dependence on the temperature is also similar to the two-dimensional case; i.e., α_{\max} decreases with increasing temperature. For $G=0$ the effective potential at $T=0$ and $T=0.4$ is shown in Fig. 2. In this case, the critical temperature $T_{\text{cr}} = 1/2 \ln 2 \simeq 0.721$ and the transition is of second order.

V. CONCLUSIONS

The results of the study of the QFT of Fermi fields with a $(\bar{\psi}\psi)^2$ interaction to the first order of the OE are very similar to those obtained in the leading order of the MF expansion. The theory can be renormalized, if the space-time dimension is less than four, but the result is trivial or precarious, depending on the choice of the bare

coupling constant g .

If g is finite, the theory is noninteracting. In the MF method the triviality of the theory can be seen from the behavior of the effective potential. In the OE considering thermal properties is necessary to draw this conclusion.

If g is chosen to be infinitesimally negative, the theory appears precarious. The renormalized effective potential coincides with that obtained in the MF approximation, although the unrenormalized results are different. The finite-temperature behavior of the precarious theory suggests a second-order phase transition to a free theory of massless particle. However, it should be stressed that the conclusion is drawn using the end-point behavior of the finite-temperature effective potential when the OE, as well as the MF, are unreliable. The lack of a solution to the gap equation can be only a signal that both methods break down beyond a critical temperature.

Note added. For finite temperature similar results have been deduced¹⁹ from the Gaussian effective potential calculated in compact space (S^1), using an analogy between internal energy in the space with closed dimension and the free energy in Minkowski space.

ACKNOWLEDGMENTS

The author wishes to acknowledge M. Spalinski and R. Tarrach for reading the manuscript. Thanks are also given to M. W. Kalinowski for several comments. This work was supported in part by the Polish Ministry of Science and Higher Education under Contract No. CPBP 01.03.

¹W. Heisenberg, *Introduction to the Unified Field Theory of Elementary Particles* (Interscience, London, 1966). Earlier papers are quoted there.

²Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1960).

³D. J. Gross and A. Neveu, *Phys. Rev. D* **10**, 3235 (1974).

⁴J. I. Latorre and J. Soto, *Phys. Rev. D* **34**, 3111 (1986).

⁵A. Okopinska, in *Proceedings of the VIII Warsaw Symposium on Elementary Particle Physics*, Kazimierz, Poland, 1985, edited by Z. Ajduk (Institute of Theoretical Physics, Warsaw, 1985); *Phys. Rev. D* **35**, 1835 (1987).

⁶R. Jackiw, *Phys. Rev. D* **9**, 1686 (1974).

⁷L. Dolan and R. Jackiw, *Phys. Rev. D* **9**, 3320 (1974).

⁸F. Cooper, G. S. Guralnik, and S. H. Kasdan, *Phys. Rev. D* **14**, 1607 (1976); C. M. Bender, F. Cooper, and G. S. Guralnik, *Ann. Phys. (N.Y.)* **109**, 165 (1977).

⁹J. M. Cornwall, R. Jackiw, and E. Tomboulis, *Phys. Rev. D* **10**, 2428 (1974); T. Barnes and G. I. Ghandour, *ibid.* **22**, 924 (1980).

¹⁰P. M. Stevenson, *Phys. Rev. D* **32**, 1389 (1985).

¹¹N. N. Bogoliubov and D. V. Shirkov, *Introduction to the*

- Theory of Quantized Fields* (Wiley-Interscience, New York, 1979).
- ¹²K. Symanzik, Nucl. Phys. **B190** [FS3], 1 (1981).
- ¹³K. Wilson, Phys. Rev. D **7**, 2911 (1973); G. Parisi, Nucl. Phys. **B100**, 368 (1975).
- ¹⁴K. Gawedzki and A. Kupiainen, Phys. Rev. Lett. **55**, 363 (1985); Nucl. Phys. **B262**, 33 (1985).
- ¹⁵P. M. Stevenson, Phys. Rev. D **23**, 2916 (1981).
- ¹⁶W. A. Bardeen and M. Moshe, Phys. Rev. D **28**, 1382 (1983); **34**, 1229 (1986).
- ¹⁷I. Roditi, Phys. Lett. **169B**, 264 (1986); A. Okopinska, Phys. Rev. D **36**, 2415 (1987).
- ¹⁸H. O. Wada, Lett. Nuovo Cimento **11**, 697 (1974); L. Jacobs, Phys. Rev. D **10**, 3956 (1975); B. J. Harrington and A. Yildiz, *ibid.* **11**, 779 (1975); R. F. Dashen, S. Ma, and R. Rajaraman, *ibid.* **11**, 1499 (1975).
- ¹⁹J. Soto, Phys. Rev. D **37**, 1086 (1988).