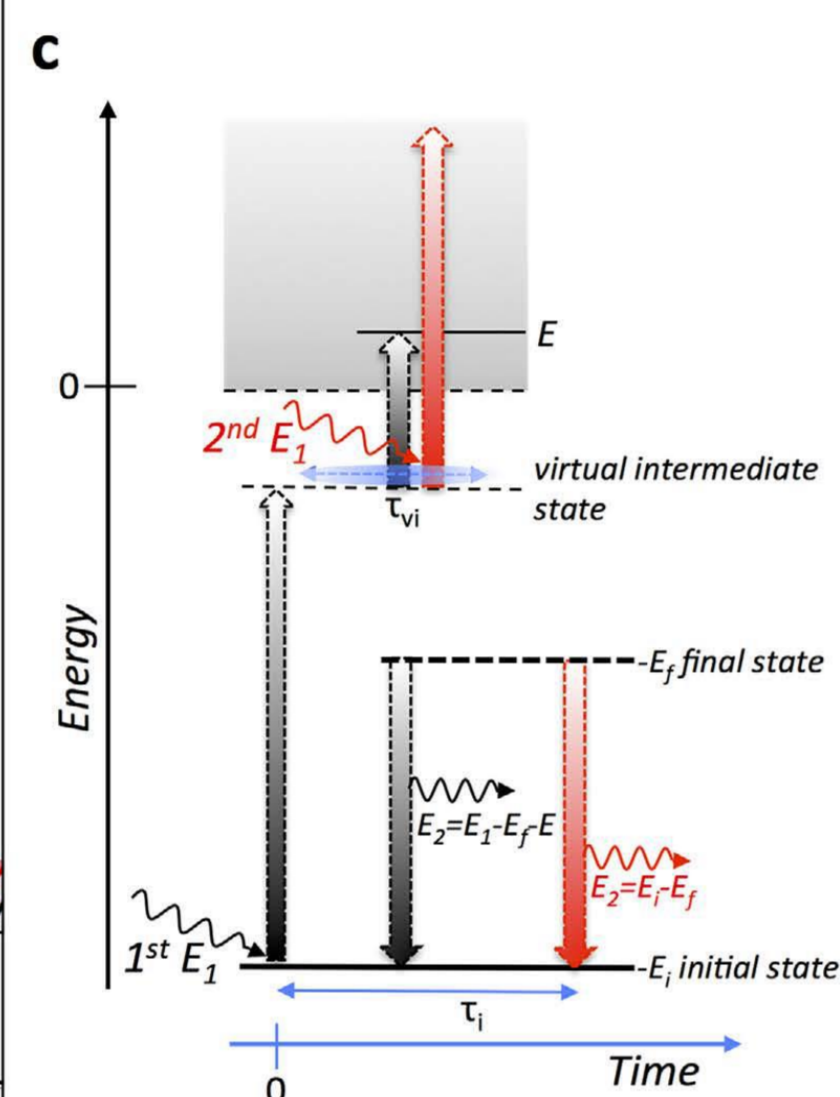
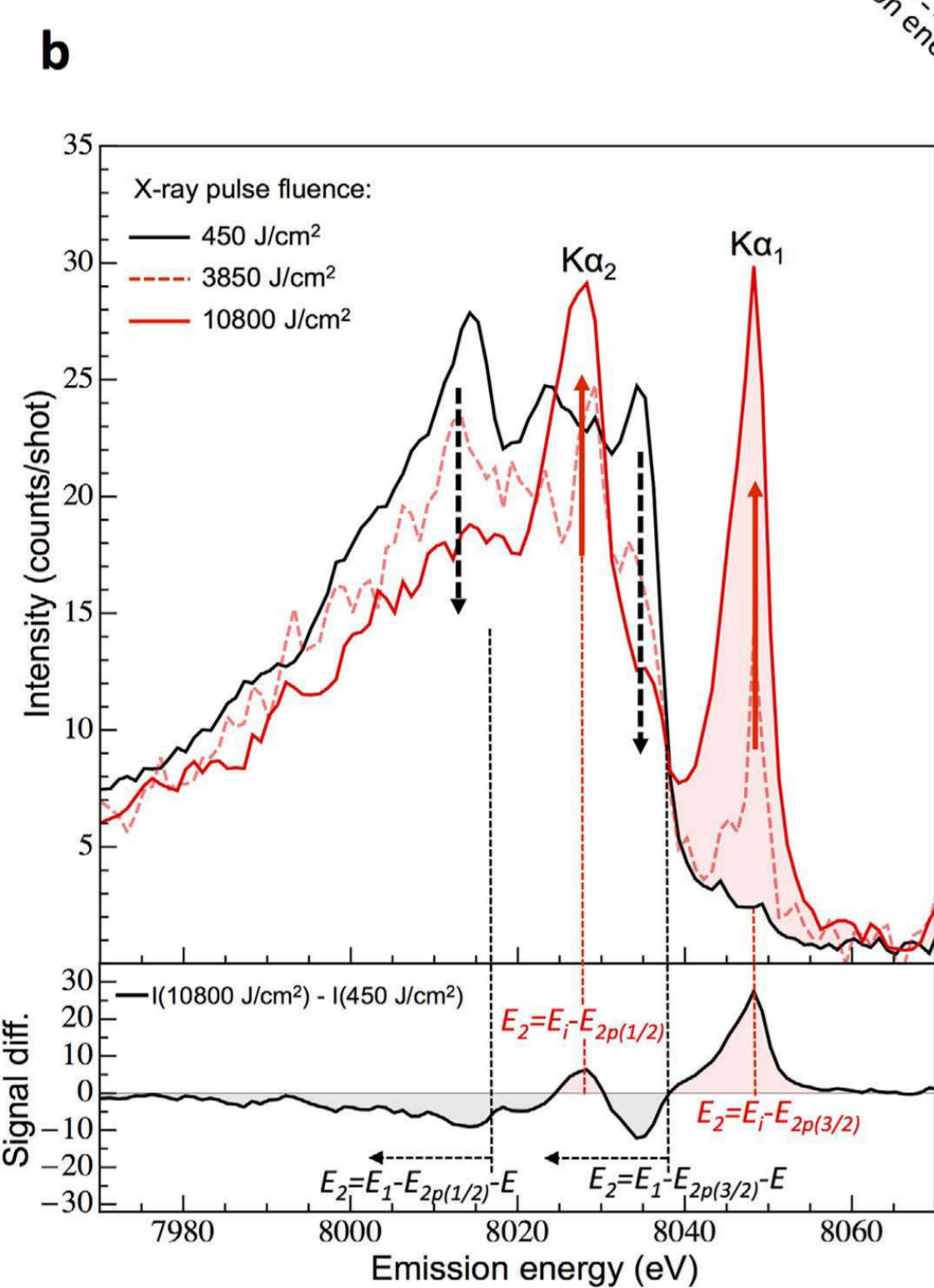
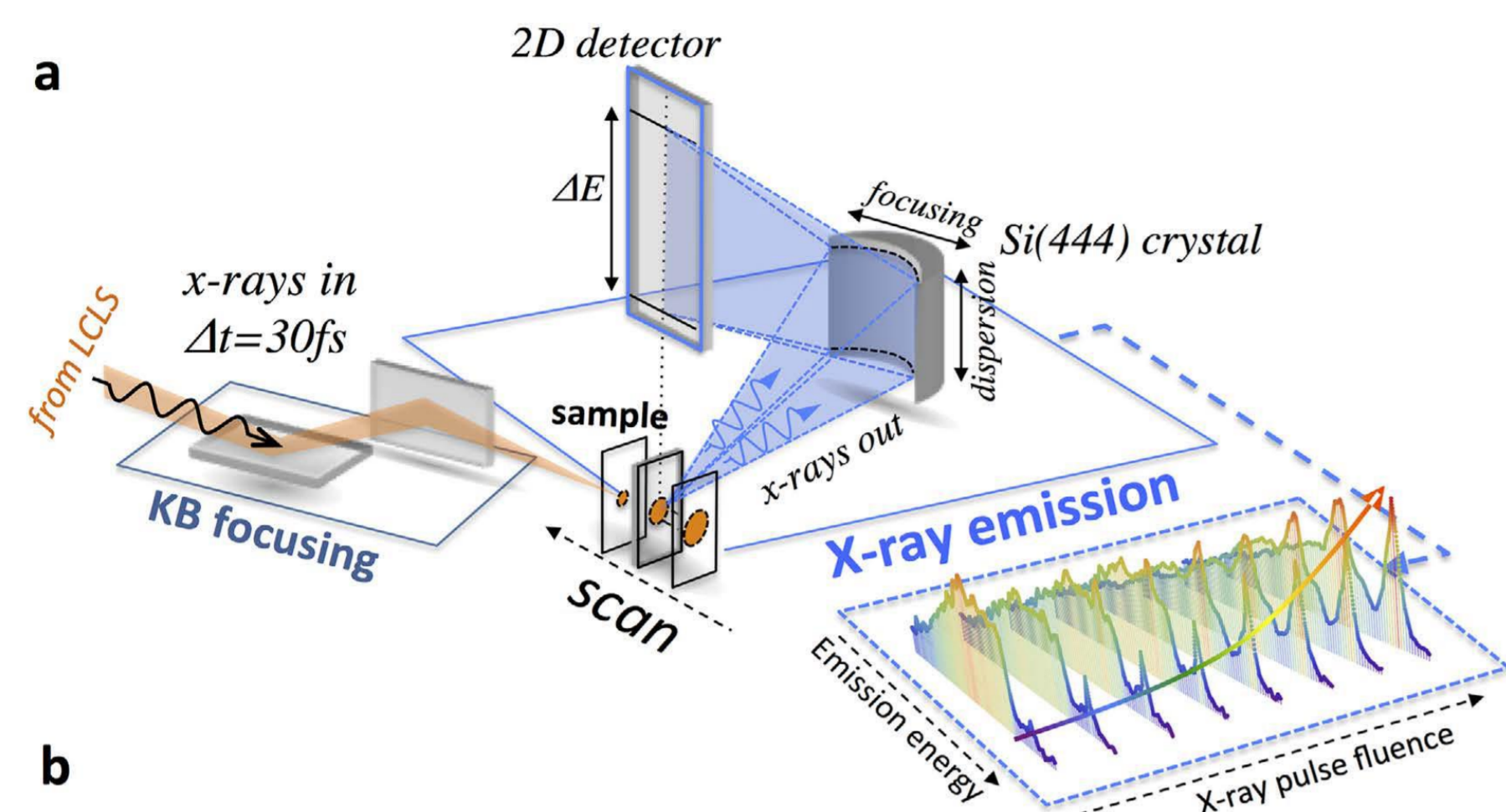


## Introduction

X-ray techniques have evolved over decades to become highly refined tools for a broad range of investigations. Importantly, these approaches rely on X-ray measurements that depend linearly on the number of incident X-ray photons. The advent of X-ray free electron lasers (XFELs) is opening the ability to reach extremely high photon numbers within ultrashort X-ray pulse durations and is leading to a paradigm shift in our ability to explore nonlinear X-ray signals. However, the enormous increase in X-ray peak power is a double-edged sword with new and exciting methods being developed but at the same time well-established techniques proving unreliable. Consequently, accurate knowledge about the threshold and saturation effects for nonlinear X-ray signals is essential. Herein we report an X-ray spectroscopic study that reveals important details on the thresholds for nonlinear X-ray interactions. By varying both the incident X-ray intensity and photon energy, we establish the regimes at which the simplest nonlinear process, two-photon X-ray absorption (TPA), can be observed. Based on time-dependent calculations in 4-level system we established intensity regimes at which TPA process should exhibit saturation effects.

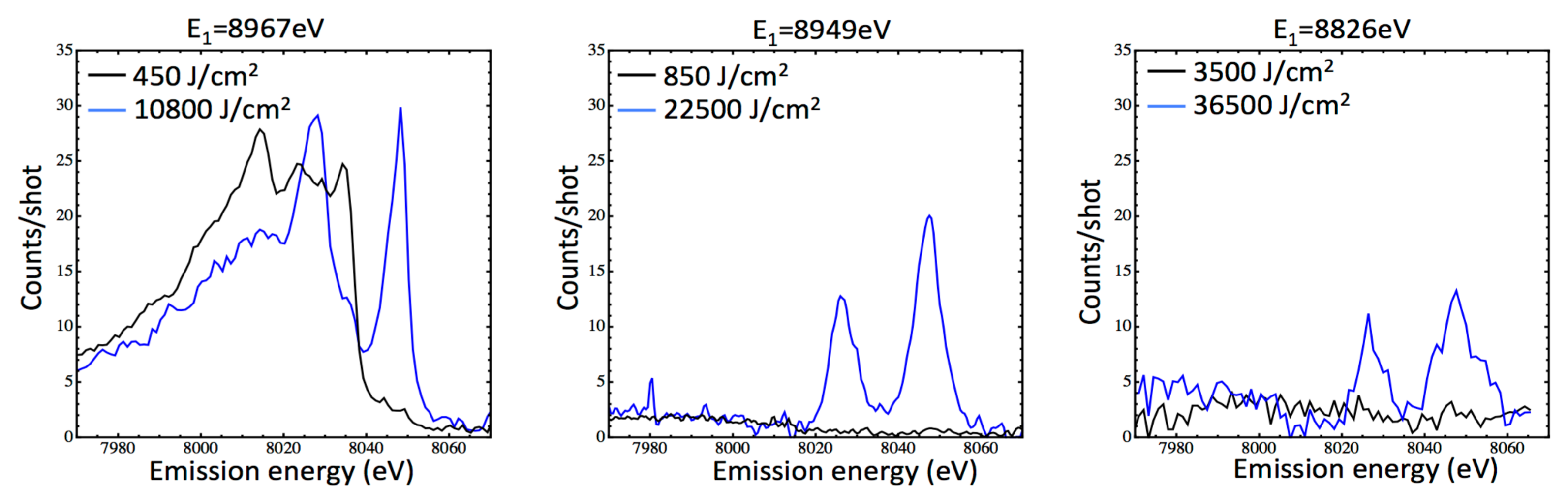
## X-Experimental setup for nonlinear two-photon ray absorption spectroscopy.



**a)** Schematics of the experimental setup showing the Kirkpatrick-Baez (KB) mirror focusing scheme and high energy resolution X-ray emission geometry. The X-ray emission data were recorded for different X-ray pulse fluence by moving the sample along the focus direction of the KB mirrors.

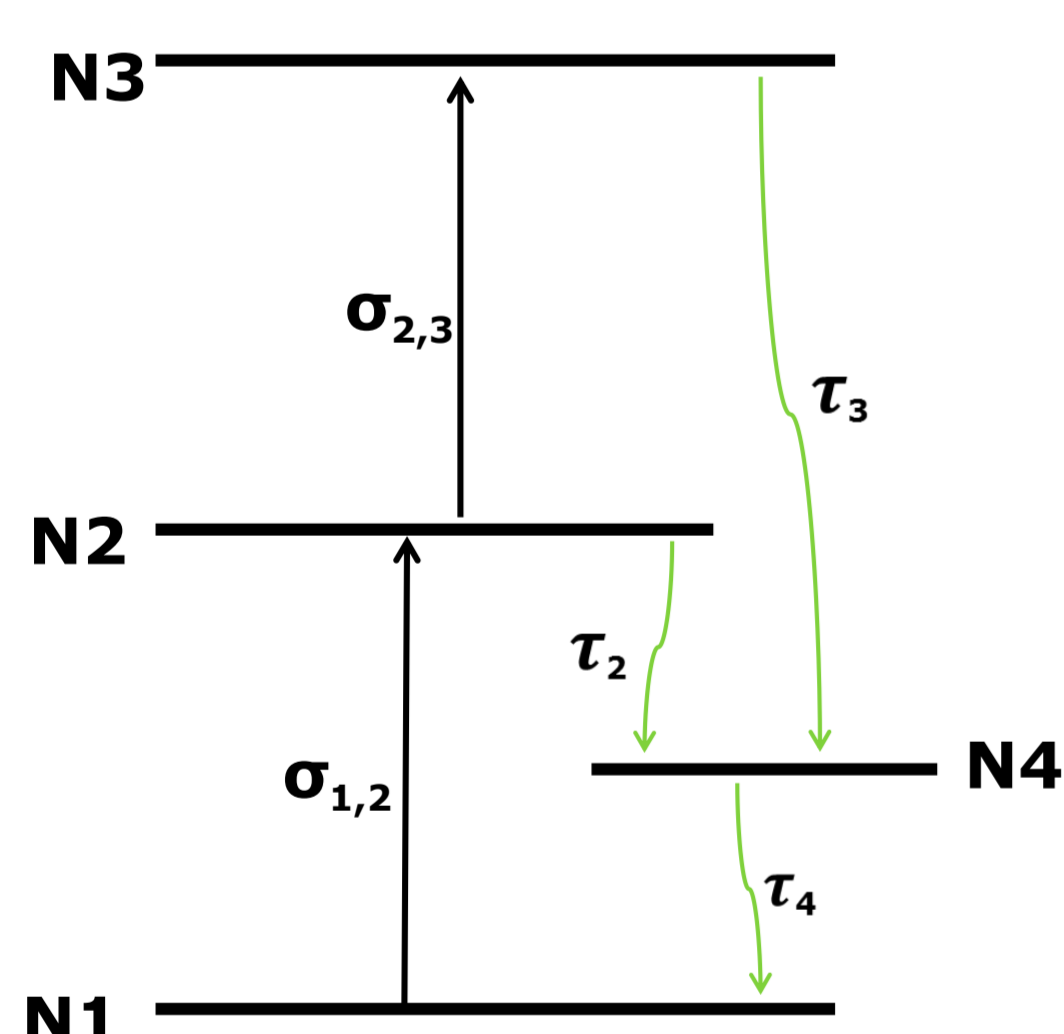
**b)** High energy resolution X-ray emission spectra recorded at an energy 12 eV below the K-shell ionization threshold of Cu for different incident X-ray fluences. The directions of spectral intensity changes with increasing X-ray pulse fluence are marked by black-dashed (HEROS) and red arrows (K $\alpha$  emission), respectively. The spectrum difference between the highest and the lowest X-ray pulse fluence is plotted in the bottom panel.

**c)** Schematic representation of radiative OPA (black arrows) and TPA (red arrows) processes in the off-resonant regime ( $E_1 < E_i$ ). While for the OPA process the emitted X-ray energy ( $E_2$ ) relates directly to the incoming X-ray energy ( $E_1$ ), the TPA mechanism leads to an ionization event and therefore the emitted X-ray energy is constant and equal to the energy difference between the initial ( $E_i$ ) and final ( $E_f$ ) electronic states. The  $E_i$  and  $E_f$  correspond to absolute values of the electron binding energies, and  $E$  is the energy of the photo-excited electron. The virtual intermediate state and the initial state are characterized by lifetimes marked by  $\tau_{vi}$  and  $\tau_i$ , respectively.



X-ray emission spectra recorded for different incoming X-ray energies tuned below the K-ionization threshold at low (black) and high (blue) X-ray fluences.

## Rate equations for 4-level system:



$$\frac{dN_1}{dt} = -I(t) \cdot \sigma_{1,2} \cdot N_1 + \frac{N_4}{\tau_4}$$

$$\frac{dN_2}{dt} = I(t) \cdot (\sigma_{1,2} \cdot N_1 - \sigma_{2,3} \cdot N_2) - \frac{N_2}{\tau_2}$$

$$\frac{dN_3}{dt} = -I(t) \cdot \sigma_{1,2} \cdot N_1 + \frac{N_4}{\tau_4}$$

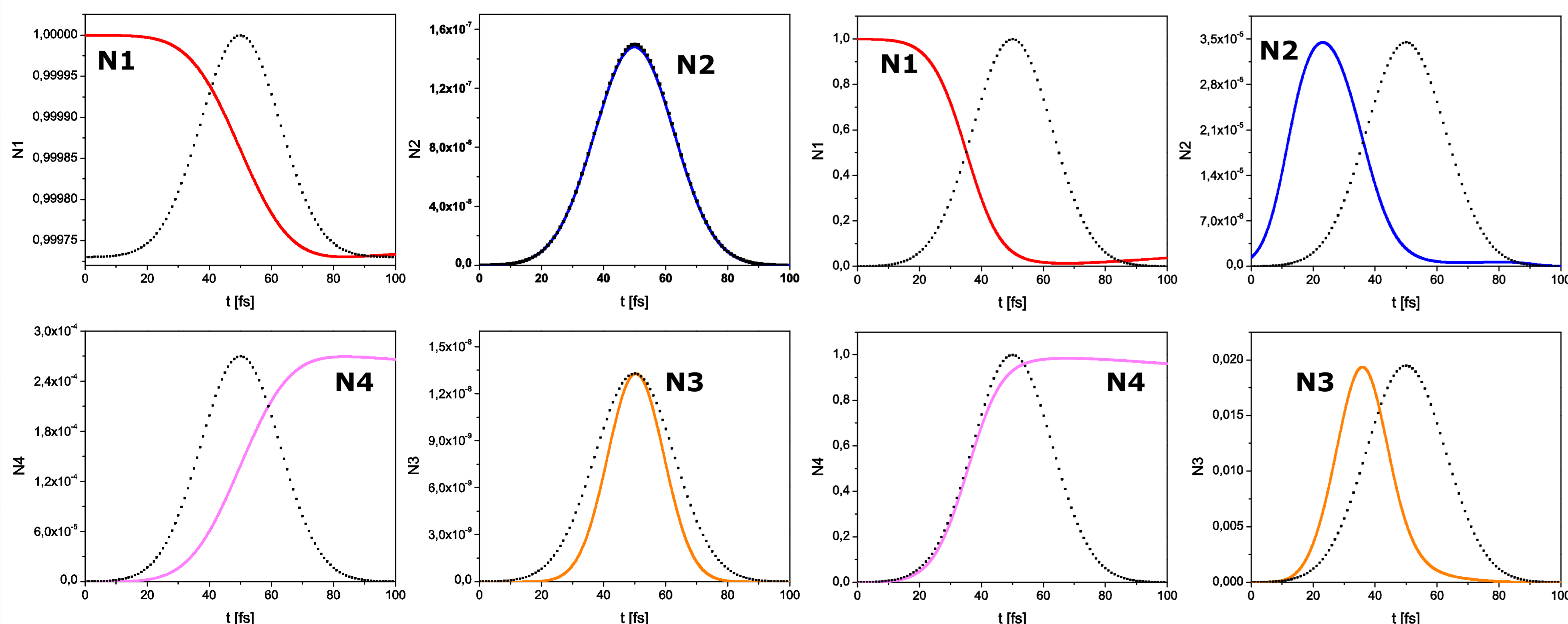
$$\frac{dN_4}{dt} = \frac{N_2}{\tau_2} + \frac{N_3}{\tau_3} - \frac{N_4}{\tau_4}$$

$$N_1 + N_2 + N_3 + N_4 = 1$$

$$\tau_4 \gg \tau_2, \tau_4$$

Where:  $N_1$  is the population of the ground state,  $N_2$  the population of the virtual intermediate state,  $N_3$  the population of continuum states of atoms (ionized states) through the second X-ray absorption and  $N_4$  represents the long living ( $\tau_4 \gg \tau_2, \tau_3$ ) intermediate state. The  $N_4$  state accounts for  $1s^2 2p^5$  electronic configuration and consecutive  $M \rightarrow L$  and valence  $\rightarrow L$  transitions.  $\tau_2, \tau_3$  and  $\tau_4$  for corresponding states. The incident X-ray flux is represented by  $I$  in photons/(cm<sup>2</sup>s) and  $\sigma_{1,2}, \sigma_{2,3}$  are the cross sections in cm<sup>2</sup> for the first and the second absorption step, respectively.

## Theoretical temporal evolution for N1, N2, N3 and N4 states :



Representation of time dependent calculations of four-level system, for two different pulse intensity. States  $N_1$  are represented as a red,  $N_2$  as blue,  $N_3$  as orange and  $N_4$  as a pink line. Photon pulse (black points) with FWHM value of  $30 \cdot 10^{-15}$  s is shown for comparison. At low pulse intensity ( $1.5 \cdot 10^{31}$  photons/cm<sup>2</sup>s;  $2 \cdot 10^{16}$  W/cm<sup>2</sup>), the maximum of states  $N_2$  and  $N_3$  coincide with pulse intensity maximum. At high pulse intensity ( $1.5 \cdot 10^{35}$  photons/cm<sup>2</sup>s;  $2 \cdot 10^{20}$  W/cm<sup>2</sup>) value the maximum positions are significantly shifted.

## Rate dependence vs. X-ray intensity:

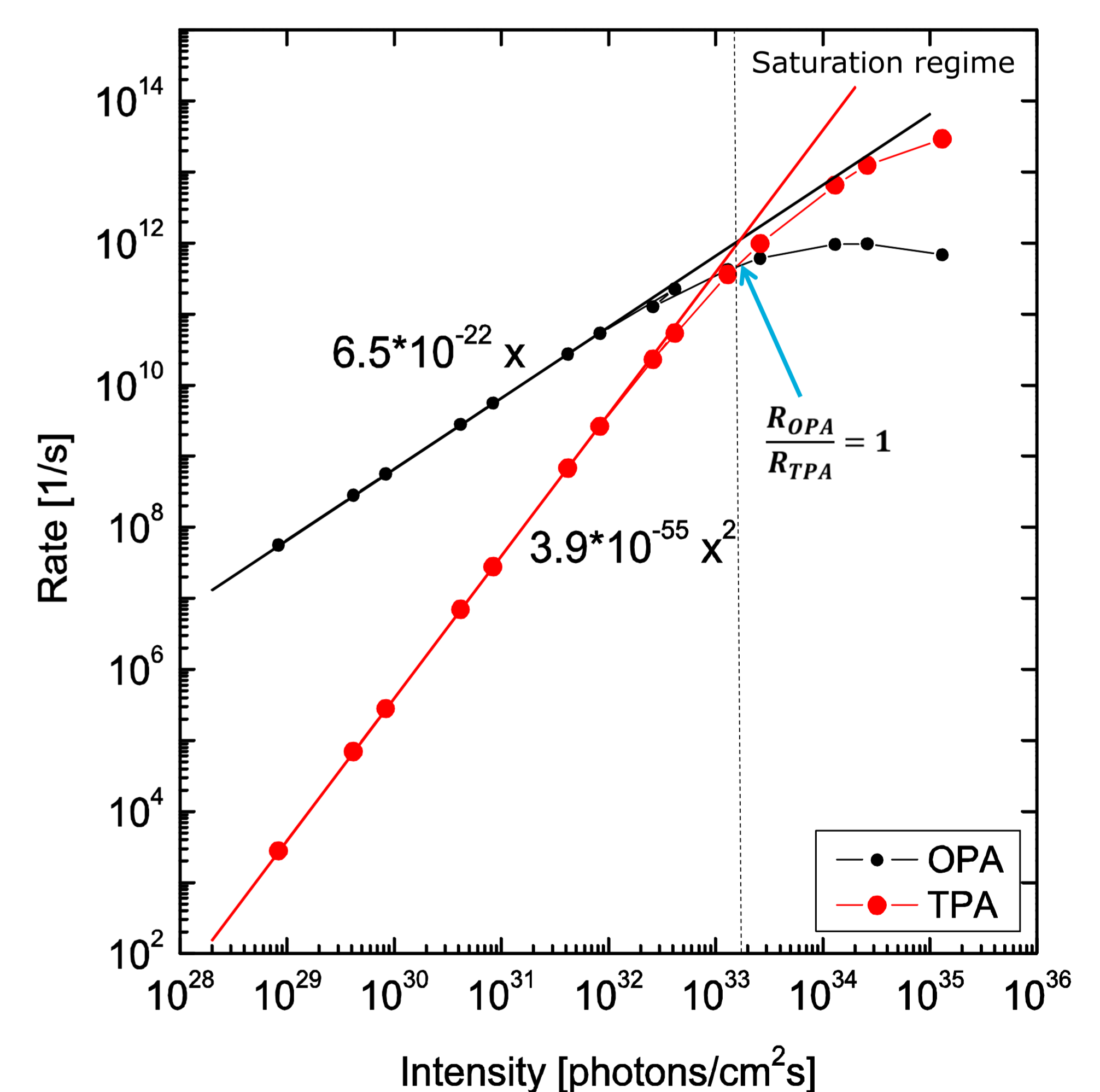


Figure above presents the dependence of rate value (Y) versus photon intensity (X). The data for OPA are plotted by black, and for TPA by red line, respectively. The straight X and X<sup>2</sup> lines correspond to the expected - linear and quadratic dependents for OPA and TPA processes. The crossing point for OPA and TPA rate curves, that corresponds to  $R_{OPA}/R_{TPA} = 1$ , is shown by an arrow at intensity value of  $2.1 \cdot 10^{33}$  photons/cm<sup>2</sup>s ( $3 \cdot 10^{18}$  W/cm<sup>2</sup>).