When small becomes large - statistical physics of small systems

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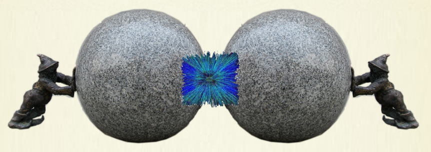


The Unreasonable Effectiveness of Mathematics in the Natural Sciences by Eugene Wigner
Communications in Pure and Applied Mathematics, vol. 13, No. I (February 1960)

From the article

- We are in a position similar to that of a man who was provided with a bunch of keys and who, having to open several doors in succession, always hit on the right key on the first or second trial. He became skeptical concerning the uniqueness of the coordination between keys and door
- This uncanny usefulness of mathematical concepts that raises the question of the uniqueness of our physical theories.
- Einstein's statement: the only physical theories which we are willing to accept are the beautiful ones.
- The concepts of mathematics, which invite the exercise of so much wit, have the quality of beauty.

Starting points

Die Mesonenausbeute beim Beschuß von leichten Kernen mit α-Teilchen

Von Heinz Koppe

Max-Planck-Institut für Physik, Göttingen

(Z. Naturforschg. 3a, 251—252 [1948]; eingeg. am 21. Juni 1948)

Mittels des neuen Berkeley-Betatrons ist es möglich gewesen, durch Beschuß von leichten Kernen (insbesondere C) mit α-Teilchen von etwa 380 MeV Mesonen zu erzeugen. Im folgenden soll eine einfache Methode angegeben werden, nach der sich die dabei zu erwartende Ausbeute abschätzen läßt.

Beim Stoß eines Kernes mit der Massenzahl M_1 und ler kinetischen Energie E auf einen ruhenden Kern mit der Masse M_2 entsteht zunächst ein Zwischenkern mit der Masse $M = M_1 + M_2$, dem die Anregungsenergie pro Nucleon

$$U = \frac{M_2}{M^2} E \qquad (1)$$

zur Verfügung steht. Nach einer bekannten Beziehung[1] 1at der Zwischenkern dann die Temperatur

$$T_0 = 3{,}8 \sqrt{U}. \qquad (2)$$

Dabei wird unter T das Produkt aus k und der absoluten Temperatur verstanden. Gl. (2) liefert T in MeV, wenn man U in MeV einsetzt.

Application of statistical physics to elementary particles is usually referred to Enrico Fermi (1950)

although it was Heinz Koppe(1948)who proposed this idea to production processes

Die Ausbeute an Mesonen ist dann gegeben durch

$$\eta = \int_0^\infty \nu(T)\, dt = \frac{O\,\mu}{\pi^2 \hbar^3} \int_0^\infty T^2\, e^{-\mu c^2 \sqrt{1/T_0^2 + 2Bt}}\, dt.$$

Unter dem Integral kann man T^2 als langsam veränderlich durch T_0^2 ersetzen und außerdem die Wurzel nach t entwickeln. Es ergibt sich

$$\eta = 0{,}031\, T_0\, M\, e^{-\mu c^2/T_0}. \qquad (7)$$

Mit den oben angegebenen Werten liefert das Stoßausbeuten $\eta = 1{,}7 \cdot 10^{-4}$.

Archive

SOVIET PHYSICS JETP VOLUME 2, NUMBER 1 JANUARY, 1956

The Statistics of Charge-Conserving Systems and Its Application to the Theory of Multiple Production

V. B. MAGALINSKII AND IA. P. TERLETSKII
Moscow State University

Submitted to JETP editor November 9, 1954
J. Exper. Theoret. Phys. USSR 29, 151-157 (August, 1955)

The quantum statistics of systems with a variable number of non-interacting particles is generalized to the case of an aggregate of oppositely charged particles, which obey the law of charge conservation. Formulas which differ from the corresponding formulas of ordinary quantum statistics are derived for the total number of particles and the total energy. The results obtained are applied to the theory of multiple production of mesons. The following questions are studied: the dependence of the energy on the relative proportions of neutral and charged mesons, the formation of nucleon-antinucleon pairs, and the relation between the yield and the primary energy. The theory is compared with the available experimental data.

1. INTRODUCTION

IN the statistical treatment of the phenomenon of multiple production of particles at high energies, proposed by Fermi[1], the total number of particles, the total energy of the system, and also the relation

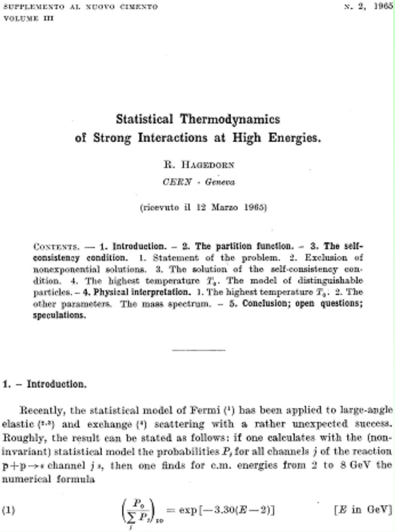
charge-conserving systems, a more detailed examination of processes of multiple production in the framework of the "thermodynamic" approximation is possible.

We make this generalization in the present paper, and as a result obtain new formulas for the total

Limiting temperature

Rolf Hagedorn was the first who systematically analyzed high energy phenomena using all tools of statistical physics. He introduced the concept of the limiting temperature $\sim 140 MeV$ based on the statistical bootstrap model.

That was the origin of multiphase structure of hadronic matter.



SUPPLEMENTO AL NUOVO CIMENTO N. 2, 1965
VOLUME III

Statistical Thermodynamics of Strong Interactions at High Energies.

R. HAGEDORN
CERN - Geneva

(ricevuto il 12 Marzo 1965)

CONTENTS. — **1. Introduction.** – **2. The partition function.** – **3. The self-consistency condition.** 1. Statement of the problem. 2. Exclusion of nonexponential solutions. 3. The solution of the self-consistency condition. 4. The highest temperature T_0. The model of distinguishable particles. – **4. Physical interpretation.** 1. The highest temperature T_0. 2. The other parameters. The mass spectrum. – **5. Conclusion; open questions; speculations.**

1. – Introduction.

Recently, the statistical model of Fermi (1) has been applied to large-angle elastic (2,3) and exchange (4) scattering with a rather unexpected success. Roughly, the result can be stated as follows: if one calculates with the (non-invariant) statistical model the probabilities P_j for all channels j of the reaction $p+p \to s$ channel j s, then one finds for c.m. energies from 2 to 8 GeV the numerical formula

$$(1) \qquad \left(\frac{P_0}{\sum_j P_j}\right)_{pp} = \exp[-3.30(E-2)] \qquad [E \text{ in GeV}]$$

Crucial question

F. Becattini, Kielce workshop, October 15 2004

What is the meaning of the statistical model ?

Theoretical description of particle production

$$\mathcal{P}_n(i \to f) = \int \prod_{f=1}^{n} \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} |\langle p'_1, \dots p'_n | \mathcal{S} | i \rangle|^2 \delta(P_i - \sum_f p_f)$$

The dynamical part

$$|\langle p'_1, \dots p'_n | \mathcal{S} | i \rangle|^2$$

The kinematical part

$$\delta(P_i - \sum_f p_f) \prod_{f=1}^{n} \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f}$$

Place for statistical physics

More particles (degrees of freedom) in the process: kinematics tends to dominate the behavior of the system

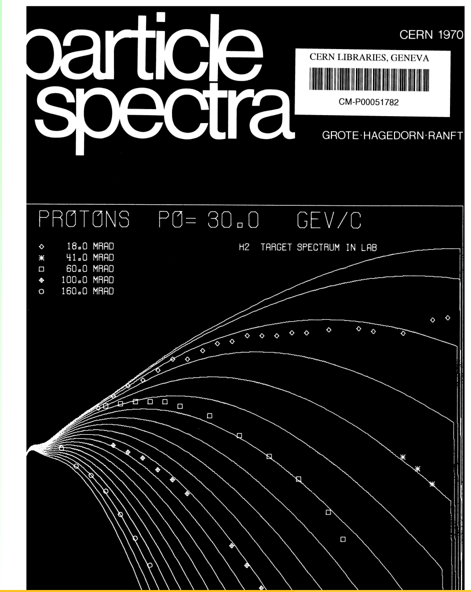
- measurable quantities are much less detailed than $\langle p'_1, \ldots p'_n | \mathcal{S} | i \rangle$
- with the integration over a large region of the phase space the dynamical details are averaged and only a few parameters remains
- restricted knowledge of $\langle p'_1, \ldots p'_n | \mathcal{S} | i \rangle$ is not needed

$$P_n = \bar{S}_n \, \mathcal{R}_n$$

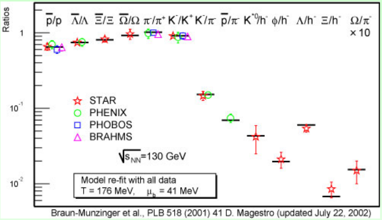
$$\mathcal{R}_n = \int \prod_{f=1}^{n} \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \delta(P_i - \sum_f p_f)$$

Arguments work if the thermodynamic equilibrium is reached

It works


particle spectra
CERN 1970
CERN LIBRARIES, GENEVA
CM-P00051782
GROTE·HAGEDORN·RANFT
PROTONS P0= 30.0 GEV/C
18.0 MRAD
41.0 MRAD
60.0 MRAD
100.0 MRAD
160.0 MRAD
H2 TARGET SPECTRUM IN LAB

It works



although still there are discussions is this "the real" temperature or "a fake" temperature due to the phase space dominance effect.

Puzzle

Multiproduction at high energy e^+e^- and pp is still well described within statistical model.

Statistics and equilibrium

Progress of Theoretical Physics, Vol. 5, No. 4, July~August, 1950

High Energy Nuclear Events

Enrico FERMI

When two nucleons collide with very great energy in their center of mass system this energy will be suddenly released in a small volume surrounding the two nucleons. We may think pictorially of the event as of a collision in which the nucleons with their surrounding retinue of pions hit against each other so that all the portion of space occupied by the nucleons and by their surrounding pion field will be suddenly loaded with a very great amount of energy. Since the interactions of the pion field are strong we may expect that rapidly this energy will be distributed among the various degrees of freedom present in this volume according to statistical laws. One can then compute statistically the probability that in this tiny volume a certain number of pions will be created with a given energy distribution. It is then assumed that the concentration of energy will rapidly dissolve and that the particles into which the energy has been converted will fly out in all directions.

The statement that we expect some sort of statistical equilibrium should be qualified as follows. First of all there are conservation laws of charge and of momentum that evidently must be fulfilled. One might expect further that only those states that are easily reachable from the initial state may actually attain statistical equilibrium. So, for example, radiative phenomena in which photons could be created will certainly not have time to develop. The only type of

The proposed theory has some resemblance to a point of view that has been adopted by Heisenberg[1] who describes a very high energy collision of two nucleons by assuming that the pion "fluid" surrounding the nucleons is set in some sort of turbulent motion by the impact energy. He uses qualitative ideas of turbulence in order to estimate the distribution of energy of this turbulent motion among eddies of different sizes. Turbulence represents the beginning of an approach to thermal equilibrium of a fluid. It describes the spreading of the energy of motion to the many states of larger and larger wave number. One might say, therefore, in a qualitative way that the present proposal consists in pushing the Heisenberg point of view to its extreme consequences of actually reaching statistical equilibrium.

What temperature? Which temperature?

T. T. Chou, C. N. Yang and E. Yen, *"Single Particle Momentum Distribution at High-energies and Concept of Partition Temperature,"* Phys. Rev. Lett. **54**, 510 (1985).

- A concept of partition temperature T_p is introduced in high-energy collisions. It is a natural mathematical consequence of the Darwin-Fowler method, and neither requires nor implies thermal equilibrium.
- $\exp\left(-E/T_p\right)$ will always be one of the factors of the single particle distribution for the nonleading particles.

$$Z(\beta) = \int_0^\infty g(E) e^{-\beta E} dE\,; \qquad g(E) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{\log Z(\beta) + \beta E} dE$$

$$g(E) \sim \frac{1}{\sqrt{2\pi}} \left[Z(\beta^*) e^{\beta^* E} \left. \frac{\partial^2 \log Z}{\partial \beta^2} \right|_{\beta=\beta^*} \right]^{-1/2} ; \qquad \left. \frac{\partial \log Z}{\partial \beta} \right|_{\beta=\beta^*} = -E$$

What temperature? Which temperature?

J. Hormuzdiar, S. D. H. Hsu and G. Mahlon, *"Particle multiplicities and thermalization in high-energy collisions,"* Int. J. Mod. Phys. E **12**, 649 (2003)

- Any mechanism for producing hadrons which evenly populates the free particle phase space will mimic a microcanonical ensemble, and therefore yield apparently thermal results.
- This type of apparent thermalization will always yield Boltzmann weights which are functions of the free particle energy, and hence describe a non-interacting ensemble.

Temperatures always

- In multiproduction processes one always deals with nontrivial (mixed states) density matrix.
- Without full microscopic description there is nonzero *relevant* entropy

$$S(E) = -\mathrm{Tr}\varrho \ln \varrho\,; \qquad \frac{1}{T} = \frac{\partial S}{\partial E}$$

  with the entropy maximized with respect to relevant macrovariables.
- The entropy is unique for the given set of relevant variables.
- For different choice of relevant variables the temperature would be different but still consistent with the scheme of statistical physics.

Conclusions and remarks

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- A temperature is not a self consistent quantity.
- Any question about temperature is a good question, any answer about temperature is a wrong answer.

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Remarks

There is a relaxation time for physical problems - about 10 - 15 years. They are considered again and again, the same questions, the same answers.