# Universality of long wavelength dynamics

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# Universality of Hydrodynamics

## Hydrodynamics of a perfect fluid

- **macroscopic description**
- **conservation laws** (for a neutral system)

$$\partial_\mu T^{\mu\nu} = 0$$

$T^{\mu\nu}$ - energy-momentum tensor

- **local thermodynamical equilibrium:**

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

$p$ - pressure

$\varepsilon$ - energy density

$u^\mu$ - hydrodynamical velocity

**Additionally, the equation of state:**

$$f(p, \varepsilon) = 0$$

For example: $p = \frac{1}{3}\varepsilon$ - for plasma of massless constituents

**A whole dynamics is hidden in the equation of state.**

# Dissipative hydrodynamics

**there is no ideal isotropy**

⇩

**transport coefficients show up**

- **conservation laws**

$$\partial_\mu T^{\mu\nu} = 0 \qquad\qquad T^{\mu\nu} = T^{\mu\nu}_{ideal} + \Pi^{\mu\nu}$$

↓ dissipative term

- **the equation of state**

$$f(p, \varepsilon) = 0$$

**Dynamics is governed by EoS and transport coefficients.**

**The hydrodynamic evolution of two different systems is qualitatively still the same.**

The differences lie in numerical factors.

# The question

**Macroscopic** hydrodynamic behaviour of different systems is very **similar**

**Microscopic** dynamics of different systems can be very **different**

**How does the universality emerge?**

# Microscopically

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\overline{\Psi}\gamma_\mu D^\mu \Psi \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\mathcal{L}_{\text{SUSY QED}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\overline{\Psi}\gamma_\mu D^\mu \Psi + \frac{i}{2}\overline{\Lambda}\gamma_\mu \partial^\mu \Lambda + (D_\mu \phi_L)^* (D^\mu \phi_L) + (D^*_\mu \phi_R)(D^\mu \phi^*_R)$$
$$+ \sqrt{2} e (\overline{\Psi} P_R \Lambda \phi_L - \overline{\Psi} P_L \Lambda \phi^*_R + \phi^*_L \overline{\Lambda} P_L \Psi - \phi_R \overline{\Lambda} P_R \Psi) - \frac{e^2}{2} (\phi^*_L \phi_L - \phi^*_R \phi_R)^2$$

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + i\overline{\Psi}_i (\gamma_\mu D^\mu)_{ij} \Psi_j \qquad F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f^{abc} A_b^\mu A_c^\nu$$

$$\mathcal{L}_{\text{SYM}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \frac{i}{2} \overline{\Psi}_i^a (\gamma_\mu D^\mu \Psi_i)^a + \frac{1}{2} (D_\mu \Phi_A)_a (D^\mu \Phi_A)_a$$
$$-\frac{1}{4} g^2 f^{abe} f^{cde} \Phi_A^a \Phi_B^b \Phi_A^c \Phi_B^d - i\frac{g}{2} f^{abc} (\overline{\Psi}_i^a \alpha^p_{ij} X_p^b \Psi_j^c + i\overline{\Psi}_i^a \beta^p_{ij} \gamma_5 Y_p^b \Psi_j^c)$$

# Effective action

**Macroscopic description is derived in terms of effective action.**

How to find the effective action?

Self-energy constrains the form of effective action

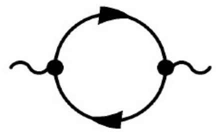
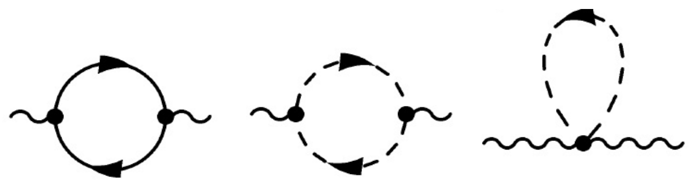
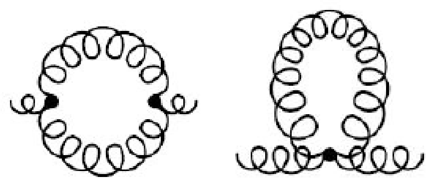
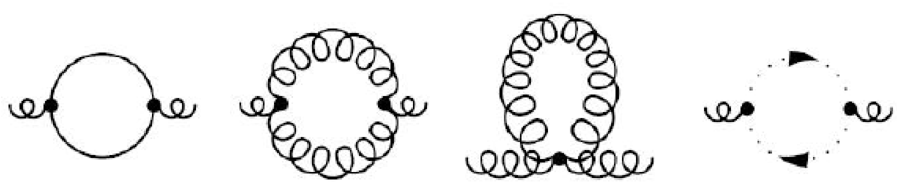
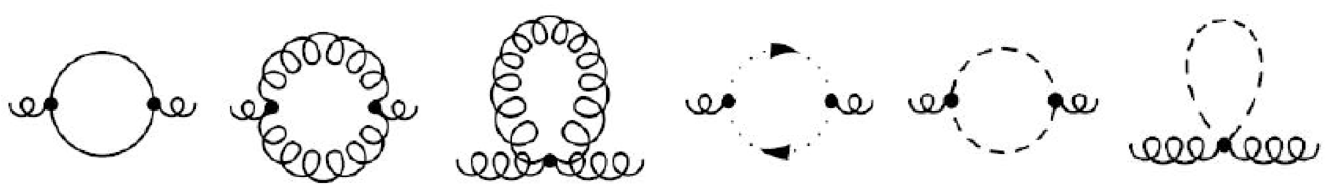
$$\mathcal{L}_2^{(A)}(x) = \frac{1}{2}\int d^4y\, A_\mu(x)\Pi^{\mu\nu}(x-y)A_\nu(y)$$

$$\Pi^{\mu\nu}(x,y) = \frac{\delta^2 S[A]}{\delta A_\mu(x)\delta A_\nu(y)}$$

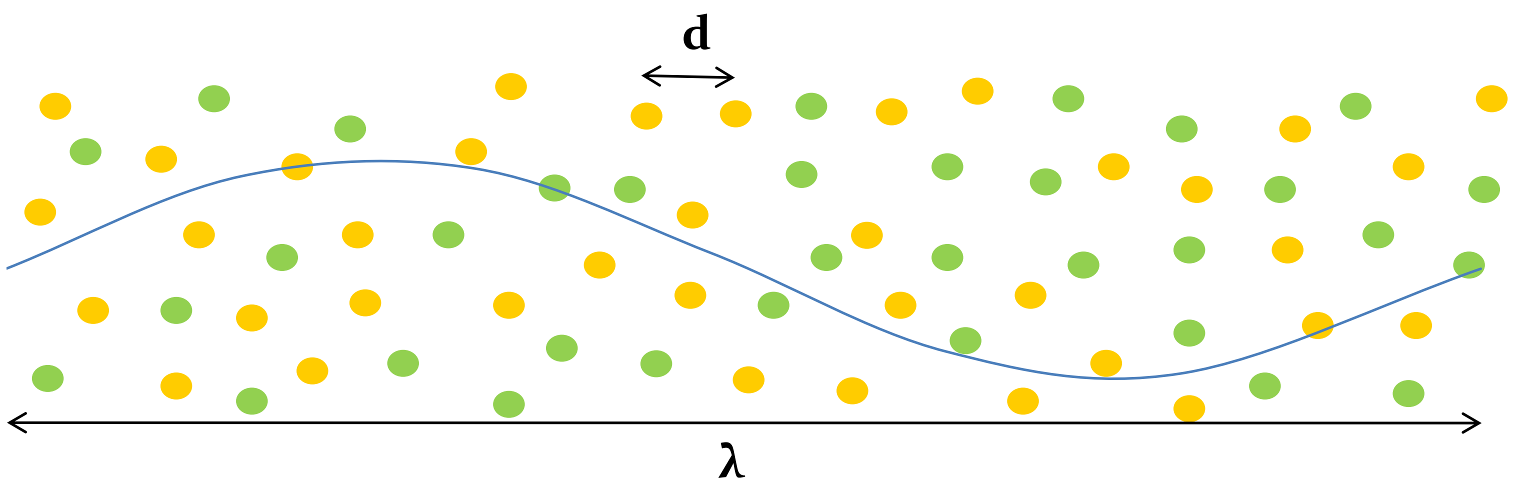
**Our strategy**

self-energy → effective action → description of the system

# Polarization tensors

| | |
|---|---|
| photon in QED |  |
| photon in SUSY QED |  |
| gluon in Yang-Mills |  |
| gluon in QCD |  |
| gluon in $\mathcal{N} = 4$ super Yang-Mills |  |

# Hard Loop Approximation




**Wavelength of a quasi-particle is much bigger than inter-particle distance in the plasma:**

$$\lambda >> d$$

$$k^{\mu} << p^{\mu}$$

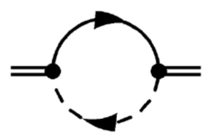
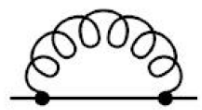
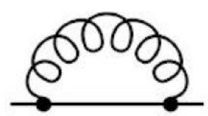
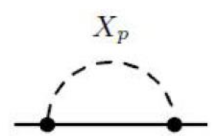
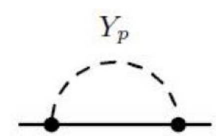
# Polarization tensor

$$\Pi(k) = \# \int \frac{d^3 p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{k^2 p^\mu p^\nu - [p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu}(k\cdot p)](k\cdot p)}{(k\cdot p + i0^+)^2}$$

$$k^\mu << p^\mu$$

After the HL approximation is applied
the polarization tensor gets
**the same structure** for the $\mathcal{N}$=4 SYM, YM, QCD,
SUSY QED and usual QED plasma.

# Fermionic self-energies

| | |
|---|---|
| electron in QED |  |
| electron in SUSY QED |   |
| photino in SUSY QED |  |
| quark in QCD |  |
| fermion in $\mathcal{N} = 4$ super Yang-Mills |      |

# Fermion self-energy

$$\Sigma(k) = \# \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{\hat{p}}{k \cdot p + i0^+}$$

$k^\mu << p^\mu$

The fermion self-energy in HL approximation has **the same structure** for the $\mathcal{N}$=4 SYM, SUSY QED and usual QED plasma.

**The structure of self-energy of every field (vector, spinor, scalar) appears to be universal.**

# From self-energies to effective action

$$\mathcal{L}_{\mathrm{HL}}^{(A)}(x) = g^2 N_c \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} F_{\mu\nu}^{a}(x) \left( \frac{p^\nu p^\rho}{(p\cdot D)^2} \right)_{ab} F_{\rho}^{b\mu}(x)$$

$$\mathcal{L}_{\mathrm{HL}}^{(\Psi)}(x) = g^2 N_c \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \overline{\Psi}_i^{a}(x) \left( \frac{p\cdot\gamma}{p\cdot D} \right)_{ab} \Psi_i^{b}(x)$$

$$\mathcal{L}_{\mathrm{HL}}^{(\Phi)}(x) = -2g^2 N_c \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \Phi_A^a(x)\Phi_A^a(x)$$

$$\frac{1}{p\cdot D}\Psi(x) \equiv \frac{1}{p\cdot\partial}\sum_{n=0}^{\infty}\left( igp\cdot A(x)\frac{1}{p\cdot\partial}\right)^n \Psi(x) \qquad \frac{1}{p\cdot\partial}\Psi(x) \equiv i\int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot x}}{p\cdot k}\Psi(k)$$

**The structure of each term of the effective action appears to be unique.**

**The universality of the effective action does not require an equilibrium.**

# Conclusions

- Universality of hydrodynamics is a consequence of its macroscopic character combined with local equilibrium.

- Universality at the macroscopic level appears as a result of long wavelength limit.
  **There is no need for the equilibrium requirement.**

- From our point of view, statistical models are so efficient mostly because of their macroscopic character.